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Accentuate of moiré in an interference pattern by defocusing

Yusupzhan Kh. ISMANOV*, Tolgonay D. TYNYSHOVA

Kyrgyz State Technical University named after I. Razzakov,
66 Ch.Aitmatov Ave., 720044, Bishkek, Kyrgyzstan

*Corresponding author: yusupzhan.ismanov@gmail.com

The article considers a mathematical model of an incoherent optical system, into which defocusing is introduced as an aberration. When modeling, it is assumed that if non-coherent lighting is used to illuminate an object, then the transforming optical system should be considered as linear with respect to the intensity of the light. An analysis of the optical system of a general form was carried out, and relations were obtained for a system with a circular pupil, who allowed obtaining defocused images at the output of the optical system, and the defocusing value was rather simply adjustable. The proposed model can be used either as a low-pass filter for preprocessing of interferograms, or as a method for extracting informative image fragments, using which it is possible to synthesize the original image.

Keywords: incoherent optical system, defocusing, interferogram, frequency spectrum, optical transfer function.

1. Introduction

The task of studying complex phase objects, especially of a dynamic type, most often boils down to the need to accumulate data on changes in optical inhomogeneity with time in these media. Moreover, if we are talking about interferometric studies, this should be a rather detailed interferometric database, that is, a record of the interferograms of an object or medium taken at different, consecutive moments of time and at different values of the interferometer sensitivity [1-4]. In order to more accurately judge the temporal changes occurring in the phase medium under study, it is necessary to take similar pictures with fairly short time intervals. However, when processing all this information, especially if it is necessary to compare the interferograms of the phase medium under investigation obtained at different moments of time, one has to deal with a huge amount of data [5-8]. When entering data into a computer, it is necessary to use methods that can significantly reduce their total volume. In this case, the method should allow not to lose an important part of the information during the reduction and, also, to realize the ability to restore all the original amount of information carried by the interferograms. As such a method is proposed to use the defocusing. The aberration of the defocusing type allows you to eliminate the high-frequency component from the interferogram, to simplify it as much as possible, that is, ultimately, to significantly reduce the amount of information entered into the computer. However, in the case of processing images like interferograms, that is, having a complex structure, it is quite difficult to identify the defining characteristic points. Constructing stylized complex images from its simplest "informative fragments" [9], which are the most characteristic features of a given image, is a fairly effective way to circumvent the problem discussed above. The defocusing of images by optical methods allows you to select these "informative fragments", which are further recognized by the holographic correlator. From the output of the holographic correlator information arrives at the computer. The computer synthesizes the original image using informative fragments [10, 11]. Theoretically and experimentally, it was shown that defocusing is accompanied by the release of informative elements with contrast reversal only in optical systems using non-coherent lighting. The degree of defocusing is selected in each case individually, depending on the characteristics of the high-frequency component of the complex interference pattern. That is, in scheme that uses a holographic correlator for recognition, it is necessary to defocus the input image using an optical system that uses incoherent illumination.

2. The transfer function of the optical system used to defocus input images

Consider the process of image formation using the optical system when the object is illuminated with incoherent and non-monochromatic light. A schematic diagram of an optical system in which an incoherent and non-monochromatic light field is used to form an image is shown in Fig. 1.

If incoherent illumination is used to illuminate an object, then the optical system should be considered as linear with respect to the intensity of the light. With incoherent

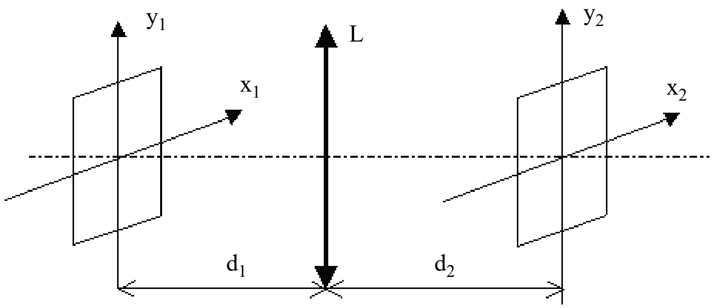

Fig. 1. The layout of the object, image and lens in the optical system. L is a lens, and d_1, d_2 are distances from the object's plane to the lens and from the lens to the image plane, respectively.

