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MODELLING EXTREME MARKET RISK OF POLISH BANKS' DEBT INSTRUMENTS' PORTFOLIOS

Abstract: The main goal of this article is to present extreme market risk evaluation methods which go beyond the standard Value at Risk methodology. Two main approaches: Expected Tail Loss (ETL) and Extreme Value Theory (EVT) are presented and then applied to simulate interest risk stemming from government debt portfolio held by Polish banks. The two methods seem to be very useful to estimate real market risk exposures during the times of distress on the financial markets.

Keywords: market risk, Value at Risk, Expected Tail Loss, Extreme Value Theory.

1. Introduction

Market risk is one of the main classes of risk to which financial institutions are exposed. In the case of investment banks, which hold large trading books, this kind of risk can be perceived as a substantial one, in common with counterparty (credit) and liquidity risk. In the Polish banking sector dominated by universal banks, market risk is not perceived as a main threat to sector stability. Besides, Polish banks' trading books are packed mainly with local government and central bank debt instruments, which makes this risk relatively easier to hedge and manage. However it should be remembered that this market risk exposure still constitutes a big fraction of local banks assets and can influence their profitability, especially in cases of extreme turbulence on financial markets.

The main goal of this article is to present extreme market risk evaluation methods which go beyond the standard Value at Risk methodology. Two main approaches: Expected Tail Loss (ETL) and Extreme Value Theory (EVT) are presented and then applied to simulate interest risk stemming from debt portfolio held by Polish banks.

The rest of the article is organized as follows. In the first part, literature of extreme market risk evaluation is discussed. In the next section, general market risk definition is presented. The third section consists of Value at Risk, Expected Tail Loss and Extreme Value Theory background description. The empirical part of the

article is devoted to a presentation of the surveyed portfolio which was constructed to mimic the general structure of the Polish banks' trading book portfolio. After that, a detailed analysis of the simulation exercise used to achieve extreme market risk measures was performed. The last part was reserved for conclusions.

2. Literature overview

A definition of market risk and a general description of its measures computation can be found in the series of Jajuga books [2000; 2007a; 2007b], in Hull's book [2007] and in the informative materials of the Bank of International Settlements [BIS 2003]. The standard method of market risk evaluation, value at risk method (and its estimations), is broadly discussed in the articles of Duffie and Pan [1997], Holton [2003], Dowd [2005] and Danielsson [2011]. The alternative methods of the VaR estimation use, including the historical approach, are presented in the works of Jorion [2007], Hull and White [1998] and Pritsker [2006]. The simulation approach to VaR modelling is explained in detail in the works of Glasserman [2004] and Mentel [2012].

Going beyond the standard VaR approach of market risk modeling, the theoretical background of Expected Tail Loss and Extreme Value theory can be found in the book of Embrechts, Kuppelberg and Mikosch [1997]. The articles of McNeil and Frey [2000] and the work of Longin [2005] have a more practical character. A useful survey of the EVT method of evaluation effectiveness of market risk in the emerging markets was done by Gencay and Seluck [2004]. Recent developments in ETL and EVT, with special attention to the consequences of the last financial crisis, can be found in Mapa and Suaiso [2009] and Ze-To [2012].

3. Definition of market risk

Market risk is defined in the literature as a risk generated with financial instruments' price changes on the primary or derivative markets [Jajuga 2007] . BIS [BIS 2003] defines market risk as "the risk of losses in on- and off-balance sheet positions arising from movements in market prices". Financial instruments held as assets by financial institutions, are prone to distress on financial markets due to mark-to-market mechanism. Taking into account the types of instruments which are traded on financial markets, four main types of market risk can be distinguished:

1. Interest rate risk.
2. FX risk.
3. Stock price risk.
4. Commodity price risk.

The Polish banks' traditional business model makes them prone mainly to interest rate risk. According to Jajuga [2007] this type of risk can be determined as a risk

stemming from changes of assets or liabilities prices due to market interest rates' fluctuations. In the empirical part of the survey, debt instruments' portfolio mimicking Polish banks expositions will be built to compare different Value at Risk measures.

4. Market risk measures: Value at Risk, Expected Tail Loss and distributions from Extreme Value Theory

4.1. Value at Risk

Value at Risk (VaR) of a financial instruments portfolio is defined as the maximum value of the loss, which potentially can be taken by the owner in a certain time horizon with a certain confidence level (responsible to certain risk distributions quantile). From a statistical point of view, VaR can be defined as a quantile risk measure. For a given continuous distribution of stochastic variable R representing return from the mentioned portfolio and quantile $q : q \in [0,1]$ VaR is a certain value such as:

$$P\left(R_t < VaR_{t,q}\right) = q\,. \tag{1}$$

Hence if for quantile $q = 5\%$ VaR of a portfolio of assets' returns in the perspective of 10 days is reported to be –3 PLN million, it can be assumed that the maximal loss from this portfolio in a given horizon with probability 0.95 should not exceed the mentioned VaR value. In cases when the functional form of the portfolio's total return distribution is known ($F(\cdot)$), the VaR of the portfolio for a quantile q at a time t is given as:

$$VaR_{t,q} = F^{-1}(q)\,. \tag{2}$$

To compute the VaR measure of market risk in practice, very strict assumptions about variable R and the structure of the analysed portfolio are usually considered:

1. Variable R should be identically and independently distributed across the time (i.i.d.).
2. Variable R should have Gaussian distribution.
3. Weights of the portfolio's components should be adjusted for changes of the instruments' value changes (dynamic VaR) to achieve comparable risk measures.
4. For scaling VaR from one time horizon to another one rule of square root should be applied.

