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TABLES OF CONTENTS

INAUGURAL LECTURE FOR OPENING THE ACADEMIC YEAR 1994/1995

<i>Bogusław Fiedor</i> ECOLOGICAL ASPECTS OF ECONOMIC RELATIONSHIPS BETWEEN POLAND AND EUROPEAN UNION	7
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I. ARTICLES

<i>Jerzy Rymarczyk</i> MODIFICATION OF PROTECTIVE INSTRUMENTS IN INTERNATIONAL TRADE AS A RESULT OF THE URUGUAY ROUND – GATT	19
--	----

<i>Stanisław Czaja, Bogusław Fiedor, Andrzej Graczyk</i> THE LINKAGES BETWEEN TRADE AND ENVIRONMENT. A CASE OF POLAND	29
---	----

<i>Jerzy Czupiał, Jolanta Żelezik</i> FOREIGN DIRECT INVESTMENT IN POLAND	59
--	----

<i>Bożena Klimczak, Bożena Borkowska, Andrzej Matysiak, Grażyna Wrzeszcz-Kamińska,</i> MICROECONOMIC PHENOMENA ACCOMPANYING THE PRIVATIZATION PROCESS OF STATE-OWNED ENTERPRISES (RESULTS OF RESEARCH OF 1990-1993)	67
--	----

<i>Aniela Styś</i> STRATEGIC MARKET PLANNING AND THE EFFECTIVENESS AND EFFICIENCY OF THE ORGANIZATION'S ACTIVITY	85
--	----

<i>Paweł Dittmann</i> SALES FORECASTING IN A TELECOMMUNICATION COMPANY	93
---	----

<i>Andrzej Baborski</i> ON SOME MORAL, LEGAL AND ECONOMIC PROBLEMS RELATED TO COMMUNICATION NETWORKS	101
--	-----

<i>Andrzej Malachowski, Elzbieta Niedzielska</i> NEW COMMUNICATION TECHNOLOGIES AS THE CHALLENGES FOR THE CONTEMPORARY CIVILISATION	113
<i>Adam Nowicki, Jacek Unold</i> COMPUTER REPRESENTATION OF THE INFORMATION SYSTEM FOR THE HOUSING SECTOR	123
<i>Bożena Baborska</i> THE FATE OF STATE OWNED FARMS IN POLAND	133
<i>Marian Kachniarz</i> AGROTOURISM AS AN ELEMENT OF RURAL AREAS DEVELOPMENT STRATEGY FOR THE SUDETY MOUNTAINS	143
<i>Ryszard Antoniewicz, Władysław Bukietyński, Andrzej Misztal</i> ON A JUST DISTRIBUTION WITH PREFERENCES	151

II. REVIEWS AND NOTES

Andrzej Baborski (ed.): EFEKTYWNE ZARZĄDZANIE A SZTUCZNA INTELIGENCJA [EFFECTIVE MANAGEMENT AND ARTIFICIAL INTELLIGENCE]. Wrocław 1994. (<i>Henryk Sroka</i>)	163
Zygmunt Bartosik, Bogumił Bernaś, Stefan Forlicz, Andrzej Kaleta: ZMIANY STRUKTURALNE W PRZEMYSŁE POLSKI – SPOJRZENIE PROSPEKTYWNE [STRUCTURAL CHANGES IN POLISH INDUSTRY – A PROSPECTIVE VIEW]. Wrocław 1994. (<i>Ryszard Broszkiewicz</i>)	164
Krzysztof Jajuga (ed.): EKONOMETRYCZNA ANALIZA PROBLEMÓW EKONOMICZNYCH [ECONOMETRIC ANALYSIS OF ECONOMIC PROBLEMS]. Wrocław 1994. (<i>Teodor Kulawczuk</i>)	165
Danuta Misińska: PODSTAWY RACHUNKOWOŚCI [THE ELEMENTS OF ACCOUNTING]. Warszawa 1994. (<i>Kazimierz Sawicki</i>)	167
Edward Nowak: DECYZYJNE RACHUNKI KOSZTÓW. (KALKULACJA MENEDŻERA) [DECISIONAL COST ACCOUNT (MANAGER'S CALCU- LATION)]. Warszawa 1994. (<i>Kazimierz Zajęc</i>)	168
Stanisław Nowosielski: PODSTAWY KONTROLINGU W ZARZĄDZANIU PRODUKCJĄ [THE ELEMENTS OF CONTROLLING IN PRODUCTION MANAGEMENT]. Wrocław 1994. (<i>Adam Starostecki</i>)	169

