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APPLICATIONS OF  
MATHEMATICS AND STATISTICS  
IN ECONOMICS

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## HEALTHCARE SYSTEMS EFFICIENCY IN THE VISEGRÁD GROUP

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### Abstract

*We examine healthcare systems efficiency in four countries of the Visegrád group during the period 2004-2010. For this purpose, we apply the Data Envelopment Analysis on data of 18 European countries. The number of practising physicians per 100 000 inhabitants, the number of hospital beds per 100 000 inhabitants and the share of health expenditures to GDP are considered as inputs, and infant mortality and life expectancy for males at birth are considered as outputs. We calculate Super-SBM efficiency scores to rank countries and to suggest improvements for the Visegrád group. To measure efficiency changes over specified time period, Malmquist productivity indexes are computed from DEA scores. Standard decomposition of Malmquist index into frontier shift and catching up effect gives a deeper insight into the issue.*

**Key words:** *Healthcare system efficiency, DEA, Super-SBM, Malmquist productivity index, Visegrád group.*

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### 1. Introduction

The efficiency of healthcare system can be viewed from social, medical and economic points. Social efficiency pursues the fundamental objective of the development of people as individuals and the development of society as a whole. We evaluate the medical efficiency by improving the health status of the patient after consumption of adequate health services, but it is not possible to quantify this efficiency with economic indicators. Economic efficiency can be evaluated through the development of labor productivity, gross domestic product, or reproduction of human potential through the improvement of the population health.

Peacock, Chan, Mangolini, Johansen (2001) distinguish allocative and technical efficiency in relation to the better use of healthcare resources. Technical efficiency focuses on minimizing inputs to achieve an expected performance. Allocative efficiency is aimed at improving the performance indicators in health care.

Benčo, Šebo and Štrangfeldová (2011) distinguish technical, allocative and cost efficiency. Technical efficiency targets on minimizing the volume of inputs or on maximizing the volume of outputs. Cost efficiency is based on technical efficiency, but it takes into account the relative costs of different inputs and requires minimizing the cost of inputs. Allocative efficiency connects technical and cost effectiveness with producing outputs for which there is the highest demand. Efficient allocation of resources meets all three requirements. Technical and cost efficiency draw attention to the production. Allocative efficiency focuses on the consumption, which brings together supply and demand.

Economic efficiency in health care is a combination of allocative and technical efficiency of the use of available resources. In a recent study of OECD countries it was found that the efficiency of Slovak healthcare system is at 61.2%. It follows that from every euro in the system we lose 39 cents, which are not used to improve population health (Bezekova, 2007).

Many researchers have focused attention on the measurement of efficiency in the healthcare. Hollingsworth (2008) reviewed 317 references on frontier efficiency in the context of health services delivery. But just a few studies have recently implemented the efficiency measurement techniques under a macro level perspective, i.e. evaluating the overall healthcare system's performance. Different approaches were found regarding the inputs and outputs incorporated into the models. Authors used to examine the impact of socioeconomic determinants, the social environment, population lifestyles, attitudes and behaviors, available medical care services, and health expenditure on healthcare outcomes. In most of these studies, the healthcare outcome was measured by life expectancy at birth, while expenditure on health per capita was included as one of the inputs. Only few studies (Bhat, 2005, Afonso et al., 2005) included the number of inpatient beds and levels of health employment as inputs rather than health expenditure.

According to Hadad et al. (2013), various Data Envelopment Analysis (DEA) approaches have been widely developed and applied in the evaluation of the health services performance.

Our study focuses on the development of the healthcare systems efficiency in the countries of the Visegrád group during the period 2004-2010. In order to observe relative efficiencies, we apply DEA on data of 18 European countries. Firstly we calculate the super-efficiency scores to compare and rank the healthcare systems of the countries of the Visegrád group within Europe. Secondly, we analyze their healthcare system efficiency development using Malmquist productivity index and its decomposition into frontier shift and catch up effect. The results of the analysis contribute to the deeper understanding of the healthcare systems efficiency and the success of national healthcare policies of the Visegrád countries.

## 2. Methodology

We apply DEA to measure technical efficiency of healthcare systems in Visegrád group regarding to 18 European countries. This approach allows us to deal with several indicators of different nature as the inputs and outputs of the healthcare systems.

