ARGUMENTA OECONOMICA

1 • 1995

Academy of Economics in Wrocław
Wrocław 1995

TABLE OF CONTENTS

ARGUMENTA OECONOMICA
No 1 • 1995
PL ISSN 1233-5835

Andrzej Baniak

COMPETITION BETWEEN THE STATE AND THE PRIVATE SECTOR AND THE EFFECTS OF PRIVATIZATION[1]

1. INTRODUCTION

In central and eastern Europe the difficult process of transforming economies towards a decentralized market economy continues – with the private sector playing a major role in that process. However, state enterprises will continue for a long time to be a constant element on the economic scene. As a matter of fact they can no longer be called 'state enterprises', as their employees now have a greater part in decision-making processes. For example, in Poland the Employees Council has the power to dismiss a manager of an enterprise and also has the right to veto any fundamental decisions concerning that enterprise. It seems that we have here a case of a labour-managed firm. The aim of such a company is not to maximize profits but the average individual income, which is the sum of the average salary and the average share of profits per employee. If the average employee's income is defined as y therefore:

$$y = w + (\pi/L)$$

where w = salary, π = profit and L = size of employment. If we assume that $\pi = pX - wL - H$ (where X is the size of production and H = permanent cost, not connected with work) therefore the aim of a labour-managed firm can be put as:

[1] This paper was published firstly in: Prace Naukowe Akademii Ekonomicznej we Wrocławiu [Research Works of Wrocław Academy of Economics (RW of WAE)] 1993, No 665.

$$y = (pX - H) / L$$

Because the condition necessary for optimum: $pX_L = y = (pX - H) / L$ leads to dependence $dL / dp = (X/L - X_L) pX_{LL} < 0$ and $dL/dH = -1/pX_{LL} > 0$ (Laidler, Estrin 1991, Chapter 18), it can be said that a labour-managed firm functions in the opposite way to one which maximizes profits: it increases employment (and production) when permanent costs rise, it decreases employment (and production) when prices grow. The purpose of this article is to analyse the situation where private firms and labour-managed ones compete on the market with the same product. This problem is of both theoretical and practical importance: one can observe markets where the private and state sector compete, for example in the food, textile and building industries. Such mixed 'state-private' markets display some particular characteristics. Firstly, because of the historically determined monopolist position of the state company and the limited possibilities of access to the market by the private sector, one can assume a state of imperfect competition on the market. Secondly, the state-owned firm has capital larger than any private company. The difference in the amount of capital causes differences in own costs: every private firm's fixed costs are lower then – and total costs higher than – those of state-owned enterprises.

We examine here the consequences of changes in property structure within one sector of industry. The object of our interest is fragmentary balance; the analysis is short-term. We consider here two types of privatization involving an immediate (and unburdened by costs) transfer of capital from the state to the private sector. At first we consider the situation where the state-owned company becomes a private one and in its activities concentrates on maximizing profits. Later we consider partial privatization, in which a part of the state-owned firm is privatized and the rest stays in the state's hands. The aim of this article is to compare the production levels before and after privatization. In particular we shall try to answer the question of whether, after privatization, industrial production increases. Which of those two types is better from the point of view of the size of production?

2. MODEL

Let us consider an industry in which $(n + 1)$ firms produce a homogenous product. A zero firm is state-owned, acts as labour-managed and maximizes average income y. The rest n firms are private and maximize profits. The size of production of i firm will be marked x_i, $i = 0,\dots n$. The function of demand is given in the formula:

$$p = D(X) = a - X, \quad a > 0,$$

where p is price, and $X = \sum_{i=0}^{n} x_i$ denotes production in the whole industry.

