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QUANTILE ESTIMATION OF PROBABILITY DISTRIBUTIONS FOR MAXIMUM DAILY PRECIPITATION AND SHORT TIME SERIES OF OBSERVATIONAL DATA FOR ENGINEERING DESIGN

Knowledge of the distribution quantiles of precipitation maximum amounts is required in many fields concerning engineering design or hydrological risk assessment. When the number of observation years is small, it is not possible to fit the probability distribution function to maximum values and to calculate quantiles. This paper presents a procedure for calculating the quantiles of the probability distribution of daily precipitation maximums over a year using stochastic convergence of distributions. The distribution series of random variables, defined based on the cut-off sample with the elimination of the smallest values, made it possible to determine the quantiles for times series of order α of the distribution. These values were approximated by a function from the exponential class and then extrapolated to obtain quantiles for the distribution of maxima. The resulting quantile estimates, for short time series, were corrected using the kurtosis of the data used for estimation, which leads to a very large error reduction.

1. INTRODUCTION

Urban development leads to a constant expansion of urban areas, which consequently by anthropogenic activities lead to changes in the environment, especially in the atmosphere. As a result, there are greater amounts of precipitation recorded in urbanized areas than in non-urban areas. Surface runoff is much higher and retention, evaporation, and underground runoff are much lower. Already today it is estimated that about 25% of the annual sum of the amount of precipitation can be caused by the influence of the city, and precipitation may increase with urban expansion and climate change [1–6, 10, 16].

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The planned infrastructure, drainage systems, especially for new residential areas must be properly designed and dimensioned taking into account the precipitation potentially exceeding the capacity of the drainage system, local land slopes or areas with sealed surface and limited retention [7, 8] – regardless of the current problem in Poland, which is the use of rainwater and environmental pollution. Thus, information on the probability of occurrence of high or extreme daily precipitation in urban areas is very important in the broad context of planning, dimensioning, construction, and operation of technical infrastructure, but also in minimizing the effects of precipitation floods or expert valuation of potential damages [3, 4, 11, 12, 15, 17].

On the other hand, in urban agglomerations, there are no long-term series of meteorological observations which would allow proper estimation of the probability of occurrence of extreme daily amounts of precipitation. Moreover, even longer observation series are of limited use for fitting probability distributions and determining their parameters. This is due to changes over the years in the amount and structure of precipitation caused by potential climate changes, relocation of meteorological stations, or changes in measurement techniques [12, 14, 15]. It is also worth noting that in the issues described above, the choice of distribution is less important than the proper estimation of quantiles of distributions of random variables (i.e., fitting the tails of the distribution) illustrating probabilities of high and extreme precipitation appearance [25, 26].

This paper considers the problem of estimating quantiles of probability distributions for short observation series. The research is an extension of earlier work [15] based on simulation calculations using a meteorological data generator [18–20]. However, in the presented study, calculations were performed on real data – daily precipitation over 30 years of observation (1989–2018) for Wrocław using independent tests and cross-validation techniques. An innovative method was also introduced to correct the error of maximum precipitation quantiles approximation, for short time series, using a linear function and kurtosis as a cut-off parameter of insignificant errors. The procedure presented was applied to estimate the probability distribution quantiles of maximum daily precipitation for annual periods.

2. METHOD FOR ESTIMATING QUANTILES OF EXTREME PRECIPITATION DISTRIBUTIONS

The estimation of maximum daily precipitation distributions quantiles as a random variable $X_{\max}$ is based on the construction of probability distributions series of random variables $\{X_k\}$, each successive distribution of which is formed based on the elimination of an increasing fraction of the smallest values in each year. In the notation adopted for the random variable X_k, the symbol k ($0 \le k < 100$) denotes the fraction of removed observations from the sample, which is used to define the probability distribution. For example, the random variables X_0, X_{50}, X_{95}, X_{99} indicate that probability distributions were constructed using, all observations with recorded precipitation, 50%, 5%, and 1% of those with the highest values in each year used together to fit the distribution or determine the distribution parameters. The random variables of the series $\{X_k\}$ may be defined for any period: a year, a summer period, a month, or other. According to the procedure described above, the series $\{X_k\}$ is expected to converge stochastically to the distribution $X_{\max}$.

Since in practice the sample is limited to n years, where n may be relatively small, e.g., 5–15 years, the question arises of accurately determining the probability distribution of the random variable $X_{\max}$ or (more importantly) its quantiles of a given order – based on a series of distributions of random variables $\{X_k\}$.

Thus, the task can be reduced to determining the quantile $X_{\max,\alpha}$ of order $1-\alpha$ for the distribution of the random variable $X_{\max}$ based on a series of quantile values $\{x_{k,\alpha}\}$ that is the limit $X_{\max} = \lim\limits_{k\to 100} x_{k,\alpha}$.

The quantile value $X_{\max,\alpha}$ for a given α can be estimated by extrapolating a function of the form:

$$f(x) = b_1 + \exp(b_2 + b_3 x) \tag{1}$$

where b_1, b_2, b_3 are the estimated parameters for the approximated values of $(k, x_{k,\alpha})$ – as shown in Fig. 1.