If the first two assumptions are fulfilled ($R \overset{i.i.d.}{\sim} N(m,s)$), the VaR of the portfolio at time t can be computed according to the formula:

$$VaR_{t,q} = \sigma\Phi^{-1}(1-q) + \mu, \tag{3}$$

where $\Phi(\cdot)$ is Gaussian distribution.

The fourth assumption is derived from the first three ones. This defines the way in which the time perspective of VaR can be rescaled. If the portfolios' returns are i.i.d. Gaussian and portfolio's components' weights are constant across the time (dynamic portfolio) the 1-period VaR can be scaled to h-period VaR with the formula:

$$VaR_{t,q} = \sqrt{h}\sigma\Phi^{-1}(1-q) + h * \mu. \quad (4)$$

Breaking the described assumptions leads usually to achieving unreliable VaR market risk evaluation. Moreover in times of serious distress on financial markets, extreme changes of financial instruments prices can happen. They can be interpreted as phenomena from tail risk distribution. Such risk events are not caught by VaR measures. If a risk exceeds the VaR threshold value, very little can be said about what real risk value is expected to materialize. Hence additional risk measurement methods, which are able to analyse extreme events breaking the VaR assumption and materializing beside the VaR threshold are needed. One of the possible methods which can be applied in such cases is Expected Tail Loss (ETL), known also as Expected Shortfall (ES), another one is the Extreme Value Theory (EVT) approach allowing to model extreme tails of market risks.

4.2. Expected Tail Loss

One of the possible solutions to the lack of adequacy of the VaR measures in the case of extreme values occurrence is to report the tail distribution of portfolio's returns. Such a possibility is supported by the Extreme Value Theory method, which will be discussed in the following chapters of the article. However chief risk officers (CRO) and policy makers in commercial banks and supervisory institutions, being accustomed to the single "standard" risk measure reported with VaR, usually demand an analogous single risk measure for extreme market risk to open the possibility of comparing both measures. In such cases, the Expected Tail Loss (ETL) approach (known also as Expected Shortfall /ES/, or distribution tail VaR, Tail VaR) is applied.

ETL is defined as the conditional expected loss from the portfolio measured at time t+1 determined at time t for q-quantle VaR:

$$ETL_{t+1,k} = E_t\left[R_{t+1} \middle| R_{t+1} < VaR_{t+1,q}\right]. \quad (5)$$

This formula can be expanded into:

$$E_t\left[R_{t+1} \middle| R_{t+1} < VaR_{t+1,q}\right] = \frac{1}{f\left(VaR_{t+1,q}\right)} \int_{F(-\infty)}^{F\left(VaR_{t+1,q}\right)} F^{-1}(q)d(q)dq\,. \quad (6)$$

The ETL measure informs us what is the most typical value of the portfolio's loss when the value of this loss has exceeded q-quantile VaR. For a Gaussian distribution of the portfolio's returns (with expected value standardized to 0), ETL can be computed as:

$$ETL_{t+1,q} = \sigma * \frac{\phi^{-1}\left(^{VaR_{t+1,q}}/_{\sigma}\right)}{\Phi^{-1}\left(^{VaR_{t+1,q}}/_{\sigma}\right)} \tag{7}$$

where $\Phi(\cdot)$ is as before the distribution function of normal distribution and $\phi(\cdot)$ is the distribution density function.

Applying the VaR definition described with equation (3), the ETL for $\mu = 0$ can be written as:

$$ETL_{t+1,q} = \sigma * \frac{\phi^{-1}\left(\Phi^{-1}(1-q)\right)}{q}. \tag{8}$$

The last formula is often applied to compute the ETL/VaR ratio with allows us to compare easily these two market risk measures

$$\frac{ETL_{t+1,q}}{VaR_{t+1,q}} = \frac{\phi^{-1}\left(\Phi^{-1}(1-q)\right)}{k\Phi^{-1}(1-q)}. \tag{9}$$

In the case of Gaussian distribution, if q converges to zero, the values of ETL/VaR ratio converge to one. For fat tail portfolio's distribution, the convergence is however not observed.

4.3. Extreme Value Theory

The second approach to examine extreme market risk characteristics is to model whole tails of portfolio's returns characteristics. In such a situation the Extreme Value Theory (EVT) is applied. According to this theory, a broad spectrum of empirical market risk distributions can be modelled with generalized Pareto distribution. Relaxing the assumption about the normal distribution of the risk EVT still keeps restrictions saying that the portfolio's returns need to be i.i.d. Confronting this requirement with the empirical portfolio's returns characteristics output of GARCH model is used in empirical applications. Analysts should be aware that this trick does not eliminate the whole problem connected with the observation's variance grouping.