lighting, the light intensity conversion can be represented as a convolution of the following form [12]:

$$I_2(x_2, y_2) = g\iint_{\infty} |h(x_2 - x_1, y_2 - y_1)|^2 I_k(x_1, y_1)\mathrm{d}x_1\mathrm{d}y_1 \tag{1}$$

Here I_k is the approximation of geometrical optics for the intensity of an ideal image; h is the impulse response, *i.e.* the amplitude in the plane of the image for a point having coordinates (x_2, y_2), when it is exposed to a point source, which is located at a point (x_1, y_1); g is a real constant; I_2 is the intensity of the image at a point with coordinates (x_2, y_2).

As the frequency analysis of these optical systems shows, linear transformation for the intensity values of the light field is implemented in them. Given this fact, you can enter the frequency spectra of the distribution of values I_k and I_2, which are normalized relative to the ideal spectrum, and the values of which are given by the relations [13]:

$$\Omega_k(\xi, \eta) = \frac{\iint_{\infty} I_k(x_1, y_1)\exp\left[-i2\pi(\xi x_1 + \eta y_1)\right]\mathrm{d}x_1\mathrm{d}y_1}{\iint_{\infty} I_k(x_1, y_1)\mathrm{d}x_1\mathrm{d}y_1} \tag{2}$$

$$\Omega_2(\xi, \eta) = \frac{\iint_{\infty} I_2(x_2, y_2)\exp\left[-i2\pi(\xi x_2 + \eta y_2)\right]\mathrm{d}x_2\mathrm{d}y_2}{\iint_{\infty} I_2(x_2, y_2)\mathrm{d}x_2\mathrm{d}y_2} \tag{3}$$

Here ξ and η are the coordinates in the frequency domain. Similarly, the transfer function of the system can be normalized:

$$\Omega_2(\xi, \eta) = \frac{\iint_{\infty} |h(x_2, y_2)|^2\exp\left[-i2\pi(\xi x_2 + \eta y_2)\right]\mathrm{d}x_2\mathrm{d}y_2}{\iint_{\infty} |h(x_2, y_2)|^2\mathrm{d}x_2\mathrm{d}y_2} \tag{4}$$

If we apply the convolution theorem to the expression (1), then we obtain the relation:

$$\Omega_2(\xi, \eta) = \Omega(\xi, \eta)\Omega_k(\xi, \eta) \tag{5}$$

The function $\Omega(\xi, \eta)$ is the transfer function of the optical system. $\Omega(\xi, \eta)$ it is, in fact, a weighting factor of a complex type for a frequency variable at a point (ξ, η), which is introduced by the optical system and which is assigned to a weighting factor of a variable that has zero frequency. For the known optical coherent transfer function $G(\xi, \eta)$ of the system, it is possible to calculate the optical transfer function using the following formula:

$$\Omega(\xi,\eta) = \frac{\iint\limits_{\infty} G(\mu',\tau')G^{*}(\mu'+\xi,\tau'+\eta)\,\mathrm{d}\mu'\,\mathrm{d}\tau'}{\iint\limits_{\infty} |G(\mu',\tau')|^{2}\,\mathrm{d}\mu'\,\mathrm{d}\tau'} \tag{6}$$

Here $G(\xi, \eta) = \mathrm{Fu}\{h\}$ is the Fourier image of the response function.