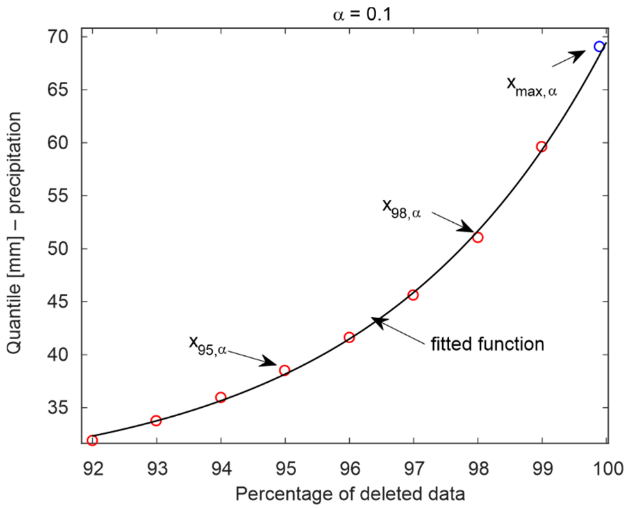

Fig. 1. approximation of quantiles $x_{k,\alpha}$ determined from the distribution of random variables X_k (X_{95}, X_{96}, …, $X_{99.5}$) and the approximation of the quantile $X_{\max,\alpha}$ of the $X_{\max}$ distribution with the quantile $x^{\text{pred}}_{\max,\alpha}$ calculated using the exponential function (X axis shows the percentage of the k removed smallest values of the amount of precipitation in the considered period, Y axis – the values of the quantile of fixed order $1-\alpha$ for the distributions of random variables X_k)

Function fitting was performed based on the method of least squares verifying the condition that the approximation of the function $f(x)$ does not "over-parameterize" the equation [19]. This means keeping the right proportions between the number of observations and the number of parameters and verifying the function fit using a cross-validation test with the possibility of approximation on an extended set of points $(k, x_{k,\alpha})$ with additional k values (the removed fraction of observations).

3. SELECTION OF THE PROBABILITY DISTRIBUTION OF MAXIMUM PRECIPITATION AMOUNTS

At the first stage, using the assumption of stochastic convergence of maximum distributions, the optimum theoretical distribution was selected, which could be used to describe the distribution of successive fractions of the observed highest daily precipitation [15]. In other words, the theoretical distribution that best represents the "tail" of the maximum precipitation distributions and shows the greatest conformity with the values extrapolated by the function (1) was selected. The basic rationale for the choice of distributions was based on previous simulations performed on randomly generated data [15, 19, 23] for 500 years, that is, for 182 500 daily "observations". According to these simulations, four distributions were chosen to estimate the quantiles of the maximum values: empirical, Gamma, GEV, and Pareto. The empirical distribution was treated as an elementary one, being a background for comparison with the other distributions, while the Gamma distribution was treated as a typical distribution used in determining precipitation for variously defined random variables [25]. From the family of extreme value distributions – EV (Gumbel, GEV, Weibull), described by Gumbel [24] the GEV distribution was selected as the one that in hydrometeorology corresponds to descriptions of processes such as the occurrence of maximum rainfall or floods during the year or the lowest flows. As a fourth distribution, the Pareto distribution was chosen – as a very flexible and physically valid distribution for modelling many phenomena, especially events that exceed a certain threshold value [25].

For the empirical distribution, the empirical quantiles of the corresponding fractions were determined, the exponential function (1) was fitted, and the quantile values of the maxima were extrapolated using this function. For the other distributions, instead of empirical quantiles, quantiles were determined from the Gamma, GEV, and Pareto theoretical distributions fitted to the empirical distributions of the fractions.

For quantiles of the order $1 - \alpha$, determined from the empirical distribution and approximated using an exponential function, underestimations occur in each of the cases considered: $\alpha = 0.1, 0.05, 0.02, 0.01$ (Fig. 2). The differences do not change monotonically, but it should be remembered that empirically determined quantiles are subject to significant error when the sample is small and the number of data is limited. The differences in the quantile value of the maxima extrapolated by the exponential function and the empirical value range from 1 to 8. This means that according to the definition: $\Pr(X_{\max} > x) = 1 - \alpha$ for a given probability $1-\alpha$ the differences reach 8 mm (per day).