Extreme values of the analysed distribution can be modelled according to the Extreme Value Theory with Peak Over the Threshold (POT) method. Let us assume that the time series (with N observations) of certain portfolio's standardized returns (taken from the GARCH model) is available to analysis ($\bar{R}$). The first step is to select (and then consider in further analysis) only those NTR observations which exceed the (arbitral) selected threshold ($\bar{R} > TR$). Then for a determined group of observations distribution of probability that difference between standardised returns and mentioned threshold is smaller than x, assuming that this standardized return exceeded the threshold value, is determined as:

$$F_{TR}(x) \equiv P(\bar{R} - TR \leq x | \bar{R} > TR). \tag{10}$$

Using the conditional probability theorem, the distribution above can be written as a function of the portfolio's standardized distribution $F(\cdot)$:

$$F_{TR}(x) = \frac{F(x + TR) - F(TR)}{1 - F(TR)}. \tag{11}$$

According to the Extreme Value Theory for the majority of standardized returns' distribution, for a properly set TR value, distribution $F_{TR}(x)$ converges to Generalized Pareto distribution $GP(x; \alpha, \beta)$. GP distribution can be described with the formula:

$$GP(x; \alpha, \beta) = \begin{cases} 1 - (1 + \alpha \frac{x}{\beta})^{-\frac{1}{\alpha}} \; if \; \alpha \neq 0 \\ 1 - e^{-\frac{x}{\beta}} \; if \; \alpha = 0 \end{cases} \tag{12}$$

for $\beta > 0$ and having in mind that $\begin{cases} x \geq 0 \; for \; \alpha \geq 0 \\ TR \leq x \leq TR - \frac{\beta}{\alpha} \; other \; case \end{cases}$. Fitting proper GP distribution to empirical data is performed with α parameter (shape parameter). Parameter α is estimated with optimization procedures applied to variable $Y = \bar{R} - TR$ given $\bar{R} - TR > 0$. This procedure often uses the Maximum Likehood Estimation (MLE) approach. In such cases the optimized function is given with the expression:

$$\max_{\beta > 0, \frac{\alpha(\bar{R} - TR)}{\beta} > 0} \begin{cases} -NTRln(\beta) - \left(\frac{1}{\alpha} + 1\right) \sum_{i=1}^{NTR} \left(1 + \frac{\alpha}{\beta}(\bar{R}_i - TR)\right) \; if \; \alpha \neq 0 \\ -NTRln(\beta) - \frac{1}{\alpha} \sum_{i=1}^{NTR} (\bar{R}_i - TR) \; if \; \alpha \neq 0 \; other \; case \end{cases}. \tag{13}$$

Another way to obtain estimators of shape parameter is to use the Hill tail index. The estimator is computed as:

$$\hat{\alpha} = \frac{1}{NTR} \sum_{i=0}^{NTR-1} (\log(X_{N-i+1}) - log(X_{N-NTR})). \tag{14}$$

A crucial issue of the POT estimation of the portfolio's return loss is the selection of proper threshold value (TR). The optimal choice of TR is a trade-off between the estimated distribution bias and its variability. If the TR value is too small, too many observations are selected as extreme and the application of ETV loses its justification. In the other case, if the value of TR is too high, a very small sample of observations enters the analysis and the consistency of the estimator can be threatened. Very often

the rule of thumb is applied, which states that for a sample of 250 returns, the value of TR should be close to 95 percentile of empirical distribution of these returns.

An alternative method of distribution's tail estimation is the Block Maxima Method (BMM). According to this method, N maximum values of standardized portfolio's returns are recursively selected on the growing sample:

$$\begin{aligned} M_1 &= \bar{R}_1 \\ M_2 &= \max\{\bar{R}_1, \bar{R}_2\} \\ &\dots \\ M_n &= \max\{\bar{R}_1, \bar{R}_2, \dots, \bar{R}_n\}. \end{aligned} \tag{15}$$

The Fisher and Tippett and Gnedenko theorem introduces for sequence of sample maxima (M_n) non-degenerate limit distribution H. For $n \to \infty$, distribution of M_n converges to H

$$\frac{M_n - d_n}{c_n} \xrightarrow{d} H \tag{16}$$

where $(c_n), (d_n) \in \mathbb{R}$ are sequences of constant corresponding to values of M_n.

The limit distribution function can take the form of one of the three following functions:

1) Frechet: $\Phi_\alpha(x) = \begin{cases} 0 \; if \; x \le 0 \\ e^{(-x)^{-\alpha}} \; if x > 0 \end{cases}$ for $\alpha > 0$

2) Weibull: $\Psi_\alpha(x) = \begin{cases} e^{-(-x)^{\alpha}} \; if \; x \le 0 \\ 1 \; if x > 0 \end{cases}$ for $\alpha > 0$

3) Gumbell: $\Lambda(x) = e^{-e^{-x}}$.

A unified one-parameter representation of the three above distribution functions can be written as the Generalized Extreme Value distribution (GEV):

$$H_\xi(x) = \begin{bmatrix} e^{-(1-\xi x)^{-\frac{1}{\xi}}} if \; \xi \neq 0 \\ e^{-e^{-x}} \; if \; \xi = 0 \end{bmatrix} \tag{17}$$

with x such that $1 + \xi x > 0$. Frechet distribution can be achieved from GEV by setting $\xi = \alpha^{-1}$, Weibull by applying $\xi = -\alpha^{-1}$ and Gumbel with $\xi = 0$. For a standardized variable x, $\frac{x-\alpha}{\beta}$ the distribution (17) can be parameterized as:

$$H_{\xi,\alpha,\beta}(x) = H_\xi\left(\frac{x-\alpha}{\beta}\right) \tag{18}$$

where ξ is distribution's shape parameter, α is location parameter and β is scale parameter.