### 2.1 DEA model

We utilize a non-radial and non-oriented DEA model. A non-oriented model is appropriate in our context, since healthcare system desires both to maximize health gains and minimize inputs. Generally, DEA model allows us to determine efficient and inefficient systems. Efficient systems are assigned the unit efficiency scores, while inefficient systems are

assigned efficiency scores less than one. In order to distinguish the efficient healthcare systems, we apply so-called super-efficiency. The super-efficiency method for radial models was developed by Andersen and Petersen (1993) to rank the efficient decision-making units (DMUs). This approach allows these DMUs to receive an efficiency score greater than one. Tone (2002) introduced non-radial super-efficiency models using the slack-based model (SBM).

## 2.2 Malmquist index

The Malmquist index (MI) evaluates the efficiency change of a DMU between two time periods. It differs from other index numbers in its ability to distinguish between improved possibilities of performance (e.g., due to technological change) and the degree to which each of the evaluated entities has taken advantage of these possibilities, in both periods. It is an index representing Total Factor Productivity (TFP) of the DMU, in that it reflects progress or regress in efficiency of the DMU along with progress or regress of the frontier technology. It is defined as the product of “Catch-up” and “Frontier-shift” terms.

The catch-up term is related to the degree of efforts that the DMU attained for improving its efficiency, while the frontier-shift term reflects the change in the efficient frontiers surrounding the DMU between the two time periods 1 and 2.

In a non-parametric framework the Malmquist index (MI) is constructed by means of DEA technologies. There are a number of ways to compute MI. We computed MI from the non-radial and non-oriented SBM model.

## 3. Data and variables

In the present study, we measure the efficiency of healthcare systems in four countries of the Visegrád group during the period 2004-2010 in relation to other European countries. DEA analysis requires that the number of DMUs should be at least three times greater than the total number of inputs and outputs used in the computation. Therefore we include to our comparison 18 countries for which the data is available during the period, i.e. Austria, Belgium, Cyprus, Czech Republic, Estonia, Finland, Germany, Hungary, Latvia, Lithuania, Luxembourg, Norway, Poland, Romania, Slovakia, Slovenia, Spain and Sweden. These countries are considered to be DMUs for DEA.

Data has been collected from two sources: the Eurostat Statistics Database [9] and the OECD Health Data [10]. Two outputs were chosen – life expectancy at birth of males and infant mortality rate (transformed to correspond with the DEA assumptions as follows). The DEA technique implies that outputs are measured in such a way that “more is better”. Since infant mortality rate (IMR) does not meet this rationale, we calculate the modified infant mortality rate (MIMR) by the formula  $MIMR = 1000/IMR$ . The inputs included in our analysis are the number of practising physicians per 100 000 inhabitants, the number of hospital beds per 100 000 inhabitants and the share of public health expenditures in relation to GDP. Since data on practising physicians in Slovakia and Finland is not available for some of observed years, we estimate the missing values based on the number of professionally active physicians [9]. Input and output data definitions and sources are presented in Table 1.

Table 1. Definition and source of variables included in the DEA models.

Variable	Role	Definition	Data source
Practising physicians	Input	The total number of practising physicians (medical doctors) in a given calendar year, per 100,000 inhabitants. Practising physicians provide services directly to patients.	[9]
Hospital beds	Input	The total number of hospital beds in a given calendar year, per 100,000 inhabitants.	[9]
Share of GDP	Input	Share of public health expenditures in relation to GDP.	[10]
Modified mortality	Output (transformed)	1 000 live births over the number of deaths of infants under 1 month of age.	[9]
Life expectancy	Output	Life expectancy in absolute value at birth - males.	[9]

Source: Author's work.

The data is analyzed using DEA-Solver software (version 7.0, [www.saitech-inc.com](http://www.saitech-inc.com)).

Table 2 shows the development of selected input and performance indicators for Visegrád (V4) countries over 7 years (2004-2010) as well as descriptive statistics for 18 countries in comparison.

The Czech Republic reports the highest number of practising physicians throughout the period with a continued moderate increase. Poland has the lowest number of practising physicians with a declining trend.