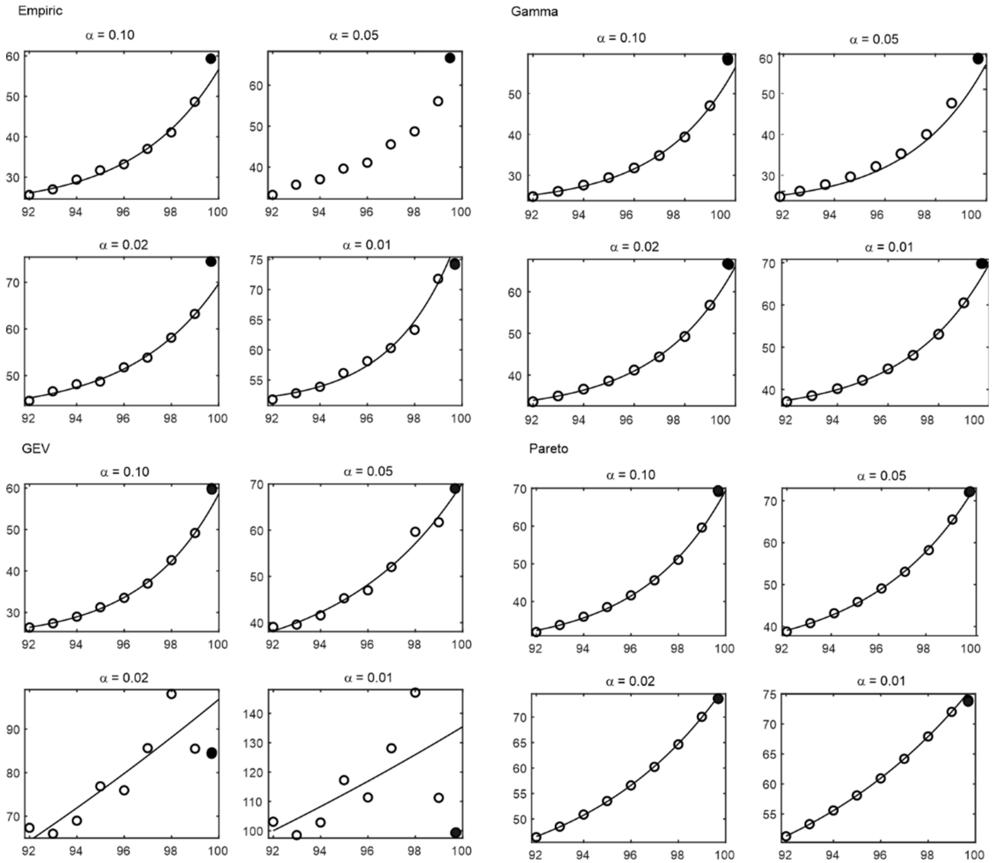

Fig. 2. Approximation of the quantile values $x_{k,\alpha}$ (k = 92, ..., 99; α = 0.1, 0.05, 0.02, 0.01) for order $1-\alpha$ and approximation of the quantile value $X_{\max,\alpha}$ (•) of the distribution of maximum daily precipitation amounts in a year by an exponential function for four probability distributions

The Gamma distribution also underestimates the quantiles of maximum precipitation amounts. However, at a lower level and with monotonicity as the $1 - \alpha$ quantiles increase. On the other hand, when the GEV distribution is chosen, a large variation is observed when fitting the tail distribution which causes the effect of stochastic convergence of the extreme value distributions to be lost and this is the rationale for using the less sensitive Pareto distribution. When the Pareto distribution was used, the best results were obtained for the goodness of fit of the quantiles determined from the distribution and those estimated using the approximation function. This fact is confirmed in the literature, as this distribution is also often used to model the tails of other random variables (e.g., studies of financial markets indicate that the use of GPD (generalized Pareto distribution) is a very good choice in extreme risk analysis). In addition, the quantiles of the maxima are consistent with the quantiles derived from the empirical distribution, as confirmed by using the Chi-square and Kolmogorov–Smirnov goodness of fit test. Comparison of quantiles determined from the probability distributions and those estimated using the exponential function are given in Table 1.

Table 1

Table 1. Comparison of quantiles $x^{\text{pred}}_{\max,\alpha}$ calculated using an exponential function for four probability distributions and quantiles $x^{\text{pdf}}_{\max,\alpha}$ determined from the distributions of maximum daily precipitation $X_{\max}$ according to the equation $\Pr\left(X_{\max} > x^{\text{pdf}}_{\max,\alpha}\right) = 1-\alpha$

Distribution	α for quantile order $1-\alpha$	$x^{\text{pdf}}_{\max,\alpha} - x^{\text{pred}}_{\max,\alpha}$	$\frac{x^{\text{pdf}}_{\max,\alpha} - x^{\text{pred}}_{\max,\alpha}}{x^{\text{pdf}}_{\max,\alpha}} \times 100\%$
Empirical	0.10	6.2	10.6
	0.05	5.3	8.6
	0.02	8.2	11.1
	0.01	0.8	1.0
Gamma	0.10	5.1	9.0
	0.05	4.4	7.4
	0.02	3.6	5.6
	0.01	3.0	4.5
GEV	0.10	3.3	5.7
	0.05	1.2	1.7
	0.02	–4.8	–5.7
	0.01	–16.2	–15.7
Pareto	0.10	4.4	6.3
	0.05	2.1	2.8
	0.02	0.1	0.07
	0.01	–0.8	–1.0

The results presented in this section indicate that the quantile estimates of the distributions (right tails) for real data are better for the Pareto distribution than those for other distributions (in particular, for the Gamma distribution used in the paper [15]). It was also found in the calculations that the GEV distribution for good quantile estimation requires a certain minimum number of observations to estimate the distribution parameters and is therefore not suitable for use with small samples. It is worth noting that Hosking et al. [13] observed a similar effect indicating that good quantile estimates are obtained when estimating the distribution parameters for more than 50 observations.

4. EVALUATING THE DISTRIBUTION QUANTILE ESTIMATION USING A CROSS-VALIDATION TEST

The distribution quantiles estimation procedure for maximum precipitation amounts for short time series with an exponential function was verified using real data with a Pareto distribution and a leave-($n - k$)-out version of the cross-validation test. Subsequently, k-year series (k = 5, 6, ..., 20) were repeatedly created by moving the k-year window by one-year increments in the 30-year observation series. In this way, 26 5-year sequences, 25 6-year sequences, ..., 11 20-year sequences were created.

Short time series were used to evaluate the quantile estimates of the maxima using errors: absolute and relative. The quantile of the appropriate order estimated from the fitted Pareto distribution for maximum precipitation and over 30 years of observations (1989–2018) for Wrocław, was used as the reference quantile. For each k-year series, the procedure of estimating $X_{\max}$, α quantiles of order $1 - \alpha$ was applied and the mean error and its standard deviation were estimated. Changes in mean errors for successive k-year periods of calculated $1 - \alpha$ quantiles of both methods (approximation using the exponential function and using Pareto distribution) are shown in Fig. 3, while relative errors are shown in Fig. 4 and, in addition to relative errors, standard deviations of error differences are shown in Fig. 5.