Like in the POT procedure, all these three parameters are estimated with optimization the algorithm, which tries to fit best theoretical distribution with real data. Again the Maximum Likehood Method can be used:

$$\max_{1+\xi x>0,\alpha,\beta>0} \frac{1}{\alpha}\left(1+\xi\left(\frac{x-\beta}{\alpha}\right)\right)^{\left(-\frac{1}{\xi}\right)^{-1}} e^{-\left(1+\xi\left(\left(\frac{x-\beta}{\alpha}\right)^{\frac{1}{\xi}}\right)\right)}. \quad (19)$$

Taking into account the characteristics of the GEV formula yields asymptotic results (asymptotic normality and obtainable information matrix) for $\xi > -0.5$, for $-1 < \xi < -0.5$ asymptotic properties are not available and for $\xi < -1$ MLE parameters' estimator does not exist (regularity conditions). The values of MLE parameters are applied to select appropriate distribution according to the Fisher-Tippett theorem.

In trying to compare both the described methods (POT and BMM), it is hard to recommend clearly the best one. In some publications (e.g. Allen, Kramadibrata, Singh and Powell, AKSP, 2010), the POT method is proven to be superior over BMM, in other ones BMM seems to be more efficient in the estimation of tail loss for really extreme events [Bystroem 2004].

5. Data used in the survey

An empirical survey of ES and EVT application to market risk evolution was based on the trade portfolios of the 40 biggest Polish banks. The structure of this portfolio was built with the information taken from supervisory data collected by the National Bank of Poland.

Table 1. The structure of the 40 biggest Polish banks debt instruments portfolio in the last quarter of 2010

Issuer's type	Resident	Non-resident
Fraction of residents/non-residents' papers	99%	1%
Fraction of issuer's type in a whole portfolio		
Central banks (money bills)	34%	0%
Central government institutions	56%	25%
– of which treasury bills	6%	0%
– of which government bonds	50%	25%
Local government institutions	4%	0%
Other financial institutions	3%	70%
Other non-financial institutions	3%	5%

Source: NBP data gathered for UKNF.

Looking at the structure of the bank's trading portfolio, we can easily notice that it consists almost in 100% of debt issued by residents. Moreover, 92% of domestic debt instruments in this portfolio was issued by central government institutions. Hence in the following analysis only these papers were taken into consideration. In the next step, the bank's different maturities debt expositions to interest risk were mapped into five buckets of time structure (0-6M), (6M-3Y), (3Y-7Y), (7Y and more). Information about banks' hedging positions (interest rate swaps, future tare agreements) from off-balance sheet statements were taken into consideration as well, but the general assumption was made that the analysed bond positions have not been hedged. In empirical computations, daily adjustments of portfolio structure were used to keep constant portfolio weights. The ideal partition of instruments and the lack of transaction costs were assumed as well.

Bearing in mind the portfolio's time structure, its different maturity components were mapped on four factors of interest rate risk, which were: short term interest rate (WIBOR 3M), two mid-term interest rates (2Y and 5Y government bonds' yields respectively) and long term interest rate (10Y government bonds' yield). The following panel of figures shows the log returns of the mentioned risk factors' last 2000 observations (June 2005-February 2013, the last observation was recorded for 12 February 2013) taken from Bloomberg.

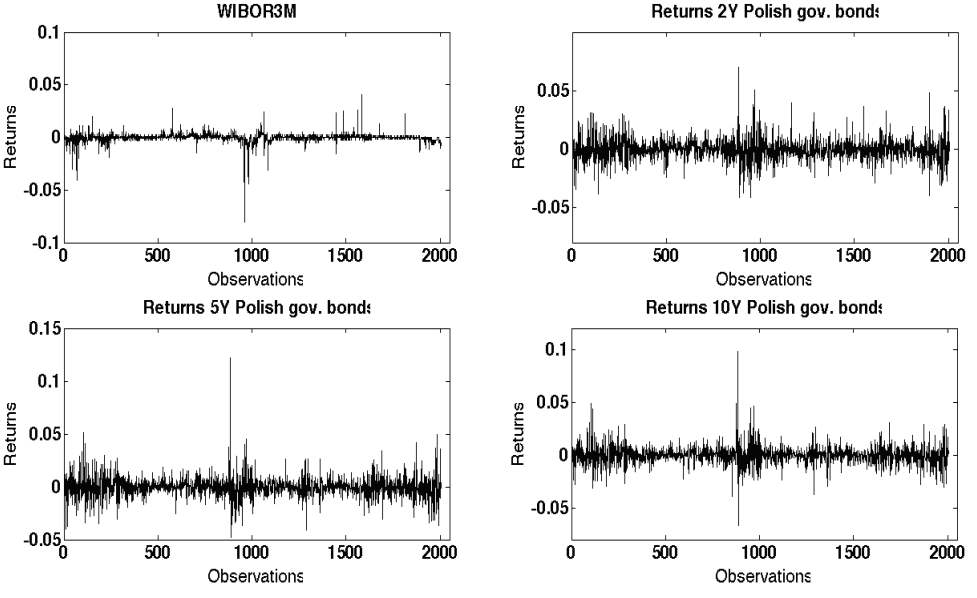

Figure 1. Log returns of the four main risk factors

Source: [Bloomberg].