Stanisława Ostasiewicz, Wanda Ronka-Chmielowiec: RACHUNEK UBEZPIECZENIOWY [INSURANCE ACCOUNT]. Wrocław 1994. (<i>Tadeusz Stanisz</i>)	171
Andrzej Rapacz: PODSTAWY EKONOMIKI PRZEDSIĘBIORSTWA [THE ELEMENTS OF ECONOMY IN A TOURIST ENTERPRISE]. Wrocław 1994. (<i>Władysław Włodzimierz Gaworecki</i>)	173
Jerzy Sokołowski: STRATEGIA PODATKOWA PRZEDSIĘBIORSTWA. JAK ZMNIJSZYĆ OBCIĄŻENIA PODATKOWE [TAX STRATEGY OF AN ENTERPRISE. HOW TO DECREASE TAX BURDENS]. Warszawa 1994. (<i>Ryszard Wierzba</i>)	174
THE WROCLAW SCHOOL OF ECONOMIC POLICY IN MARKET ECONOMY (<i>Janusz Kroszel</i>)	175
Stefan Wrzosek: OCENA EFEKTYWNOŚCI RZECZOWYCH INWESTYCJI PRZEDSIĘBIORSTW [THE APPRAISAL OF THE EFFECTIVNESS OF MATERIAL INVESTMENT OF ENTERPRISES]. Wrocław 1994. (<i>Lesław Martan</i>)	177
III. HABILITATION MONOGRAPHS 1994-1995 (summaries)	179
IV. LIST OF PUBLICATIONS BY THE ACADEMIC STAFF OF THE WROCLAW ACADEMY OF ECONOMICS 1994-1995	183

Ryszard Antoniewicz, Władysław Bukietyński, Andrzej Misztal

ON A JUST DISTRIBUTION WITH PREFERENCES

The paper „On a just distribution with preferences” presents a method for the just distribution of a certain whole into a finite number of elements. It is assumed that just distribution without preferences is a distribution into n equal elements. The just distribution with preferences is such a distribution fulfilling given conditions (called preferences) which least deviates from the uniform distribution. The general solution to the problem is illustrated with selected examples.

1. INTRODUCTION

One of the main problems in economy, both on a micro and on a macro scale, is the distribution of material goods among particular economic agents. Let us assume that the value of those goods can be expressed in money. Thus, we shall deal with a distribution of a certain amount of money. On a macro scale it refers for example to the distribution of the national income among particular sectors of the national economy, i.e. industry, consumption, agriculture, transportation or trade. On a micro scale of a single household it refers for example to the construction of a family budget. Without any additional assumptions, a just distribution would give everyone the same amount, regardless of the size or importance of the given sector. Usually however, we deal with the situation when some sectors are preferred to others. It manifests itself usually in such a way that one sector should receive some multiple of the amount given to another one.

2. THE GENERAL STATEMENT OF THE PROBLEM. PRINCIPLE OF THE MINIMUM

Without loss of generality we can assume that the amount of a good to be distributed is equal to 1. A distribution will be called any vector $p = (p_1, p_2, \dots, p_n)$ with positive components for which the following equality is true:

$$\sum_{i=1}^n p_i = 1 \quad (1)$$

Preferences will be called each one of $k-1$ equations:

$$W_i(p) = 0 \quad \text{dla } i = 2, \dots, k \quad (2)$$

Let us assume that the just distribution without preferences is a uniform distribution, i.e.:

$$s = \left(\frac{1}{n}, \dots, \frac{1}{n}\right)$$

Other distributions deviate more or less from the just distribution. Now a question arises as to how a deviation between two distributions p and q should be measured. Since we are in R^n space, we may take the Euclidean distance or any other equivalent metric. However, intuition advises not to do so, because in this way we do not take into account the proportions between the respective components of p and q . For example, the distributions $p=(2\varepsilon, 1/2-\varepsilon, 1/2-\varepsilon)$ and $q=(20\varepsilon, 1/2-10\varepsilon, 1/2-10\varepsilon)$ which lie close to each other in the sense of the Euclidean metric for small ε , should be quite distant, as q_1 received 10 times more than p_1 did. Therefore, the following formula seems

to be more appropriate: $d(p, q) = \sum_{i=1}^n \left(\frac{p_i - q_i}{p_i} \right)^2$.

On the other hand though, one component should not decide alone on the deviation, thus let us assign proper weights to all the components. This results in the formula for deviation:

$\Delta(p, q) = \sum_{i=1}^n p_i \left(\frac{p_i - q_i}{p_i} \right)^2$, and after simplifying (Antoniewicz 1975):

$$\Delta(p, q) = \sum_{i=1}^n p_i^{-1} (p_i - q_i)^2 \quad (3)$$

It is easy to observe that the following conditions are satisfied:

$$\begin{aligned} \Delta(p, q) &\geq 0 \quad \text{for } p, q \in R_+^n, \\ \Delta(p, q) &= 0 \iff p = q \end{aligned}$$

Definition: A just distribution with preferences will be called a distribution \mathbf{p} satisfying conditions (1) and (2), which is closest to a uniform distribution in the sense of the above given deviation (Antoniewicz, Misztal 1993).

We should therefore seek a constrained minimum of the function of variable \mathbf{p} : $\Delta(p, s) = \sum_{i=1}^n p_i^{-1} (p_i - n^{-1})^2$. Since,

$$\Delta(p, s) = \sum_{i=1}^n (p_i - 2n^{-1} + n^{-2} p_i^{-1}) = \sum_{i=1}^n p_i - 2 + n^{-2} \sum_{i=1}^n p_i^{-1} = n^{-2} \sum_{i=1}^n p_i^{-1} - 1$$

hence it is equivalent to examine the function $f(p) = \sum_{i=1}^n p_i^{-1}$.

Generally, when nonlinear preferences are allowed, the problem becomes very complex and one may only hope to find a solution by means of approximate methods. The case when the constraints are given as the linear equations seems to be important, both theoretically and in practical terms, because the preferences applied in practice are mainly expressed as linear relationships. We will not consider a nonlinear case.

3. A SOLUTION TO A LINEAR CASE

Lagrange's method of indefinite multipliers

The additional conditions (linear preferences) can be written in the form:

$$a_{11}p_1 + \dots + a_{1n}p_n = b_1, \dots, a_{k1}p_1 + \dots + a_{kn}p_n = b_k \quad \text{or in short:}$$

$\mathbf{A}\mathbf{p} = \mathbf{b}$, where $\mathbf{A} = (a_{ij})$ is a $(k \times n)$ -matrix and $\mathbf{b} = (b_i)$ is a vector of k components. We form the auxiliary function:

$$F(p, \lambda_1, \dots, \lambda_k) = f(p) + \sum_{j=1}^k [\lambda_j \cdot (\sum_{i=1}^n a_{ji} p_i - b_j)]$$

and we equate to zero the partial derivatives: $\frac{\partial F}{\partial p_i} = -p_i^{-2} + \sum_{j=1}^k \lambda_j a_{ji} = 0$.

Then we have a formula:

$$p_i = \left(\sum_{j=1}^k \lambda_j a_{ji} \right)^{-1/2} = \frac{1}{\sqrt{\lambda_1 a_{1i} + \dots + \lambda_k a_{ki}}} \quad (4)$$

The multipliers $\lambda_1, \dots, \lambda_k$ will be found by substituting the values of p_i calculated above, to the equation $\mathbf{A}p = \mathbf{b}$. Let $W_i(p) := \sum_{i=1}^n p_i - 1$ and the conditions (1), (2) be satisfied. Then, by (1), (2) and (4) we obtain a system of equations:

$$W_i(p_1(\lambda_1, \dots, \lambda_k), \dots, p_n(\lambda_1, \dots, \lambda_k)) = 0 \quad \text{for } i = 1, 2, \dots, k$$

or in short:

$$V_i(\lambda_1, \dots, \lambda_k) = 0 \quad \text{for } i = 1, 2, \dots, k$$

This is a nonlinear system of equations whose solution is generally obtainable using approximate methods only. Note: The solution (4) is really a minimum (if it exists, of course).