After, changing the variables $\mu = \mu' + \dfrac{\xi}{2}$ and $\tau = \tau' + \dfrac{\eta}{2}$ we have the expression:

$$\Omega(\xi,\eta) = \frac{\iint\limits_{\infty} G\left(\mu-\frac{\xi}{2},\tau-\frac{\eta}{2}\right)G^{*}\left(\mu+\frac{\xi}{2},\tau+\frac{\eta}{2}\right)\mathrm{d}\mu\,\mathrm{d}\tau}{\iint\limits_{\infty} |G(\mu,\tau)|^{2}\,\mathrm{d}\mu\,\mathrm{d}\tau} \tag{7}$$

In the case of a coherent system [14] $G(\xi, \eta) = Z(\lambda d_2\xi, \lambda d_2\eta)$. Here $Z(x, y)$ is the function of the pupil; λ is the average wavelength; d_2 is the distance between the lens and the image.

$$\Omega(\xi,\eta) = \frac{\iint\limits_{\infty} Z\left(\mu-\frac{\lambda d_2\xi}{2},\tau-\frac{\lambda d_2\eta}{2}\right)Z\left(\mu+\frac{\lambda d_2\xi}{2},\tau+\frac{\lambda d_2\eta}{2}\right)\mathrm{d}\mu\,\mathrm{d}\tau}{\iint\limits_{\infty} Z(\mu,\tau)\,\mathrm{d}\mu\,\mathrm{d}\tau} \tag{8}$$

here Z^2 in the denominator is replaced by Z, given that the function Z is 1 or 0.

Consider the optical system in the presence of aberration. Defocusing is a fairly common case of aberration. If we consider the phase error at a point with coordinates (x, y) in the plane of the exit pupil as $kV(x, y)$, where k is the wave number, and V sets the effective path length error, then the transmitting coefficient, which in general has a complex form, can be written as:

$$Z_j(x,y) = Z(x,y)\exp\left[ikV(x,y)\right] \tag{9}$$

where $Z_j(x, y)$ is recording of the function of the pupil in general. If we consider aberration, then for a coherent transfer function, we can write the relation:

$$G(\xi,\eta) = Z(\lambda d_2\xi,\lambda d_2\eta)\exp\left[ikV(\lambda d_2\xi,\lambda d_2\eta)\right] \tag{10}$$

The area of overlap of functions

$$Z\left(\mu-\frac{\lambda d_2\xi}{2},\tau-\frac{\lambda d_2\eta}{2}\right) \text{ and } Z\left(\mu+\frac{\lambda d_2\xi}{2},\tau+\frac{\lambda d_2\eta}{2}\right)$$

can be represented as some function $B(\xi, \eta)$.

The integrand function is the product of the pupil functions, which are shifted relative to each other in the directions of the axes of coordinates μ and τ on $\lambda d_2 \xi$ and $\lambda d_2 \eta$, respectively. As can be seen from (8), this function is not zero only in the area where the pupil functions overlap. This is due to the fact that in other places the value of one of the functions is 1, and the other is 0, or both values are 0. That is, the transfer function of the optical system in the absence of aberrations is

$$\Omega(\xi, \eta) = \frac{\iint\limits_{B(\xi, \eta)} \mathrm{d}\mu \, \mathrm{d}\tau}{\iint\limits_{B(0, 0)} \mathrm{d}\mu \, \mathrm{d}\tau}$$

Integral $\iint\limits_{B(0, 0)} \mathrm{d}\mu \, \mathrm{d}\tau$ is the integral of the pupil function in the absence of displacement. However, we believe that the pupil function in the overlap area is equal to 1, *i.e.* the overlap area is in this case the pupil area. All this means that the considered integral is numerically equal to the area of the pupil.

Accounting for the presence of aberrations gives us the following expression for the optical transfer function:

$$\Omega(\xi, \eta) = \frac{\iint\limits_{B(\xi, \eta)} \exp\left\{ ik\left[V(\mu - m_x, \tau - m_y) - V(\mu + m_x, \tau + m_y) \right] \right\} \mathrm{d}\mu \, \mathrm{d}\tau}{\iint\limits_{B(0, 0)} \mathrm{d}\mu \mathrm{d}\tau} \tag{11}$$

Here $m_x = \dfrac{\lambda d_2 \xi}{2}$ and $m_y = \dfrac{\lambda d_2 \eta}{2}$.