Each firm produces, using the same technology, described by the Cobb–Douglas function with constant returns to scale:

$$x_i = f(K_i, L_i) = K_i^{0,5} L_i^{0,5}$$

where L = labour, K = capital; capital K_i is a constant factor, labour is a variable factor. All the private firms have the same amount of capital K, which is smaller than the capital of state-owned firm, K_0:

$$K = K_1 = ... = K_n < K_0.$$

Using this assumption the functions of costs are:

$$c_0(x_0) = (wx_0^2/K_0) + rK_0 \text{ (for a state-owned firm)}$$

$$c_i(x_i) = (wx_i^2/K) + rK \text{ (for a private firm)}$$

We can see that despite the same technology the functions of costs differ: the state-owned firm has lower total costs but higher fixed costs. Similar expression of the problems are presented in (Delbono, Rossini 1992, 226–240) but there it is assumed that both kinds of firms have identical functions of costs. First let us examine the existence of the Nash equilibrium in the game of $(n + 1)$ persons. We will obtain it solving the following system of equations:

$$\delta y/\delta x_0 = 0; \ \delta \pi_i/\delta x_i = 0, \quad i = 1, ..., n.$$

Because all the private firms are identical, it is easy to prove that in the Nash equilibrium their production will also be the same:

$$x_1 = ... = x_n \text{ (see proof of theorem 1)}$$

That is why we shall consider one representative private firm with production x.

Theorem 1

If a state-owned firm maximizes the average income and private firms maximize profits, the functions of reactions look as follows:

$$x_0(x) = 2rK_0 (a - nx)^{-1} \text{ (for a state-owned firm)}$$

$$x(x_0) = (a - x_0)(1 + n + 2wK^{-1})^{-1} \text{ (for a private firm)}$$

Nash equilibrium (x_0^L, x^L) exists and is explicitly set if, and only if, $a^2 \geq 2rK_0$.

The comparative statistics for Nash equilibrium is as follows:

change

influence	Δa	Δn	Δr	Δw	ΔK	ΔK_0
x_0^L	–	+	+	–	+	+
x^L	+	–	–	–	+	–

proof: see Appendix

We can see that the labour-managed firm in conditions of imperfect competition also functions in the reverse way: it reduces employment (and production) in response to increased demand (increase of parameter a) and increases employment (and production) when fixed costs rise.

3. RESULTS OF PRIVATIZATION

Privatization is at present one of the most important problems of transforming economies. It should increase effectiveness, competitiveness and enable the better allocation of resources. In the presented model, privatization means instant and free capital transfer from the state sector (dominated by the labour--managed firm) to the private one. In effect K_0 gets smaller, nK grows and $\Delta K_0 + \Delta nK = 0$.

We can start from the analysis of the situation where a state-owned company becomes totally privatized and its purpose is to maximize profits. We shall concentrate on Nash equilibrium obtained by solving a set of equations:

$$\delta\pi_i/\delta x_i = 0, \quad i = 0, 1, ..., n.$$

Theorem 2

If all the firms maximize their profits, then the functions of reaction are as follows:

$$x_0(x) = (a - nx)\,(2 + 2wK_0^{-1})^{-1};\; x\,(x_0) = (a - x_0)\,(1 + n + 2wK^{-1})^{-1}.$$

Nash equilibrium (x_0^P, x^P) always exists and is set explicitly where $x_0^P > x^P$.

Results of the comparative statistics:

change

influence	Δa	Δn	Δr	Δw	ΔK	ΔK_0
x_0^P	+	–	0	?	–	+
x^P	+	–	0	?	+	–

proof: see Appendix

In Nash equilibrium, the production of the *zero*th firm is greater than the production of the *i*th firm because of the difference in their capital: $K < K_0$. Generally speaking, it is not easy to tell whether the production of a state-owned firm increases when privatized. Because of algebraic problems it is also difficult to form opinions on production changes in the whole industry. We shall present some hypotheses further on. Apart from 'total' privatization we can also observe partial privatization, where a part of the state-owned firm is privatized and supports the creation of the new private company. In such cases $K_0 \rightarrow K_0 - K$ and $n \rightarrow n + 1$. The number of private firms is increased by one and $\Delta K_0 = -K$ i $\Delta n = 1$. And again because of the algebraic problems it is very difficult to compare the production levels after total and partial privatization. We shall tackle this problem from a numerical point of view.