In order to prove it, we evaluate the second derivative:

$$\frac{\partial^2 F}{\partial p_i \partial p_j} = 2p_i^{-3} \delta_{ij}, \quad \text{where } \delta_{ij} = \begin{cases} 1 & \text{for } i = j \\ 0 & \text{for } i \neq j \end{cases}$$

It turns out that F'' is positive definite, thus completing the proof.

A reduction to the seeking of the ordinary minimum

Let us recall that the additional conditions form a linear system $\mathbf{A}p = \mathbf{b}$. If it is an indeterminate system of independent equations (it makes sense to consider independent equations only), then we can assume that the parameters are the variables p_1, p_2, \dots, p_m for $m = n - k$. After solving the system $\mathbf{A}p = \mathbf{b}$ we determine a new system:

$$p_{m+i} = \sum_{j=1}^m d_{ij} p_j + e_i \quad \text{for } i = 1, 2, \dots, k. \quad (5)$$

Hence, we obtain:

$$g(p_1, p_2, \dots, p_m) = f(p_1, p_2, \dots, p_n) = \sum_{i=1}^m p_i^{-1} + \sum_{i=1}^k \left(\sum_{j=1}^m d_{ij} p_j + e_i \right)^{-1}$$

By equating to zero the partial derivatives of the function g and multiplying the equation obtained by (-1) , we obtain the formula for $s = 1, 2, \dots, m$:

$$p_s^{-2} + \sum_{i=1}^k d_{is} \left(\sum_{j=1}^m d_{ij} p_j + e_i \right)^{-2} = 0 \quad (6)$$

The solving of the above system gives the desired distribution p . It is worth observing that by multiplying the formula (6) by $(p_s p_{m+1} p_{m+2} \dots p_n)^2$ and using (5), we obtain the system of algebraic equations of degree $2k$ in variables p_1, \dots, p_m .

4. EXAMPLES

Example 1. Let us assume that the management of a certain enterprise want to grant a special bonus to four of their workers: A_1, A_2, A_3, A_4 . The total amounts to Zl 1800. Also, the worker A_4 , who had been very helpful to A_1 , should receive fourfold as much as A_1 , with his consent.

Solution: The unit here is 1800, therefore the results obtained should be multiplied by this number. By assumption we have $p_4 = 4p_1$ and $p_1 + p_2 + p_3 + p_4 = 1$, thus yielding the system: (5) $p_3 = 1 - 5p_1 - p_2, p_4 = 4p_1$. Besides, $m = k = 2$. Hence, (6) follows: $p_1^{-2} - 5(1 - 5p_1 - p_2)^{-2} + 0.25p_1^{-2} = 0, p_2^{-2} = (1 - 5p_1 - p_2)^{-2}$. The results are: $p_1 = \frac{1}{9}, p_2 = p_3 = \frac{2}{9}, p_4 = \frac{4}{9}$, i.e. the respective amounts are: 200, 400, 400, 800.

Example 2. A five-person limited company at the end of the business year made a Zl 1 mn profit and decided to divide a sum of Zl 300,000. How to divide the money among the partners if it was agreed that the president of the company should receive as much as the two least rewarded shareholders put together.

Solution: The system (5): $p_4 = 1 - 2p_1 - 2p_2 - p_3, p_5 = p_1 + p_2$ and (6): $p_1^{-2} - 2p_4^{-2} + p_5^{-2} = 0, p_2^{-2} - 2p_4^{-2} + p_5^{-2} = 0, p_3^{-2} - p_4^{-2} = 0$.