Hungary leads in hospital beds and in spite of the significant decrease in the year 2007 still has the highest number of hospital beds among the V4 countries. Poland shows the lowest number of hospital beds at the beginning of the period, but in the last three years Slovak Republic has the lowest values of this indicator.

The share of financial resources directed to the health sector in gross domestic product in the V4 countries varies from 4.0% in Poland up to 6.4% in the Czech Republic. The share in individual countries is increasing except Hungary, where the share in 2007 fell down from 5.5% to 5.0% and remains at this level till the end of the period. In 2007 we observe a similar decrease of the share in the Czech Republic, but in the following years the share gradually increases and reaches the previous level.

As performance indicators we take child mortality and average life expectancy of males at birth in absolute values.

We modified the indicator of child mortality so as to maximize its value (to be a desired output of the DEA method used). We see that the most unfavorable situation in child mortality is in the Slovak Republic and in Hungary. The best outcomes of this indicator are in the Czech Republic.

The average life expectancy of men at birth increases in all V4 countries. Men in the Czech Republic and in Hungary are expected to live to the highest and lowest age, respectively.



Table 2. Data and descriptive statistics.

Indicator	geo\time	2004	2005	2006	2007	2008	2009	2010
Practising physicians	Czech Republic	352	355	356	356	354	356	359
	Hungary	334	278	304	280	309	302	287
	Poland	229	214	218	219	216	217	217
	Slovakia	315	289	301	300	320	313	319
	Mean - V4	307	284	295	289	300	297	295
	Mean - 18 countries	304	305	309	313	318	319	324
	Minimum - 18 countries	208	214	216	212	216	217	217
	Maximum - 18 countries	420	432	445	454	460	468	478
Hospital beds	Czech Republic	764	755	743	731	719	711	703
	Hungary	783	785	792	719	711	714	718
	Poland	667	652	647	642	662	665	656
	Slovakia	690	677	671	675	656	651	644
	Mean - V4	726	717	713	692	687	685	680
	Mean - 18 countries	631	617	607	597	592	580	567
	Minimum - 18 countries	301	293	289	286	281	276	273
	Maximum - 18 countries	858	847	830	824	821	824	825
Share of GDP	Czech Republic	5,9	5,8	5,6	5,3	5,5	6,4	6,0
	Hungary	5,5	5,7	5,5	5,0	4,9	5,0	5,0
	Poland	4,0	4,0	4,1	4,2	4,6	4,8	4,7
	Slovakia	5,1	5,1	4,9	5,1	5,4	6,0	5,8
	Mean - V4	5,1	5,2	5,0	4,9	5,1	5,6	5,4
	Mean - 18 countries	6,1	5,5	5,5	5,6	5,8	6,3	6,4
	Minimum - 18 countries	3,8	2,5	2,8	3,4	3,3	3,0	4,1
	Maximum - 18 countries	7,9	8,0	7,9	7,8	7,9	8,7	8,6
Modified mortality	Czech Republic	270	294	303	323	357	345	370
	Hungary	152	161	175	169	179	196	189
	Poland	147	156	167	167	179	179	200
	Slovakia	147	139	152	164	169	175	175
	Mean - V4	179	188	199	206	221	224	234
	Mean - 18 countries	216	235	249	263	280	275	281
	Minimum - 18 countries	60	67	72	83	91	99	102
	Maximum - 18 countries	323	417	400	556	556	417	435
Life expectancy	Czech Republic	72,5	72,9	73,5	73,8	74,1	74,3	74,5
	Hungary	68,7	68,7	69,2	69,4	70,0	70,3	70,7
	Poland	70,6	70,8	70,9	71,0	71,3	71,5	72,1
	Slovakia	70,3	70,2	70,4	70,6	70,9	71,4	71,8
	Mean - V4	70,5	70,7	71,0	71,2	71,6	71,9	72,3
	Mean - 18 countries	72,9	73,0	73,5	73,6	74,1	74,5	74,9
	Minimum - 18 countries	65,6	64,9	65,0	64,5	65,9	67,1	67,6
	Maximum - 18 countries	78,4	78,5	78,8	79,0	79,2	79,4	79,6

Source: Author's work based on [9] and [10].