4. NUMERICAL ANALYSIS

Presently we shall examine the production levels of different types of the market presented in previous paragraphs. Because of the algebraic problems we shall use numeral analysis. The suitable program has been written in TURBO-PASCAL language with the assistance of Janusz Łyko. We have compared the production levels for three types of market: with a labour-managed firm, totally privatized labour-managed firm and partially privatized labour-managed firm. There was a great number of simulations conducted, the most interesting part is presented in Tables 1–4. Symbols x_0^L, x_0^P and x_0^C denote respectively production of the labour-managed firm before privatization, after total privatization and

Table 1
Changes in production levels with the changes of K
($a = 100$, $n = 1$, $r = 1$, $w = 1$, $K_0 = 100$)

K	x_0^L	x^L	X^L	x_0^P	x^P	X^P	x_0^C	x^C	X^C
10	3,6	43,8	47,4	34,8	29,6	64,5	4,5	29,6	64,2
30	3,7	46,6	50,3	33,6	32,1	65,7	3,8	31,4	66,5
50	3,8	47,2	51	33,3	32,7	66	2,8	32	66,8
70	3,8	47,4	51,2	33,2	32,9	66,1	1,7	32,5	66,6
90	3,8	47,6	51,4	33,1	33	66,2	0,6	32,9	66,4

Table 2
Changes in production levels with the changes of n
($a = 100$, $r = 1$, $w = 1$, $K = 10$, $K_0 = 100$)

n	x_0^L	x^L	X^L	x_0^P	x^P	X^P	x_0^C	x^C	X^C
1	3,7	45,9	49,6	33,9	31,5	65,4	4,2	30,9	66
3	6,4	22,8	74,9	20,8	19,3	78,8	6,1	18,4	79,7
5	8,1	15,1	83,4	15	13,9	84,7	7,4	13,1	85,6
7	9,3	11,2	87,7	11,8	10,9	88	8,3	10,1	88,9
9	10,1	8,9	90,2	9,7	9	90,2	8,9	8,2	91

Table 3
Changes in production levels with the changes of K_0
($a = 100$, $n = 5$, $r = 1$, $w = 1$, $K = 20$)

K_0	x_0^L	x^L	X^L	x_0^P	x^P	X^P	x_0^C	x^C	X^C
100	8,1	15,1	83,4	15	13,9	84,5	7,4	13,1	85,7
150	11,1	14,6	84	15,1	13,9	84,6	10,6	12,6	86,2
200	13,7	14,2	84,4	15,2	13,9	84,7	13,4	12,2	86,6
250	16	13,8	84,9	15,2	13,9	84,7	15,9	11,9	87
300	18,2	13,4	85,3	15,2	13,9	84,7	18,2	11,5	87,3

Table 4
Changes in production levels with changes of r
($a = 100$, $n = 3$, $w = 1$, $K = 10$, $K_0 = 100$)

r	x_0^L	x^L	X^L	x_0^P	x^P	X^P	x_0^C	x^C	X^C
1	6,1	22,4	73,2	21,9	18,6	77,7	6,4	18	78,4
5	22,4	18,5	77,8	21,9	18,6	77,7	22,4	14,9	82,1
10	36,6	15,1	81,9	21,9	18,6	77,7	35,7	12,4	85,2
20	57,5	10,1	87,9	21,9	18,6	77,7	55	8,7	89,6

partial privatization. Analogously x^L, x^P, x^C denote the total product of a private firm, whereas X^L, X^P, X^C – the total production of the three considered types of markets.

Based on the simulations, we can formulate certain hypotheses.

Hypothesis 1.
Usually $x^L > x_0^L$ unless the difference $rK_0 - nrK$ is large or n is large.

The production of a private company is usually higher than that of a labour-managed firm. The exception is the case when the capital of the labour-managed firm is much larger than the capital of the private sector (see Table 3, 4) or the private sector is highly competitive.

Hypothesis 2.
If the difference $rK_0 - nrK$ is large then $x^L > x^P$. Otherwise $x^L < x^P$.

As a rule, after total privatization the level of industrial production grows unless the capital in the state-owned sector exceeds the private one. It seems that in such a case after total privatization we encounter a certain deformity in the market – one big company and several smaller ones. Such a deformity may cause a decrease in output.