Subtracting the two first equations from each other results in $p_1 = p_2$ and the latter one gives: $p_4 = p_3$. Thus, it follows from (5) that $p_4 = 1 - 4p_1 - p_3$ and $p_5 = 2p_1$, yielding $p_1^{-2} + (2p_1)^{-2} = 2(0.5 - 2p_1)^{-2}$. Finally, $p_1 = p_2 = (5 - \sqrt{10})/12, p_3 = p_4 = (\sqrt{10} - 2)/6$ and

$\rho_5 = (5 - \sqrt{10})/6$. Approximately, we obtain the amounts of money: Zł 91,886 for the president, for each of the two next shareholders Zł 58,114; and Zł 45,943 for each of the latter ones.

5. AN APPROXIMATE METHOD

In case of more complex systems of equations than those in Examples 1 and 2, an approximate method for solving nonlinear systems was applied. We used the modified Newton's method (Dennis, Schnabel 1983), which is roughly described below.

Given:

$$F \in C^1(R^n, R^n)$$

Find:

$$x_* \in R^n : F(x_*) = \bar{0}$$

(We seek a root of a function with a continuous derivative.)

Solution: Choose a starting point x_0 . In the k -th iteration:

1. Evaluate $F(x_k)$ and decide whether to stop the algorithm (in case when $\|F(x_k)\|_2 < \varepsilon$, where ε is an acceptable error) or else to continue.
2. Find $J(x_k)$ (the Jacobian of F at the point x_k).
3. Evaluate s_k from the equation $J(x_k) \cdot s_k = -F(x_k)$.
4. Decide whether to perform the step of the ordinary Newton's algorithm $x_{k+1} = x_k + s_k$ or else to apply the method of the steepest descent for the function $f(x) = 0.5\|F(x)\|_2^2$ with a step $x_{k+1} = x_k - \lambda_k \cdot g$, where $g = f'(x_k)$, $\lambda_k = \frac{f(x_k)}{g^T \cdot g}$.

The algorithm is ever convergent, but sometimes x_k may be convergent to x (a local minimum of function $f(x)$), if x_0 (starting point) lie near x . Then we must change x_0 .

Let us recall that the desired solution of the equation system (5), (6) can be represented as the system of $(m+k)$ equations:

$$p_1^{-2} + \sum_{i=1}^k d_{i1} p_{m+i}^{-2} = 0 \tag{R-1}$$

$$p_m^{-2} + \sum_{i=1}^k d_{im} p_{m+i}^{-2} = 0 \tag{R-m}$$

$$p_{m+1} - \sum_{j=1}^m d_{1j} p_j - e_1 = 0 \tag{R-m+1}$$

$$p_{m+k} - \sum_{j=1}^m d_{kj} p_j - e_k = 0 \tag{R-m+k}$$

It follows from the assumption that the condition $\sum_{i=1}^n p_i = 1$ underlies the equations (R-m+1), ..., (R-m+k) (note that $n = m + k$), hence for all $j = 1, 2, \dots, m$ we have:

$$\sum_{i=1}^k d_{ij} = -1 \text{ and } \sum_{i=1}^k e_i = 1$$

Denoting the left-hand sides of the equations (R-1),..., (R-m+k) by F_1, \dots, F_n and the vector function (F_1, \dots, F_n) by F we can observe that we seek the solution of the equation $F(x) = \bar{0}$, where $F : R^n \rightarrow R^n$. Furthermore, the function F is of class C^1 and its Jacobian has a particularly simple form:

$$J = \begin{bmatrix} -2p_1^{-3} & \dots & 0 & -2d_{11}p_{m+1}^{-3} & \dots & -2d_{k1}p_{m+k}^{-3} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & -2p_m^{-3} & -2d_{1m}p_{m+1}^{-3} & \dots & -2d_{km}p_{m+k}^{-3} \\ -d_{11} & \dots & -d_{1m} & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ -d_{k1} & \dots & -d_{km} & 0 & \dots & 1 \end{bmatrix}$$

The authors have executed the calculations using a TurboPascal program they wrote themselves. The input data follow: the matrix (d_{ij}) , the vector (e_i) and the starting point x_0 (it would be well to substitute $x_0 = (\frac{1}{n}, \dots, \frac{1}{n})$). Thus the authors reduce the after-mentioned examples to the form (5).