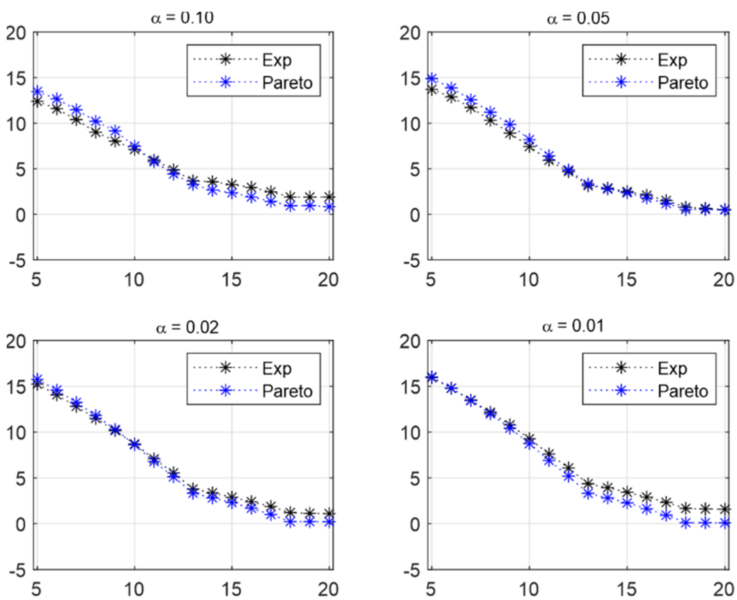

Fig. 3. Mean absolute errors for the quantile approximated using the exponential function (Exp) and the quantile approximated using the Pareto distribution (X axis indicates time series (years) for which the quantile values were determined)

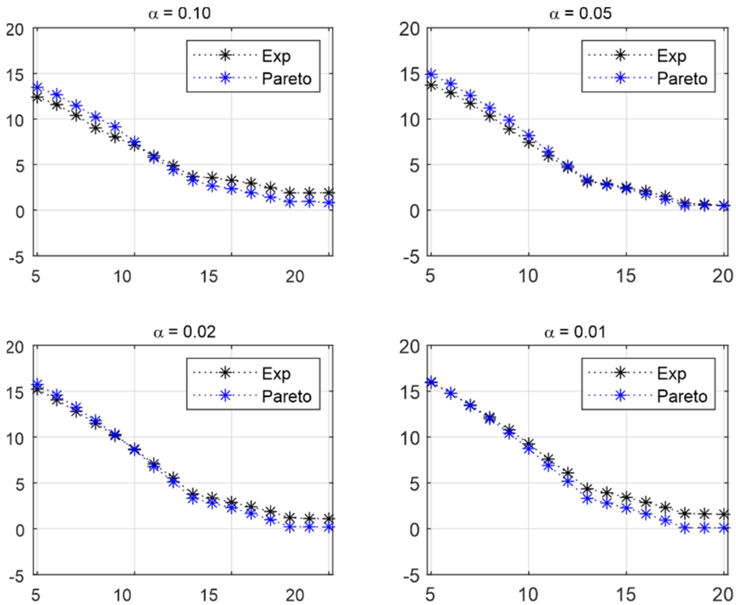

Fig. 4. Mean relative errors for the quantile approximated using the exponential function (Exp) and quantile approximated using the Pareto distribution (X axis indicates time series (years) for which the quantile values were determined)

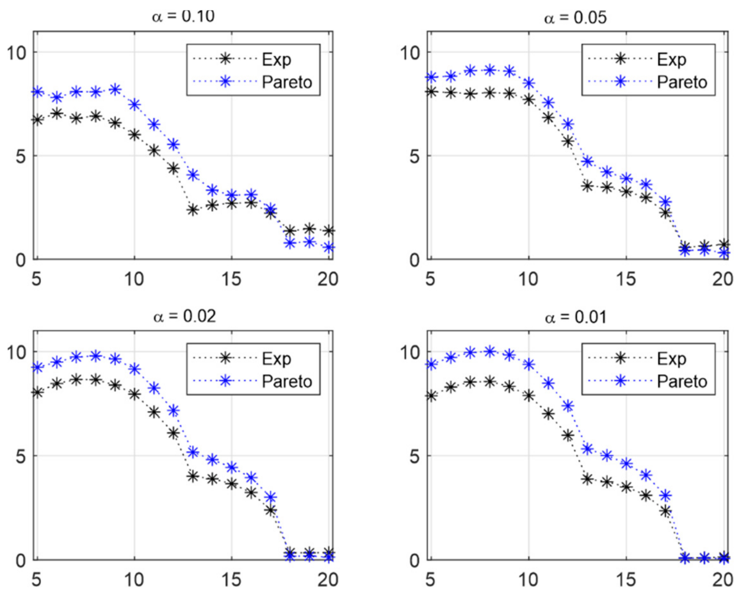

Fig. 5. Standard deviation of the absolute error for the quantiles approximated using the exponential function (Exp) and the Pareto distribution (X axis indicates time series (years) for which the quantile values were determined)

The analysis of Figs. 3–5 reveals two important facts:

- For all quantiles considered and for short time series of observations up to 10 years, both the mean absolute errors and the relative errors are smaller for the estimates using the exponential function, although in many cases the differences are small.
- For all considered periods of years, the standard deviation of errors is smaller for the method of estimating quantiles using an exponential function. This fact is important in the choice of method, indicating the stability of the prediction.

As a result, this means that approximation and prediction of quantiles using extrapolation of the exponential function is more favourable.