#### 4. Results and discussion

Evaluating super-efficiency scores for reference years in the V4 countries (for details see Table. 3), we can conclude that the best placed is the health system of Poland, although in 2005-2007 experienced some fall among the surveyed countries. In 2010, however, Poland returned to the forefront walls among the evaluated countries.

Table 3. Super-efficiency scores, ranks and projections.

		2004	2005	2006	2007	2008	2009	2010	
Czech Republic	Scores	0,75	0,71	0,70	0,61	0,69	0,69	0,79	
	Ranks	11	13	12	12	11	11	11	
	Projections (%)	Practising physicians	-35	-29	-27	-25	-25	-32	-27
		Hospital beds	-37	-23	-24	-22	-21	-35	-33
		Share of GDP	-2	0	-2	-2	0	-3	0
		Modified mortality	0	28	32	72	40	21	2
Life expectancy		1	5	4	4	4	2	1	
Hungary	Scores	0,58	0,52	0,50	0,45	0,54	0,66	0,74	
	Ranks	16	17	18	16	14	13	12	
	Projections (%)	Practising physicians	-31	-9	-14	-7	-18	-22	0
		Hospital beds	-35	-26	-29	-20	-17	-25	-22
		Share of GDP	0	0	0	0	0	0	0
		Modified mortality	65	127	128	197	119	51	48
Life expectancy		6	11	11	9	7	5	1	
Poland	Scores	1,08	1,02	1,01	1,00	1,02	1,02	1,13	
	Ranks	3	9	8	8	6	5	3	
	Projections (%)	Practising physicians	0	6	3	1	5	6	10
		Hospital beds	0	0	0	0	0	0	0
		Share of GDP	25	0	0	0	0	0	28
		Modified mortality	0	0	0	0	0	0	0
Life expectancy		0	0	0	0	0	0	0	
Slovakia	Scores	0,66	0,55	0,55	0,42	0,46	0,50	0,56	
	Ranks	13	16	17	17	17	18	15	
	Projections (%)	Practising physicians	-27	-15	-12	12	-17	-23	-14
		Hospital beds	-20	-11	-16	-15	-13	-27	-25
		Share of GDP	0	0	0	0	0	0	0
		Modified mortality	51	122	128	223	185	126	107
Life expectancy		3	7	5	8	8	6	3	

Source: Author's work.

Although the Czech Republic does not show high super-efficiency scores, it maintains its position among the surveyed countries with small fluctuations. Hungary together with the

Slovak Republic ranged in assessing the super-effectiveness the lowest positions. In Hungary, however, the health system becomes more efficient. Despite the fact that in the early years it lags behind the Slovak Republic, in the last studied years it has significantly improved its position, while the health system in the Slovak Republic has lagged. Even in 2009, Slovakia occupied the last place among the surveyed countries.

Comparing the recommended projections for individual input and output indicators between years and among the V4 countries, we can find opportunities to be done in order to achieve efficiency of health systems. We observe that Poland has the best condition and development of the monitored indicators. Even with an increase in the number of physicians and the expenditure ratio to GDP Poland is able to maintain the efficiency of its system. The Czech Republic, according to the results should reduce the number of doctors and the number of beds and slightly reduce the share of health expenditure to GDP. To achieve efficiency in output indicators should be primarily reduced child mortality. Hungary, together with the Slovak Republic, may also further reduce the number of practising physicians and hospital beds. However, problematic in these countries are performance indicators. The child mortality and life expectancy do not correspond to the level of healthcare system inputs.

When evaluating the distance of the health systems of the V4 countries to the efficiency frontier (see Figure 1.), we can conclude that only the health system in Poland reaches the maximum efficiency during the whole period. The healthcare system of the Slovak Republic has the largest fluctuations regarding to the efficiency frontier. A positive finding is that all V4 countries are moving closer to the frontier in recent years. However, we can talk just about the so-called false improving, because the border efficiency decreases.