6. EXAMPLES FOR THE APPROXIMATE METHOD

Example 3. The Abacki family's expenditure excluding constant items (housing, fuel, light) equals Zł 1000. They wish to divide the money into five parts: food, clothing, bank savings, leisure, and other goods and services. The family agreed on the following preferences:

- expenditure on food, clothing and bank savings should amount to three times the other expenditure,

- expenditure on savings should amount to half the spending on food.

They decided to divide the money as close to the equal distribution as possible.

Solution: We have $p_1 + p_2 + p_3 = 3(p_4 + p_5)$, $p_3 = 0.5p_1$. By (1) and transforming it, we obtain the system: $p_2 = -1.5p_1 + 0.75$, $p_3 = 0.5p_1$, $p_5 = -p_4 + 0.25$. The approximate method gives the results (after rounding up): Zł 340, Zł 240, Zł 170, Zł 125, Zł 125.

Example 4. Four friends started a limited liability company. After several years however, the partners decided to stop the business and liquidated the property. The amount of money obtained is equal to Zł 10 million. How to divide the money if Mr C spent twice as much time working for the company as Mr D did. The latter believes that his time is equal to one third of the total time of Mr A and Mr C. Mr B relies fully on his partners estimates.

Solution: We have: $p_3 = 2p_4$, $p_4 = \frac{p_1 + p_3}{3}$. Considering (1) and transforming we obtain: $p_2 = 1 - 4p_1$, $p_3 = 2p_1$, $p_4 = p_1$. The approximate methods gives the results: Zł 1,899,367; Zł 2,402,531; Zł 3,798,735; Zł 1,899,367.

Example 5. An eccentric millionaire willed his house to his favourite cat. Cash of \$ 1 bn was to be divided among five closest persons according to the following rules: his beloved wife Mary should receive half the total amount for all the children. Uncle Henry should receive a quarter of the amount for both sons: the elder John and the younger Mark. Daughter Agatha should receive 80 percent of the amount for Mark. Also, the distribution should be as close to the equal one as possible.

Solution: Let p_1, p_2, p_3, p_4, p_5 be the shares of the property appointed to Mark, John, Agatha, Mary and Henry, respectively. Therefore,

$p_4 = \frac{p_1 + p_2 + p_3}{2}$, $p_5 = \frac{p_1 + p_2}{4}$, $p_3 = 0.8p_1$. After transforming:

$p_2 = -\frac{59}{35}p_1 + \frac{4}{7}$, $p_3 = \frac{4}{5}p_1$, $p_4 = \frac{2}{35}p_1 + \frac{2}{7}$, $p_5 = -\frac{6}{35}p_1 + \frac{1}{7}$. Finally,

the heirs get: \$ 210,912,140; \$ 215,890,960; \$ 168,729,710; \$ 297,766,410; and \$ 106,700,780.

Example 6. During the privatization of the POLFAB company and its transformation into a joint stock company, it was agreed that 10 percent of all shares will be given free to the workers. They were divided into four groups depending on number of years worked: group I: 0-5 years, group II: 5-15 years, group III: 15-25 years, group IV: over 25 years. It turned out that the numbers of workers is the same in each group. How to divide the shares if workers of group IV should receive as many shares as workers of group II and III together, and workers of group I - two thirds of the number of shares for workers in group III.

Solution: We have: $p_4 = p_2 + p_3$, $p_1 = \frac{2}{3}p_3$. Hence, $p_3 = -2p_1 + 0.5$, $p_3 = 1.5p_1$, $p_4 = -0.5p_1 + 0.5$. Finally, the percentage distribution follows: group I - 16%, group II - 18%, group III - 24%, group IV - 42%.

Example 7. A wealthy and provident man has planned the following investments for the next year: bank deposit, foreign currency, government bonds, gold and stocks. His reason told him to engage in stocks only 30 percent of money put on less risky bank deposits and bonds combined. As the precious metals market was not favourable at the moment, he decided to spend on gold 40 percent of money placed in foreign currency. What should the percentage distribution of the investments planned look like?