Provided that we consider defocusing as an aberration, the well-known thin lens formula is converted to the following:

$$\frac{1}{d_2} + \frac{1}{d_1} - \frac{1}{f} = \delta$$

here d_1 is the distance between the object and the lens; d_2 is the distance between the lens and the plane of the focused image; f is the focal length of the lens; δ is the displacement characteristic of the plane in which we view the image, from the plane in which the image would be focused. The expression for the effective error of the path length, with aberration such as defocusing can be written as [13]

$$V(x, y) = \frac{\delta(x^2 + y^2)}{2}$$

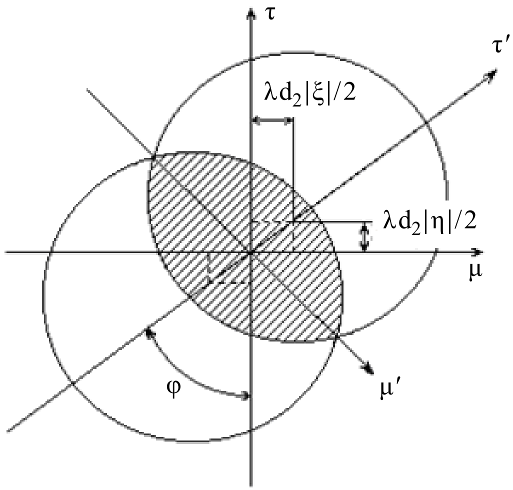

Fig. 2. The area of intersection of the functions of the pupil (shaded).

Consider an optical system having an entrance pupil in the form of a circle with a diameter b [10, 11]. For such an inlet, the pupil function can be written as

$$Z(x, y) = \operatorname{circ}\left(\frac{\sqrt{x^2 + y^2}}{b/2}\right) \tag{12}$$

The shaded area in Fig. 2 is the intersection area $B(\xi, \eta)$ of functions

$$Z\left(\mu - \frac{\lambda d_2 \xi}{2}, \tau - \frac{\lambda d_2 \eta}{2}\right) \text{ and } Z\left(\mu + \frac{\lambda d_2 \xi}{2}, \tau + \frac{\lambda d_2 \eta}{2}\right).$$

To calculate the integral over the intersection region $B(\xi, \eta)$, it is convenient to switch to another coordinate system (x', y'), which is rotated by the angle φ. The value of this angle is given by the relations:

$$\cos\varphi = \frac{m_x}{\sqrt{m_x^2 + m_y^2}} \quad \text{and} \quad \sin\varphi = \frac{m_y}{\sqrt{m_x^2 + m_y^2}} \tag{13}$$

Here $m_x = \dfrac{\lambda d_2 \xi}{2}$ and $m_y = \dfrac{\lambda d_2 \eta}{2}$.

Relationships connecting new and old coordinates are:

$$x = x'\cos\varphi - y'\sin\varphi \quad \text{and} \quad y = x'\sin\varphi + y'\cos\varphi \tag{14}$$

An expression describing the transfer function of an optical system having a circular entrance pupil is:

$$\Omega(\xi, \eta) = \frac{4}{\pi b^2}\int_{\theta_2}^{\theta_1}\int_{\gamma_2}^{\gamma_1}\exp\left\{\frac{ik\delta}{2}\left[(x'\cos\varphi - y'\sin\varphi - m_x)^2 + (x'\sin\varphi + y'\cos\varphi - m_y)^2\right]\right\}$$

$$\times \exp\left\{-\frac{ik\delta}{2}\left[(x'\cos\varphi - y'\sin\varphi + m_x)^2 + (x'\sin\varphi + y'\cos\varphi + m_y)^2\right]\right\}\mathrm{d}x'\mathrm{d}y'$$