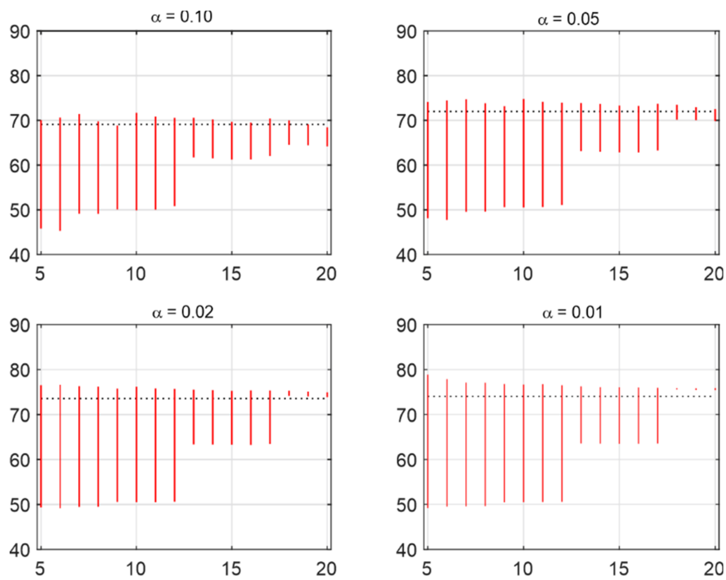

Fig. 6. 95% two-sided confidence intervals for the estimated quantiles using the exponential function for series of observations (the dotted lines are the reference quantiles)

Figure 6 shows the 95% two-sided confidence intervals for the estimated quantiles using the exponential function. The graphs illustrate the variation of the width of the intervals depending on the number of years of the time series as well as the estimated quantile and are a practical, important piece of information providing an interval estimate of the quantile of the maximum precipitation distribution.

5. APPROXIMATION ERROR CORRECTION FOR ESTIMATION USING THE EXPONENTIAL FUNCTION

The results of estimating the quantiles of the distributions using the exponential function (Figs. 3 and 4) indicate a strong underestimation of the quantiles of the maxima for short series of observations. These estimates can be improved by shifting the quantile by a fixed value. Note that the corrections make sense for a time series lasting at most 13 years when the error values Err(v) are smaller than those obtained using the Pareto distribution.

At the same time, for time series from 5 to 13 years, the mean errors decrease linearly, and the value of the absolute error can be described approximately by a function of the form (Figs. 3 and 4):

$$\mathrm{Err}(v) = 23 - \frac{8}{5}v \tag{2}$$

where v is the time period expressed in years.

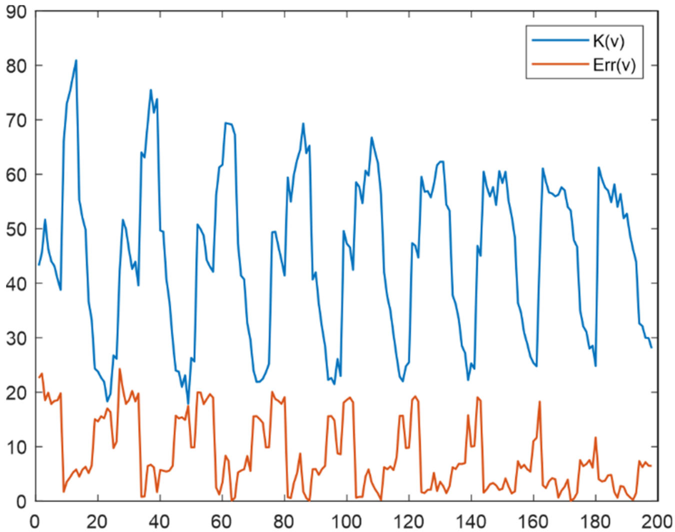

Fig. 7. Course of kurtosis $K(v)$ (upper curve) and absolute error Err(v) (lower curve) for 198 consecutive v = 5-, 6-, ..., 13-year sets

Next, we can observe the dependence of the kurtosis $K(v)$ of the precipitation (Fig. 7) for individual v = 5-, ..., 13-year time series and the computational error Err(v). This figure illustrates the behaviour of absolute error and kurtosis of 198 series, from 5 to 13 years created based on a 30-year reference set (1989–2018), according to the previously described rule (first 5-year set – years 1989–1993, second 5-year set – years 1990 –1994, ..., 26th 5-year set – years 2014–2018, first 6-year set 1989–1994, ..., last analyzed time series, i.e., 18th 13-year set – years 2006–2018).

Analysis of Fig. 7 indicates that for a low kurtosis of the series, a high error is observed, and a kurtosis value of 55 can be taken as the cut-off point. This fact allows us to determine the corrected values of the estimated quantile in the form:

Table 2

Errors of quantile estimates calculated using the corrected exponential function $x^{\text{corr}}_{\max,\alpha}$ and the Pareto distribution $x^{\text{Pareto}}_{\max,\alpha}$ for various time series relative to the reference quantile $x^{\text{pdf}}_{\max,\alpha}$ determined for the distribution of maximum daily precipitation $X_{\max}$ according to $\Pr\left(X_{\max} > x^{\text{pdf}}_{\max,\alpha}\right) = 1-\alpha$