In the next table, the basic descriptive statistics of the four selected interest rate risk factors were presented.

Table 2. Descriptive statistics of selected risk factors

	WIBOR3M	2Y PL gov. bonds	5Y PL gov. bonds	10Y PL gov. bonds
Mean	–0.0002	–0.0002	–0.0002	–0.0001
Median	0.0000	0.0000	0.0000	0.0000
Max	0.0404	0.0703	0.01216	0.0980
Min	–0.0808	–0.0416	–0.0481	–0.0663
Standard dev.	0.0045	0.0094	0.0098	0.0089
Skewness	–4.0204	0.4268	1.0739	0.6200
Kurtosis	76.4856	7.7696	17.6226	15.4415
Jarque-Bera statistics	456310	1960	18239	13053
Jarque-Bera p-value	0.000	0.000	0.000	0.000
Number of observations	2000	2000	2000	2000

Source: own computations.

The computed elementary statistics point out that the most volatile instruments were 2Y and 5Y government bonds. The greatest positive returns were generated in the analysed period by 10Y Polish bonds, the greatest negative returns by WIBOR3M. Besides that, all the instruments were characterized with excessed kurtosis ($k > 3$) what can be interpreted as a sign of extreme changes of the log returns' presence. No instrument passed the Jarque-Bera test either, which means that for all of the instruments the description of their stochastic characteristics with the Gaussian distribution hypothesis has to be rejected.

6. Empirical survey and achieved results

The models of Polish banks portfolio's returns extreme risk were built and estimated in the Matlab package. The codes for POT and GMM methods estimation were written in the Matlab quasi-C language. Upon the creation of these routines, the author used the built-in functions of the Matlab selected toolboxes, Econometrics (for computing residuals from GARCH model) and Statistics (to estimate distributions of excessed values/block maxima).

In the first step of the empirical characteristics of Polish banks – like a trading portfolio of debt instruments was analysed. Figure 2 shows the returns of the prepared portfolio. Table 3 presents its elementary characteristics.

Similarly to the risk factor time series, the portfolio's log returns descriptive statistics indicates the high possibility of extreme value occurrence. It is slightly skewed to the positive values. The exceeded kurtosis and rejection of the Jarque-

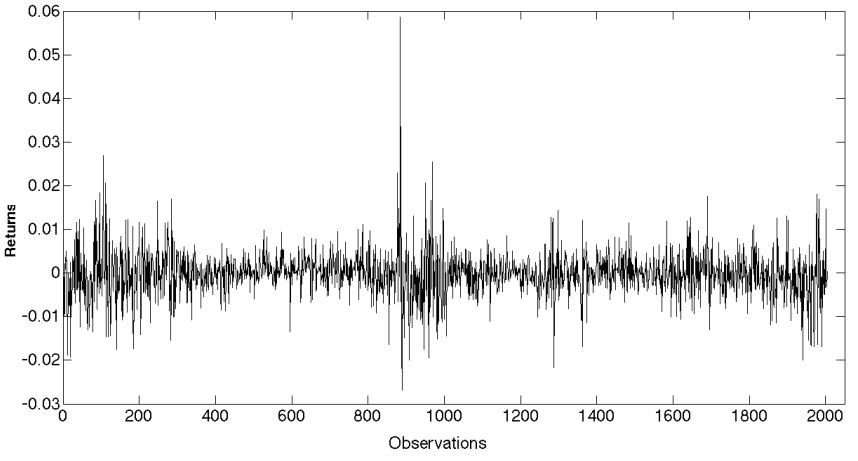

Figure 2. Log returns of the analysed portfolio (June 2005-February 2013)

Source: own computations.

Table 3. Descriptive statistics of the analysed portfolio

	WIBOR3M
Mean	–0.0002
Median	–0.0001
Max	0.0586
Min	–0.0286
Standard dev.	0.0052
Skewness	0.6194
Kurtosis	13.54
Jarque-Bera statistics	9404
Jarque-Bera p-value	0.000
Number of observations	2000

Source: own computations.

Bera null hypothesis indicate the possibility of fat tails in the empirical portfolio's returns distribution. Intuitively, the reason for the application of the Expected Shortfall and Extreme Value Theory methods has been confirmed. This conclusion is also confirmed by the QQ-plot of the portfolio's returns distribution against the Gaussian one.

As it was mentioned before, the estimation of the Expected Shortfall and tail distributions requires the assumption of the portfolio's returns being i.i.d. According to Figure 4, both portfolio's (log) returns and their squares are autocorrelated.