$$= \frac{4}{\pi b^2}\int_{\theta_2}^{\theta_1}\int_{\gamma_2}^{\gamma_1}\exp\left\{-ik\delta\left[m_x(x'\cos\varphi - y'\sin\varphi) + m_y(x'\sin\varphi + y'\cos\varphi)\right]\right\}\mathrm{d}x'\mathrm{d}y'$$

$$= \frac{4}{\pi b^2}\int_{\theta_2}^{\theta_1}\exp\left[-ik\delta x'(m_x\cos\varphi + m_y\sin\varphi)\right]\mathrm{d}x'$$

$$\times \int_{\gamma_2}^{\gamma_1}\exp\left[-ik\delta y'(m_y\cos\varphi - m_x\sin\varphi)\right]\mathrm{d}y' \tag{15}$$

Here $\theta_1 = -\theta_2 = \sqrt{R^2 - m_x^2 - m_y^2}$ and $\gamma_1 = -\gamma_2 = \sqrt{R^2 - x'^2} - \sqrt{m_x^2 + m_y^2}$.

According to (13), for the integral over x', $C = m_x\cos\varphi + m_y\sin\varphi = \sqrt{m_x^2 + m_y^2}$, for the integral over y', $m_y\cos\varphi - m_x\sin\varphi = 0$.

Considering all the above, as a result of integration over y', the expression for the optical transfer function of an optical system having a circular entrance pupil can be written as:

$$\Omega_{\kappa p}(\xi, \eta) = \frac{4}{\pi b^2}\int_{\theta_2}^{\theta_1}\exp(-ikC\delta x')2\gamma_1\,\mathrm{d}x' \tag{16}$$

Perform discretization of the integrand function:

$$C(l, s) = \sqrt{m_x^2 + m_y^2} = \sqrt{\frac{b^2(s - M/2 - 1)^2 + b^2(l - M/2 - 1)^2}{M^2}}$$

$$= \frac{b\sqrt{(s - M/2 - 1)^2 + (l - M/2 - 1)^2}}{M}$$

Discretization of (16) gives the expression:

$$\Omega_{\kappa p}(l, s) = \sum_{k=1}^{M}\exp\left[-4ik\delta C(l, s)\theta_1(l, s)\frac{k - M/2 - 1}{M}\right]16\gamma_1(l, s, k)\,\theta_1(l, s) \tag{17}$$

Here M is taken according to the sampling theorem: $M = 2\nu_0/\Delta\xi$, ν_0 is the highest spatial frequency, which is passed by the considered optical system; $l = 1, 2, ..., M$ and $s = 1, 2, ..., M$ determine the values of the sampling points for the spatial frequencies ξ and η; $k = 1, 2, ..., M$ – for the coordinate x'.

$$C(l, s) = b\sqrt{(s - M/2 - 1)^2 + (l - M/2 - 1)^2}$$

$$\theta_1(l, s) = \sqrt{\frac{b^2}{4} - \frac{\left[(s - M/2 - 1)^2 + (l - M/2 - 1)^2\right]b^2}{M}}$$

$$x' = \frac{2(k - M/2 - 1)\,\theta_1(l, s)}{M}$$

$$\gamma_1(l, s, k) = \sqrt{b^2/2 - x'^2} - C(l, s)$$

Relation (17) can be considered as a working formula for modeling the process of defocusing images.