No. of years	α for quantile order $1-\alpha$	$\left\lvert x^{\text{corr}}_{\max,\alpha} - x^{\text{pdf}}_{\max,\alpha}\right\rvert$	$\left\lvert x^{\text{Pareto}}_{\max,\alpha} - x^{\text{pdf}}_{\max,\alpha}\right\rvert$	$\frac{\left\lvert x^{\text{corr}}_{\max,\alpha} - x^{\text{pdf}}_{\max,\alpha}\right\rvert}{x^{\text{pdf}}_{\max,\alpha}}$	$\frac{\left\lvert x^{\text{Pareto}}_{\max,\alpha} - x^{\text{pdf}}_{\max,\alpha}\right\rvert}{x^{\text{pdf}}_{\max,\alpha}}$
5	0.10	4.3966	13.4688	6	20
	0.05	4.6803	14.8964	7	21
	0.02	5.5465	15.7537	8	21
	0.01	6.1409	16.0278	8	22
6	0.10	5.0134	12.6698	7	18
	0.05	5.0509	13.8715	7	19
	0.02	5.362	14.5782	7	20
	0.01	5.9163	14.8001	8	20
7	0.10	4.731	11.4778	7	17
	0.05	4.8504	12.5762	7	17
	0.02	5.3049	13.2274	7	18
	0.01	5.7986	13.4333	8	18
8	0.10	4.9956	10.2089	7	15
	0.05	5.0383	11.2106	7	16
	0.02	5.4752	11.8024	7	16
	0.01	5.9696	11.9884	8	16
9	0.10	4.2574	9.1543	6	13
	0.05	4.3617	9.8659	6	14
	0.02	5.4712	10.2917	7	14
	0.01	6.0946	10.4284	8	14
10	0.10	4.1977	7.4780	6	11
	0.05	4.4443	8.1950	6	11
	0.02	5.7061	8.6144	8	12
	0.01	6.2764	8.7452	8	12
11	0.10	4.0386	5.7692	6	8
	0.05	4.2588	6.4209	6	9
	0.02	5.4511	6.7943	7	9
	0.01	5.9854	6.9066	8	9
12	0.10	3.8774	4.4298	6	6
	0.05	3.9492	4.8723	5	7
	0.02	4.9521	5.1201	7	7
	0.01	5.4832	5.1927	7	7
13	0.10	3.1385	3.2854	5	5
	0.05	3.1091	3.3263	4	5
	0.02	4.0346	3.3269	5	5
	0.01	4.6051	3.3201	6	4

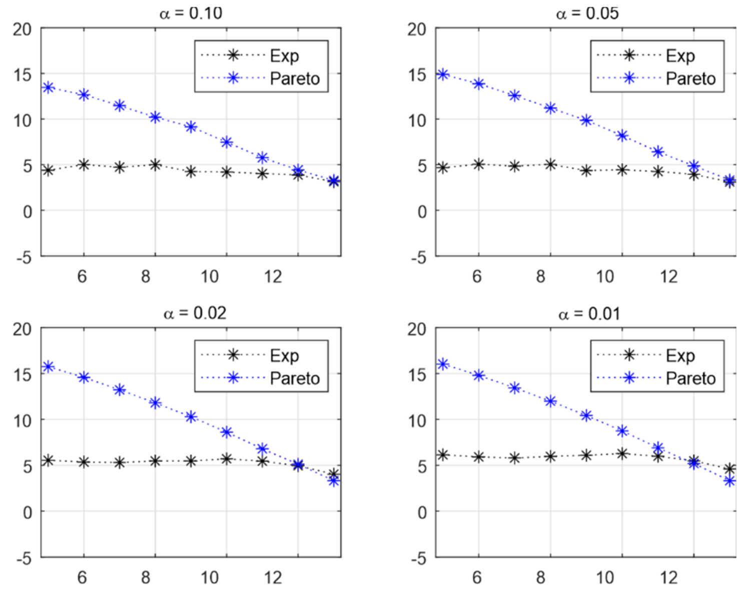

Fig. 8. Mean absolute error for the quantiles approximated using the corrected exponential function (Exp-corr) and by the Pareto distribution distribution (*X* axis indicates time series (years) for which the quantile values were determined)

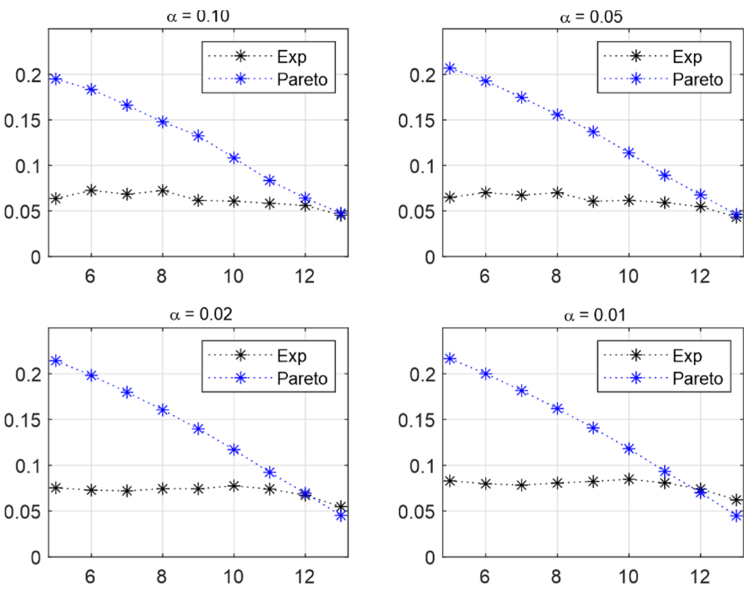

Fig. 9. Mean relative errors for the quantiles approximated using the corrected exponential function (Exp) and by the Pareto distribution (*X* axis indicates time series (years) for which the quantile values were determined)