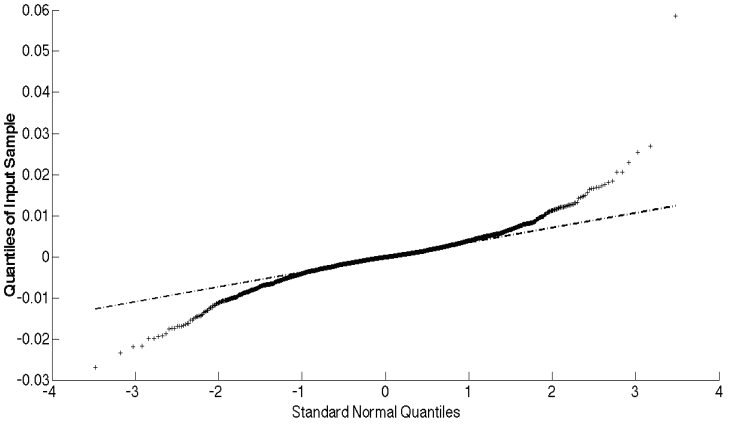

Figure 3. QQ-plot of the analysed portfolio's returns distribution against the Gaussian one

Source: own computations

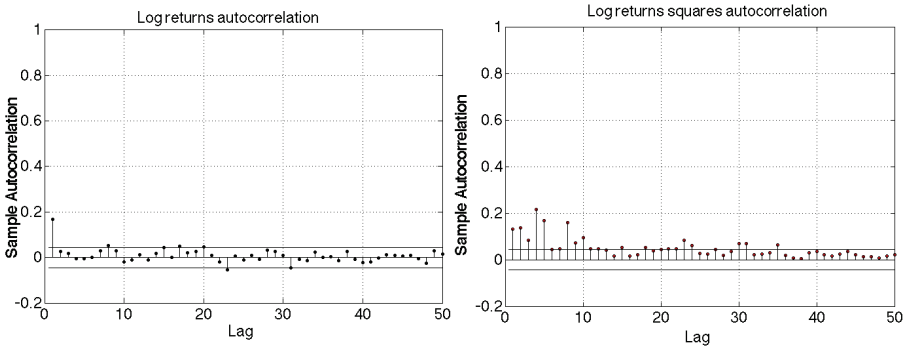

Figure 4. Autocorrelation of the analysed portfolio's returns

Source: own computations.

Hence, to achieve the i.i.d. distributed input to the ES and EVT computations, additional filtering is needed. Data filtering is applied with GARCH(1,1) analytical structure and output of this procedure is just residual of this model. The residuals and their volatility are presented in Figure 5.

The filtering portfolio's returns removes autocorrelation from the initial data. This is proved by the residuals and squared residuals autocorrelation plots depicted in Figure 6.

With the computed i.i.d. analysed portfolio's returns' residuals the appropriate extreme value analysis could be started. As the first method, the Peak Over Threshold (POT) approach was applied. The crucial prerequisite of this method's application is

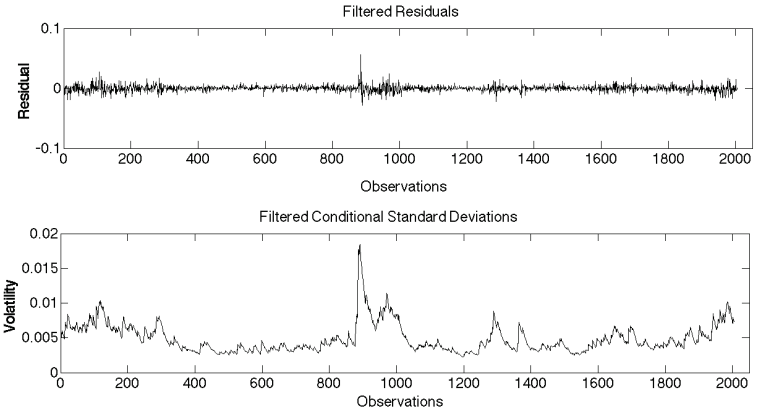

Figure 5. Portfolio's returns residuals from the GARCH(1,1) model

Source: own computations.

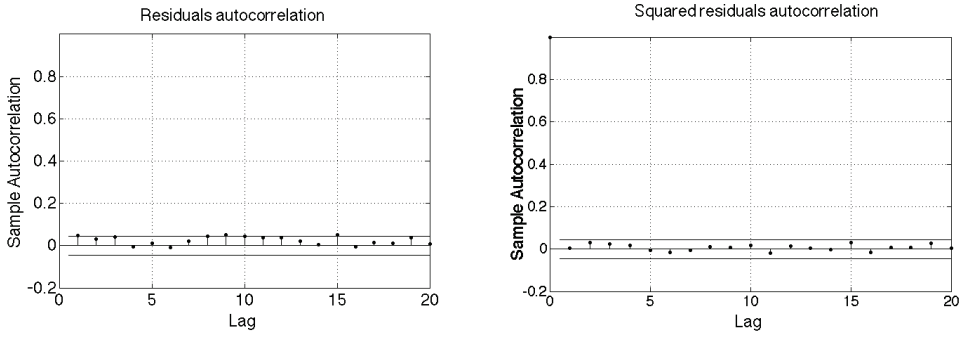

Figure 6. Autocorrelation of the analysed portfolio's returns' residuals

Source: own computations.

to choose properly the threshold level. Hence three different threshold levels were taken into consideration: 10%, 5% and 1%. In the first case, 245 portfolio's returns residuals were detected as threshold exceedances, for 5% threshold 131 residuals were chosen and for the third one only 24. The last dataset was classified as definitely too small and the first one as a too broad definition of extreme risk. Consecutively the 5% threshold value was chosen as the proper one for the POT distribution estimation.