3. The results of the simulation of defocusing process

Figures 3 and 4 show the results of defocusing of interferograms for a phase object such as a thin lens, which were obtained, respectively, in channels 3 and 4 of the holographic interferometer [1-4]. In all the figures, the inner square represents the interference pattern in the presence of a phase medium. Outside the internal square, we have the interference pattern, which is obtained in the absence of the phase medium, with all other parameters of the interferogram being unchanged. Interferograms are presented sequentially at different defocusing values ($\delta = 0$, 0.002, and 0.006). It can be seen from the figures that the appropriate choice of the defocusing value of the interferograms makes it possible to quite clearly distinguish the moiré pattern, that is, the low-

(a) (b) (c)

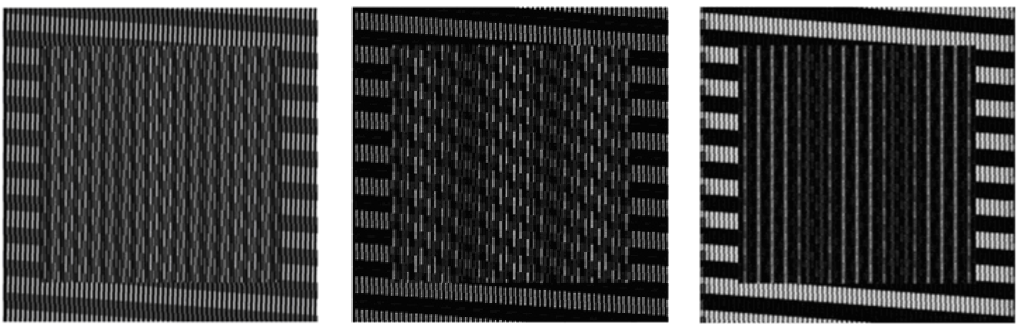

Fig. 3. Interferograms of a lens. Channel 3 of the multichannel holographic interferometer [1]. (a) Defocus parameter $\delta = 0$ (defocus is absent), (b) defocus parameter $\delta = 0.002$, and (c) defocus parameter $\delta = 0.006$.

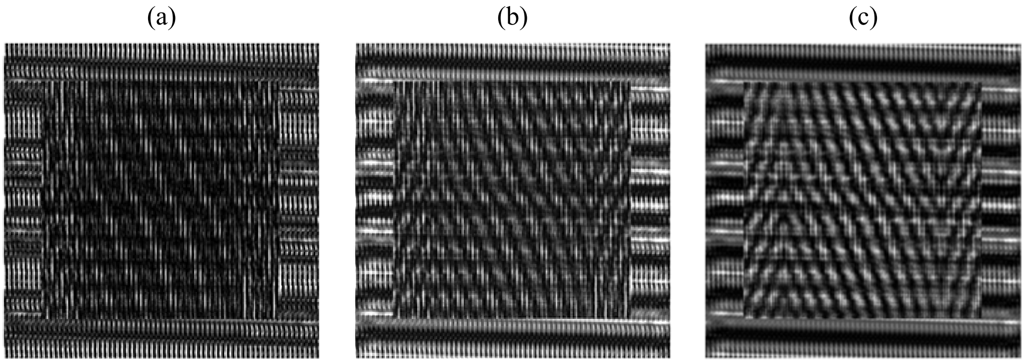

Fig. 4. Interferograms of a lens. Channel 4 of the multichannel holographic interferometer [1]. (a) Defocus parameter $\delta = 0$ (defocus is absent), (b) defocus parameter $\delta = 0.002$, and (c) defocus parameter $\delta = 0.006$.

frequency component. In this case, defocusing works as a spatial filter that removes high frequencies in complex interference patterns that are produced in the output channels of a multichannel holographic interferometer.

4. Conclusions

A mathematical model is developed that describes the process of defocusing images for the case of an incoherent optical system and non-monochromatic light.

The proposed model can be used as a method of preliminary preparation of images (interferograms) in computer systems for processing complex interferograms without the use of optical defocusing devices. The defocusing method can be quite effectively used in two cases:

a) As a low-pass filter, which, with an appropriate choice of the defocus parameter, reduces the impact of the high-frequency components of the complex interference pattern and more clearly distinguishes the moiré pattern.

b) As a way to select informative fragments of images, by which it is possible, if necessary, to synthesize the original image. This method of preprocessing of images makes it possible to drastically reduce the amount of input data during their computer processing, classification and decoding of interferograms.

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