Hypothesis 3.
Always $x^C \geq x^P$.
It is the most suprising result. It turns out that after partial privatization,

production increases more than after total privatization. The explanation can be similar to the one in the previous case. The market is deformed after total privatization but after a partial one the market is not deformed to such a degree and is also more competitive. Therefore the increase in production is larger.

5. CONCLUSION

This paper examines the consequences of different kinds of privatization for the private and state sectors. We have obtained a rather paradoxical result: after partial privatization, the increase of production is larger than after a firm has been totally privatized. This may be connected with the fact that total privatization creates some tensions on the market which are connected with the difference of resources in the state and private sectors. Moreover, as total privatization undoubtedly involves higher costs, it can make partial privatization a more acceptable choice. We have assumed here that the state-owned company functions like the labour-managed one, i. e. it maximizes the average income. However, the most recent empirical studies (Prasnikar et al 1991) show that a labour-managed firm not only maximizes the average income but also the level of employment. It seems therefore that the more suitable objective function for the labour-managed firm is:

$$f_0(x_0) = \alpha y + (1-\alpha)\, L_0 \quad \text{or} \quad f_0(x_0) = y^{\alpha} L_0^{1-\alpha}, \quad \text{where } 0 < \alpha < 1.$$

The author currently researches the state of imperfect competition with the above mentioned objective functions.

APPENDIX

Proof of the theorem 1.

A labour-managed firm maximizes

$$y = ((a-X)x_0 \; - rK_0)\, /L_0 = ((a-X)\, x_0 - rK_0)\, K_0/x_0^2.$$

From the necessary condition we obtain the function of reaction for x_0:

$$\delta y/\delta x_0 = -x_0^{-1} - (a-X)\, x_0^{-2} + 2rK_0 x_0^{-3} = 0.$$

We shall write $x_{-i} = X - x_i$, $i = 0,\ldots, n$. We can now write differently the necessary condition (*NC*):

$$x_0^{-3}\; (x_0(x_{-0} - a) + 2rK_0) = 0.$$

The function of reaction looks as follows:

$$x_0 = 2rK_0 (a - x_{-0})^{-1}. \tag{1}$$

Sufficient condition (*SC*):

$$\delta^2 y/\delta x_0^2 = 2x_0^{-4} ((a - x_{-0}) x_0 - 3rK_0).$$

For x_0 from the equation (1) we have $\delta^2 y/\delta x_0^2 = -2x_0^{-4} rK_0 < 0$.

A private firm maximizes profits.

$$NC: \delta\pi_i/\delta x_i = -x_i + a - X - (2wx_i/K) = a - x_i(2 + (2w/K)) - x_{-i} = 0.$$

$$SC: \delta^2\pi_i/\delta x_i^2 = -2 - (2w/K) < 0.$$

Function of reaction: $x_i = (a - x_{-i}) (2 + (2w/K))^{-1}$, $i = 1,..., n$.

To obtain the Nash equilibrium we have to solve the set of equations $\delta y/\delta x_0 = 0$, $\delta\pi_i/\delta x_i = 0$, $i = 1,..., n$ or

$$\begin{cases} x_0(a - x_{-0}) = 2rK_0 \\ x_i (2 + (2w/K)) + x_{-i} = a, \qquad i = 1, \ldots, n. \end{cases} \tag{2}$$

We show now that if the above set has a solution, then $x_1 = ... = x_n$, i.e. production of all private firms is the same in Nash equilibrium.

Suppose $x = (1/n) x_{-0}$. Let's write down differently the set (2):

$$\begin{cases} x_0(a - nx) = 2rK_0 \\ x_i (1 + (2w/K)) + x_0 + nx = a, \qquad i = 1, \ldots, n. \end{cases} \tag{3}$$

If we add up the sides of the equation (3) for $i = 1,..., n$, then we obtain $nx (1 + (2w/K)) + nx_0 + n^2x = na$ or $x (1 + n + (2w/K)) + x_0 = a$. Therefore

$$x = m^{-1} (a - x_0), \text{ where } m = 1 + n + (2w/K). \tag{4}$$

From (3) and (4) we obtain

$$x_i (1 + (2w/K)) + x_0 + nm^{-1} (a - x_0) = a.$$

It turns out, that all x_i are the same ($i = 1,..., n$) therefore if private firms maximize profits, their production is the same in Nash equilibrium; the aim of a state-owned firm is not important here.