The parameters of POT distribution were computed with the Maximum Likehood Estimation (MLE) procedure described in the theoretical part of the article. The achieved results (shape parameter $\hat{\alpha} = 0.146$, scale parameter $\hat{\beta} = 1.024$ of Generalized Pareto Distribution) pointed out that calibrated distribution is

characterized with fat tails, hence the extreme value analysis approach was fully justified. A histogram of the exceedance portfolio's returns residuals and calibrated GPD distribution are presented in Figure 7.

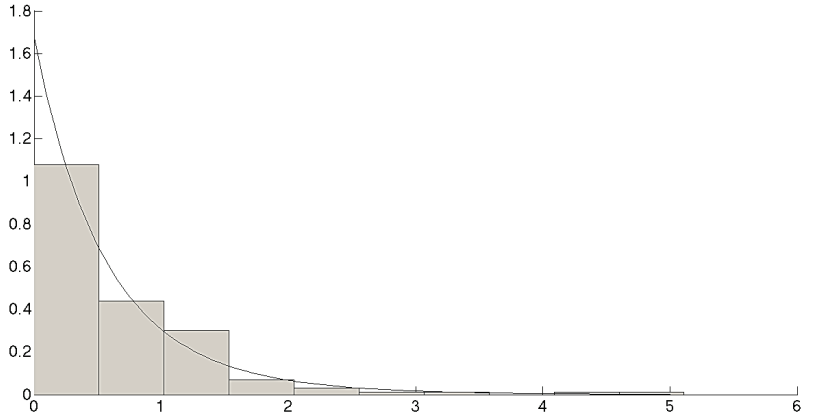

Figure 7. Histogram of the exceedance portfolio's returns residuals (5% threshold) and calibrated Gen. Pareto Dist.

Source: own computations.

As can be seen from the above picture, the POT distribution almost perfectly fits the empirical histogram of tail observations. The same high coincidence is observed in the case of the empirical and theoretical cumulative distribution functions (CDFs) comparison.

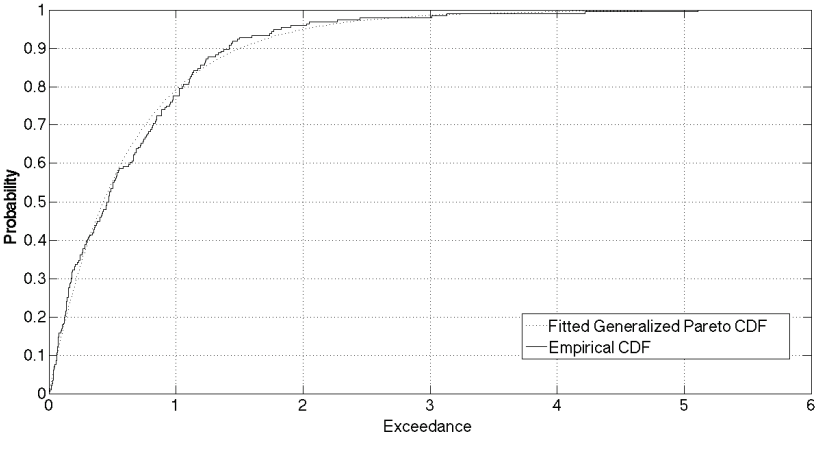

Figure 8. Empirical CDF of exceedance portfolio's returns residuals (5% threshold) and CDF of calibrated GPD

Source: own computations.

As the last step of the POT analysis Value at Risk (VaR) and Expected Tail Loss were computed. The value of "POT VaR" was computed on the level of 2.94% of the initial portfolio value and "POT ES" as 3.35% respectively.

In the second part of the empirical survey, the Block Maxima Method of Extreme Value Theory (EVT) was applied to check the tail characteristics of Polish banks' portfolio's returns' residuals. The key question which needs to be answered at the beginning of the BMM analysis is about the length of block used for selecting the local maxima. As in the case of the Pot analysis, three different scenarios were considered: 10, 20 and 40 observations in each block. As before, the appropriate block's granularity was achieved for the middle block length, 20 observations.

Given this value, the GEV distribution was fitted to the extracted local maxima. One more time the MLE procedure was applied to estimate the three parameters of this distribution. The achieved results, shape parameter $\hat{\xi} = 0.154$, location parameter $\hat{\alpha} = 1.611$ and scale parameter $\hat{\beta} = 0.5679$ allowed to construct the calibrated GEV distributions, which showed the presence of fat tails. Similarly to the POT case, a histogram of the block maxima histogram and the calibrated GEV distribution are shown in Figure 9.

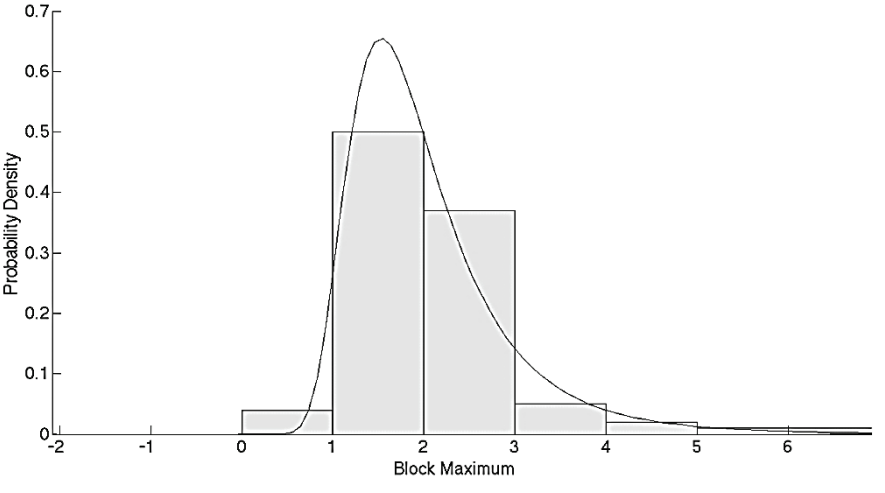

Figure 9. Histogram of the block maxima portfolio's returns' residuals (20 elements blocks) and distribution of the calibrated GEV

Source: own computations.