Solution: Let p_1, p_2, p_3, p_4, p_5 denote the shares of money invested in bank, bonds, currency, gold and stocks. Then, $p_5 = 0.3(p_1 + p_2)$, $p_4 = 0.4p_3$. Hence, $p_3 = -\frac{13}{14}p_1 - \frac{13}{14}p_2 + \frac{5}{7}$, $p_4 = -\frac{13}{35}p_1 - \frac{13}{35}p_2 + \frac{2}{7}$, $p_5 = \frac{3}{10}p_1 + \frac{3}{10}p_2$. The final distribution follows: bank deposit - 22.4%, bonds - 22.4%, foreign currencies - 29.8%, gold - 11.9%, stocks - 13.5%.

Example 8. A country of MONEYLAND decided to aid six under-developed countries: ALAND, BLAND, CLAND, DLAND, ELAND and FLAND. The three latter countries were (under-)developed to a similar degree. The main difference was that DLAND had twice as much population as ELAND, whereas ELAND had 1.5 times more population than FLAND. Thus it was justified to give DLAND twice as much ELAND and 2×1.5 as much as FLAND. It is also known that ALAND's and BLAND's population together equal that of CLAND. How to divide the money?

Solution: Let $p_1, p_2, p_3, p_4, p_5, p_6$ denote the shares for ALAND, FLAND, BLAND, CLAND, DLAND, ELAND. We have: $p_5 = 2p_6$, $p_6 = 1.5p_2$,

$p_4 = p_1 + p_3$. After transforming: $p_3 = -p_1 - 2.75p_2 + 0.5$,
 $p_4 = -2.75p_2 + 0.5$, $p_5 = 3p_2$, $p_6 = 1.5p_2$. The final distribution follows:
 ALAND - 12,2%, BLAND - 12,2%, CLAND - 24,4%, DLAND - 27,9%,
 ELAND - 14%, FLAND - 9,3%.

Example 9. Mutual Fund POWER's portfolio had the following structure on December 31, 1994: stocks - 26%, bonds - 6%, TBs - 65%, foreign securities - 3%. Due to the booming market for stocks and bonds, the Fund decided to change the structure of investments engaging new means. There was some reasoning in favour of placing six times more money in treasury bills than in bonds (it used to be approximately 11 times more) and foreign securities with 1 percent of bonds should amount to 1 percent of the entire portfolio.

Solution: Let p_1, p_2, p_3, p_4 denote the shares of money invested in bonds, foreign papers, bills and stocks, respectively. Then we have: $p_3 = 6p_1$,
 $p_2 + 0.01p_1 = 0.01$. Therefore, $p_2 = -0.01p_1 + 0.01$, $p_3 = 6p_1$,
 $p_4 = -6.99p_1 + 0.99$. Finally, distribution follows: stocks - 39.9%, bonds - 8.5%, bills - 50,7%, foreign papers - 0.9%. The results obtained are close to those reported for the portfolio of the Mutual Fund Pioneer I in Poland on June 30, 1995 (A.S. 1995).

Example 10. The management of the SPOLKA company decided to issue 10 million shares for a closed list of their five biggest shareholders - banks A, B, C, D, E. At first, SPOLKA was to offer each bank 2 million shares. It turned out however, that bank C was willing to buy at least twice as many shares as bank A was to buy. Bank B and E, in turn, wanted to have together as many shares as A, C and D together.

Solution: Let p_1, p_2, p_3, p_4, p_5 denote the shares for respective banks A, B, C, D, E. We have then: $p_3 = 2p_1$, $p_2 + p_5 = p_1 + p_3 + p_4$ and after transforming: $p_3 = 2p_1$, $p_4 = -3p_1 + 0.5$, $p_5 = -p_2 + 0.5$. Finally, banks A, B, C, D, E will receive 1,132,705; 2,500,000; 2,265,409; 1,601,886; 2,500,000 shares, respectively.

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