We can write down $x_i = x$ and reduce (2) to:

$$\begin{cases} x_0(a - nx) = 2rK_0 \\ x_0 + mx = a. \end{cases} \tag{5}$$

Reaction functions:

$$x_0 (x) = 2rK_0 (a - nx)^{-1}, \ x (x_0) = m^{-1} (a - x_0).$$

From (5) we obtain $(a - mx)(a - nx) = 2rK_0$ and

$$mnx^2 - a(m+n)x + a^2 - 2rK_0 = 0.$$

Discriminant $\Delta = a^2(m-n)^2 + 8mnrK_0 > 0$. Let $B = \Delta^{0,5}$. We have:

$$x_1 = (a(m+n) - B)/2mn,$$

$$x_2 = (a(m+n) + B)/2mn,$$

and respectively there are two solutions for x_0:

$$x_{01} = (B - a(m-n))/2n,$$

$$x_{02} = -(B + a(m-n))/2n.$$

Because $x_{02} < 0$, let's concentrate on the first pair of roots. It can be shown easily that $x_1 \geq 0$, if and only if, $a^2 \geq 2rK_0$ and always $x_{01} > 0$. Therefore Nash equilibrium is given by x_1, x_{01}.

Total differential of the set (2) looks as follows:

$$\begin{cases} (a - nx)dx_0 - x_0 ndx = -x_0 da + x_0 xdn + 2K_0 dr + 2r - dK_0 \\ dx_0 + mdx = da - xdn - (2x/K)dw + 2wxK^{-2}dK. \end{cases} \quad (6)$$

We can see that the main determinant

$$\det\begin{bmatrix} a - nx & -x_0 n \\ 1 & m \end{bmatrix} = (a - nx)m + x_0 n$$

is greater than zero. From the set (6) one can easily obtain the results of comparative statistics presented in theorem 1.

Proof of the theorem 2.

Because in Nash equilibrium production of private firms is identical (see proof of theorem 1) we shall write further that

$$x_1 = \ldots = x_n = x.$$

The *zero*th firm maximizes profits:

$$\pi_0 = (a - X)x_0 - wx_0^2 K_0^{-1} - rK_0.$$

$$NC: \delta\pi_0/\delta x_0 = -x_0 + a - X - (2wx_0/K_0) = 0.$$

$$SC: \delta^2\pi_0/\delta x^2 = -2 - (2w/K_0) < 0.$$

Function of reaction: $x_0(x) = (a - nx)(2 + (2w/K_0))^{-1}$.

Solving a set of linear equations:

$$\begin{cases} x_0((2w/K_0) + 2) + nx = a \\ x_0 + mx = a \end{cases}$$

we obtain Nash equilibrium. The main determinant $M = 2m\ (1 + wK_0^{-1}) - n$ is greater than zero and:

$$x_0 = a\ (1 + 2wK^{-1})\ /M;$$

$$x = a\ (1 + 2wK_0^{-1})\ /M.$$

We can find the total differential of the above set:

$$\begin{cases} ((2w/K_0) + 2)dx_0 + ndx = da - xdn - 2x_0K_0^{-1}dw + 2wx_0K_0^{-2}dK_0 \\ dx_0 + mdx = da - xdn - (2x/K)dw + 2wxK^{-2}dK. \end{cases}$$

From the above one can easily obtain the results of the comparative statistics presented in the theorem 2.

REFERENCES

Delbono F., Rossini G.: *Competition Policy vs. Horizontal Merger with Public, Entrepreneurial, and Labour-Managed Firms*. 'Journal of Comparative Economics' 1992, No 16.

Laidler D., Estrin S. (1991): *Introduction to Microeconomics*. Warsaw: Gebethner & Co.

Prasnikar J. et al. (1991): *A Test of Enterprise Behaviour under Yugoslav Labour--Management*. Department of Economics. University of Pittsburgh. Pittsburgh.