Moreover, the empirical and theoretical CDFs are compared in Figure 10.

Both Figures show very close correspondence of the empirical and calibrated PDF's and CDF's.

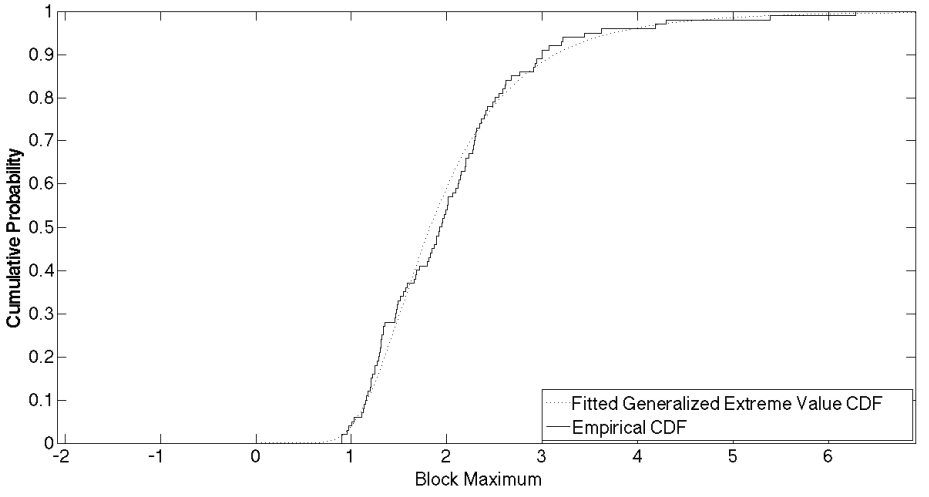

Figure 10. Empirical CDF of block maxima portfolio's returns' residuals and CDF of calibrated GPD

Source: own computations.

In the final step of the BMM analysis, Value at Risk and Expected Shortfall were computed. The value of "BMM VaR" was computed on the level of 4.63% of initial portfolio value and "POT ES" as 5.21% respectively.

7. Conclusions

This article was written to present methods of the extreme market risk valuation, which go beyond the standard Value at Risk methodology. Two main approaches: Expected Tail Loss (ETL) and Extreme Value Theory (EVT) were described and then applied to simulate interest risk stemming from debt portfolio constructed in a way to simulate the global market risk portfolio held by Polish banks.

The constructed debt portfolio characteristic shows that the application of the extreme risk evaluation method is fully justified and needed. The gained results allow to answer the question of what can happen in the case when the risk exceeds the standard Value at Risk measure. The presented methods can be interpreted as complementary approaches, especially useful in times of distress on financial markets, observed during the last crisis of 2008-2012. They provide a deeper insight into market risk developments in the fat tails of the debt instruments' returns. The obtained results show that debt instruments' portfolios of the biggest Polish banks are generally immune to financial (interest rate) risk as the EVT measures are from 2.94% in the case of "POT VaR" to 5.21% for "POT ES". These values suggest that debt instruments' portfolios should not be perceived as a serious threat to the stability

of domestic banks. However it is still strongly recommended to use methods using beyond the standard VaR methodology for monitoring market risk developments in their trading books.

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MODELOWANIE WARTOŚCI EKSTREMALNYCH RYZYKA RYNKOWEGO PORTFELI PAPIERÓW DŁUŻNYCH ZNAJDUJĄCYCH SIĘ W POSIADANIU POLSKICH BANKÓW

Streszczenie: Głównym celem artykułu jest przedstawienie metod szacowania wartości ekstremalnych ryzyka rynkowego, wykraczających poza standardową procedurę ewaluacji tego rodzaju ryzyka, za pomocą wartości narażonej na ryzyko (VaR). W pracy zaprezentowane zostaną dwa alternatywne podejścia do pomiaru ekstremalnych strat z tytułu materializacji ryzyka rynkowego: oczekiwana wartość straty w ogonie (*Expected Tail Loss*, ETL) oraz teoria wartości ekstremalnych (*Extreme Value Theory*, EVT). Oba ujęcia będą następnie użyte do oszacowania ryzyka stopy procentowego portfeli rządowych papierów dłużnych znajdujących się w posiadaniu polskich banków. Uzyskane wyniki wskazują na przydatność stosowania wspomnianych metod do oceny ekspozycji banków na ryzyko rynkowe jako uzupełnienie metody VaR, w szczególności w okresie zawirowań na rynkach finansowych.

Słowa kluczowe: ryzyko rynkowe, *Value at Risk*, *Expected Tail Loss*, *Extreme Value Theory*.