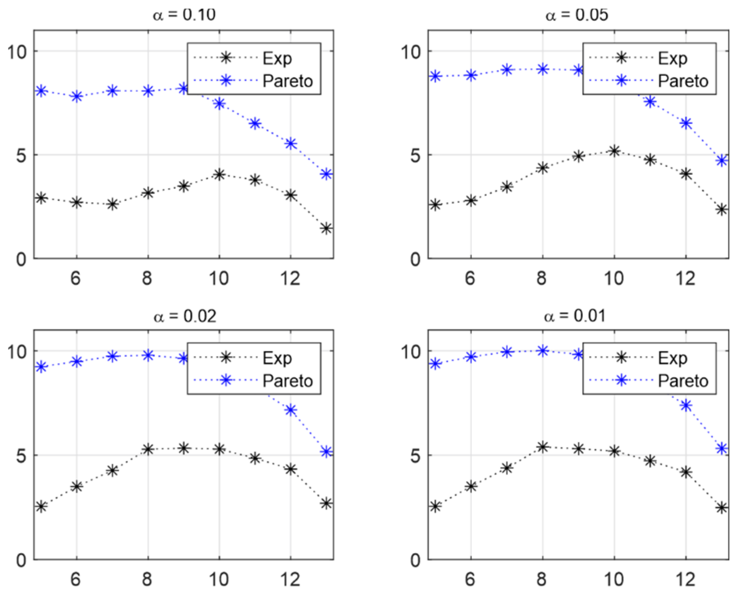

Fig. 10. Standard deviation of the absolute error for the quantiles approximated using the corrected exponential function (Exp) and by the Pareto distribution (X axis indicates time series (years) for which the quantile values were determined)

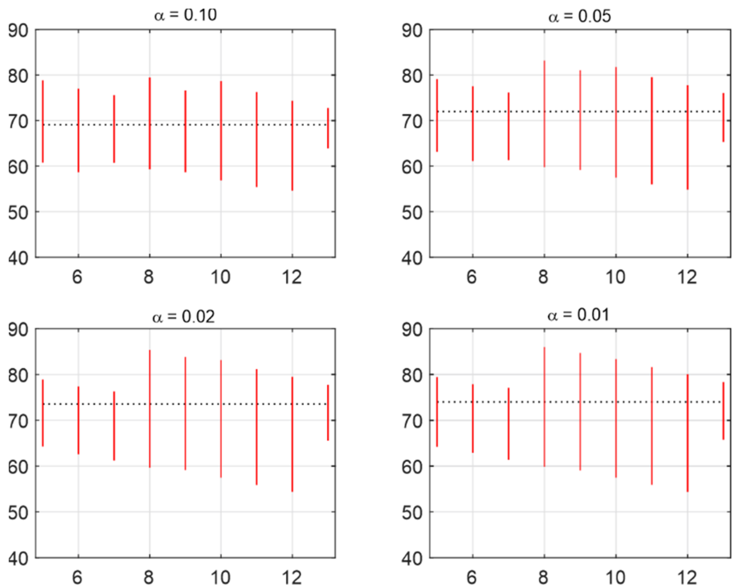

Fig. 11. 95% two-sided confidence intervals for quantiles estimated by an exponential function and modified using kurtosis for time series of 5-, 6-, ..., 13-year observations (the dotted line is the reference quantile)

$$x^{\text{corr}}_{\max,\,\alpha}(v) = x^{\text{pred}}_{\max,\,\alpha}(v) + \left(23 - \frac{8v}{5}\right)\delta\left(K(v) < 55\right) \tag{3}$$

for $v = 5, 6, ..., 13$, where δ denotes the zero-one function (1 when the condition ($K(v) < 55$) is satisfied and 0 when not).

In practice, this means that when estimating a given quantile of the distribution for a short sample (5, 6, ..., 13 years), the kurtosis in the sample is small (<55), the extrapolated value obtained from equation (1) then can be improved using equation (3).

The quantile estimate $x^{\text{corr}}_{\max,\,\alpha}$ corrected according to equation (3) is characterized by absolute and relative errors that on average are twice as small compared to the estimates obtained using the Pareto distribution (Table 2). An illustration of the errors for the increasing number of years in the sample (evolution of mean absolute and relative error) is provided in Figs. 8 and 9.

It is noteworthy that a 12–13-year series of observations is the criterion indicating the choice of quantile estimation method. For a set of more than 12–13 years, calculating the quantile of the distribution by the classical method (selection of the distribution, estimation of its parameters, calculation of quantiles) is more advantageous, that is, it gives a smaller error. From the point of view of potential applications, the error variance is very important. As a result of the approximation correction, the standard deviation of the absolute error is on average three times smaller (Fig. 10). Consequently, confidence intervals for mean quantile values are relatively small and can also serve as interval forecasts (Fig. 11).

6. CONCLUSIONS

In the study carried out on long-term data (1989-2018) for Wroclaw, it was shown that for short observation time series (from 5 to 13 years) quantiles of maximum daily precipitation heights can be approximated based on extrapolation of an exponential function, approximated on values of quantiles of the Pareto distribution obtained for truncated samples containing 95, 96, ..., 99.5 percent of the largest observations.

For short time series, less than 13 years, the average error of quantile approximation using an exponential function decreases linearly as the length of the series increases and is strongly correlated with the kurtosis of the time series and determine an effective procedure for correcting the values of the calculated quantiles.

The procedure described allows for up to a threefold reduction in the error of quantile estimation while at the same time, on average, the standard error deviation is three times lower compared to the quantiles determined from the Pareto distribution which in the conducted studies was a better fitting distribution than the GEV and Gamma distributions.

REFERENCES

[1] BZYMEK B., JAROSIŃSKA E., *The effects of catchment surface sealing on stormwater runoff*, J. Techn., 2012, 109 (4), 41–57 (in Polish).

[2] ROMANIAK A., *Assessment of the relation between atmospheric precipitation and rainwater runoff for various urban surfaces*, J. Water Land Dev., 2017, 32 (1–3), 87–94. DOI: 10.1515/jwld-2017-0010.

[3] KAŹMIERCZAK B., KOTOWSKI A., *The influence of precipitation intensity growth on the urban drainage systems designing*, Theor. Appl. Clim., 2014, 118 (1–2), 285–296. DOI: 10.1007/s00704-013-1067-x.

[4] LARSEN A.N., GREGERSEN I.B., CHRISTENSEN O.B., LINDEJ J., MIKKELSEN P.S., *Potential future increase in extreme one-hour precipitation events over Europe due to climate change*, Water Sci. Techn., 2009, 60 (9), 2205–2216. DOI: 10.2166/wst.2009.650.

[5] JENNINGS D.B., JARNAGIN S.T., *Changes in anthropogenic impervious surfaces, precipitation and daily streamflow discharge: a historical perspective in a mid-atlantic subwatershed*, Land. Ecol., 2002, 17 (5), 471–489.

[6] VALTANEN M., SILLANPAA N., SETATA H., *Effects of land use intensity on stormwater runoff and its temporal occurrence in cold climates*, Hydr. Proc., 2014, 28 (4), 2639–2650. DOI: 10.1002/hyp.9819.

[7] DIETZ M.E., *Low impact development practices: A review of current research and recommendations for future directions*, Water Air Soil Poll., 2007, 186, 351–363. DOI: 10.1007/s11270-007-9484-z.

[8] KOTOWSKI A., KAŹMIERCZAK B., DANCEWICZ A., *Modeling precipitation for sewer sizing*, Institute of Fundamental Technological Research, Polish Academy of Sciences, Warsaw 2010 (in Polish).

[9] DIETZ M.E., CLAUSEN J.C., *Stormwater runoff and export changes with development in a traditional and low impact subdivision*, J. Environ. Manage., 2008, 87 (4), 560–566. DOI: 10.1016/j.jenvman.2007.03.026.

[10] GLASER R., BRÁZDIL R., PFISTER C., DOBROVOLNY P., BARRIENDOS VALLVE M., BOKWA A., CAMUFFO D., KOTYZA O., LIMANÓWKA D., RACZ L., RODRIGO F.S., *Seasonal temperature and precipitation fluctuations in selected parts of europe during the sixteenth century*, Clim. Change, 1999, 43, 169–200. DOI: 10.1023/A:1005542200040.

[11] KHARIN V.V., ZWIERS F.W., *Estimating extremes in transient climate change simulations*, J. Clim., 2005, 18 (8), 1156–1173. DOI: 10.1175/JCLI3320.1.

[12] FRANCIS T., *Extreme Precipitation Analysis at Sizewell: Final Report*, MetOffice, UK, 2011.

[13] HOSKING J.R.M., WALLIS J.R., *Parameter and quantile estimation for the generalized Pareto distribution, technometrics*, 1987, 29 (3), 339–349, http://www.jstor.org/stable/1269343

[14] KAŹMIERCZAK B., WDOWIKOWSKI M., *Maximum rainfall model based on archival pluviographic records-case study for Legnica* (*Poland)*, Per. Polyt. Civil Eng., 2016, 60 (2), 305–312, DOI: 10.3311/PPci.8341.

[15] KUCHAR L., BROSZKIEWICZ-SUWAJ L., *Quantile estimation of probability distributions for maximum daily precipitation and short time series of observational data*, [In:] T. Kałuża (Ed.), *Modelling of hydrological processes*, *Modelling issues in the water management sector*, Bogucki Wydawnictwo Naukowe, Poznań 2020, 55–66 (in Polish).

[16] FORESTIERI A., LO CONTI F., BLENKINSOP S., CANNAROZZO M., FOWLER H.J., NOTO L.V., *Regional frequency analysis of extreme rainfall in Sicily* (*Italy)*, Int. J. Clim., 2018, 38, e698–e716. DOI: 10.1002/joc.5400.

[17] LI L., YAO N., LIU D.L., SONG S., LIN H., CHEN X., LI Y., *Historical and future projected frequency of extreme precipitation indicators using the optimized cumulative distribution functions in China*, J. Hydr., 2019, 124170, DOI: 10.1016/j.jhydrol.2019.124170.

[18] KUCHAR L., *Using WGENK to generate synthetic daily weather data for modelling of agricultural processes*, Math. Comp. Symul., 2004, 65, 69–75. DOI: 10.1016/j.matcom.2003.09.009.

[19] KUCHAR L., *Weather generation with a new approach to rainfall variance estimation and seasonal correlation of variables for crop production modelling*, Agrophys., 2011, 1 (4), 40–46.
[20] KUCHAR L., IWANSKI S., JELONEK L., SZALINSKA W., *Application of spatial weather generator for the assessment of climate change impacts on a river runoff*, Geografie, 2014, 119 (1), 1–25.
[21] MILLARD S.P., NEERCHAL N.K., *Environmental statistics with S-PLUS*, CRC Press, Boca Raton 2000.
[22] WALPOLE R.E., MYERS R.H., MYERS S.L., YE K., *Probability and statistics for engineers and scientists*, 7th Ed. Prentice Hall, New York 2002.
[23] KUCHAR L., IWAŃSKI S., *Evaluation of multisite synthetic data generated by spatial weather generator and long climate data series*, EKO-DOK 2018 E3S Web of Conferences, 2018, 44, 00083. DOI: 10.1051/e3sconf/20184400083.
[24] GUMBEL E.J., *The statistics of extremes*, Columbia University Press, New York 1960.
[25] STEDINGER J.R., VOGEL R.M., FOUFOULA-GEORGIOU E., *Frequency analysis of extreme events*, [In:] D.R. Maidment (Ed.), *Handbook of hydrology*, McGraw-Hill, 1993.
[26] KOTOWSKI A., KAŹMIERCZAK B., *Probabilistic models of maximum precipitation for designing sewerage*, J. Hydrom., 2013, 14 (6), 1958–1965. DOI: 10.1175/JHM-D-13-01.1.