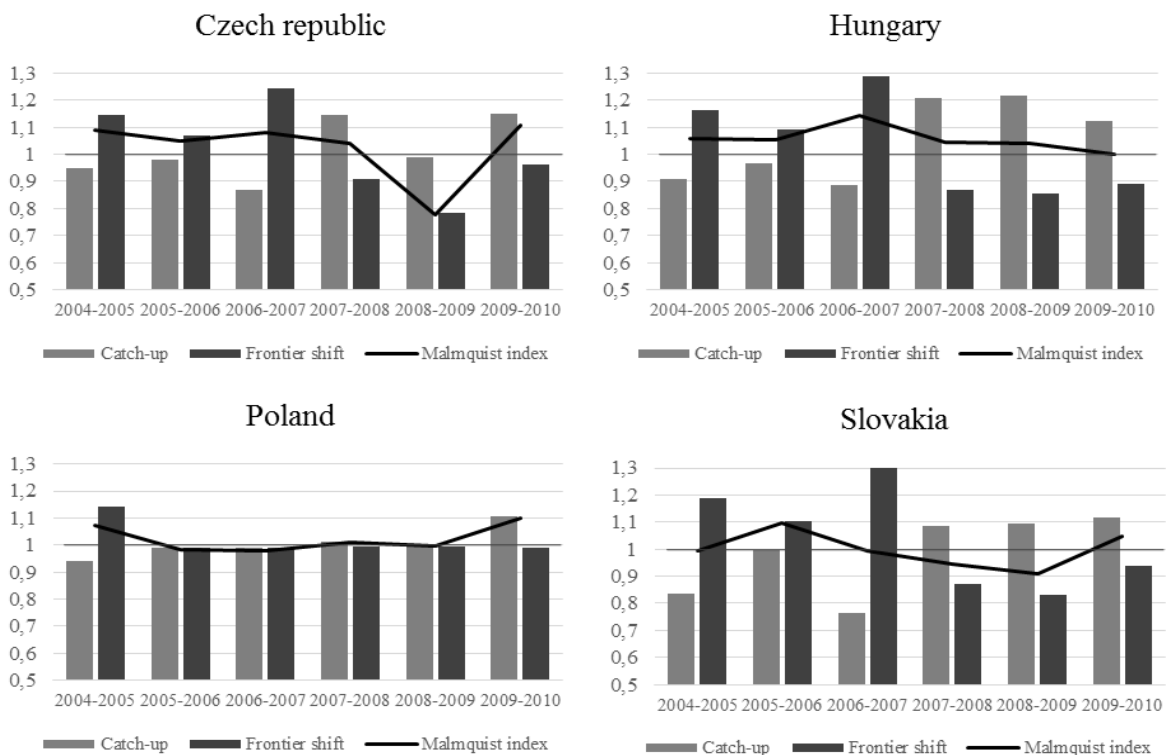


Figure 1. Development of relative healthcare efficiency in V4 countries.  
 Source: Author's work.

Malmquist index points to the fact that the Czech and Slovak Republic, despite the decreasing effectiveness of the frontier, still show a decrease in their effectiveness against it. It follows that the levels of health systems efficiency in these countries decreases. On the other hand, the development of health systems in Poland and Hungary is more stable.

## 5. Conclusion

We applied DEA to analyse the relative efficiency of healthcare systems in the V4 countries within the group of 18 European countries in the period 2004-2010. We found that the health system in Poland belongs to the most efficient within the compared countries although the deeper super-efficiency analysis shows some insufficiencies and fluctuations.

The other three countries have relatively inefficient health systems in all years. But their development is different. The Czech Republic keeps a stable position in the ranking. Hungary took one of the last places in the ranking in the first years, but has significantly improved its position at the end of the period. On the contrary, Slovakia remains at very low positions in the ranking for the whole period. The Malmquist index decomposition reveals that all the three countries approach to the efficiency frontier in the last three years (2008-2010). But this optimistic observation is only partially due to the improvement of their real efficiencies since the efficiency of reference countries significantly decreased between years 2007 and 2008. The real reasons of this development will be a matter of further research.

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## Appendix: the DEA methodology

In this section, we first present the Malmquist index, and then we present the DEA models, according to Cooper et al. (2007). Let us recall, that the Malmquist index is defined as the product of “Catch-up” and “Frontier-shift” terms.

We denote the activities of DMU<sub>o</sub> at the time period 1 and 2 by  $(x_o, y_o)^1$  and  $(x_o, y_o)^2$ , respectively. Then we develop the numerical measures for which we employ the notation  $\delta^s((x_o, y_o)^t)$  ( $t = 1, 2$  and  $s = 1, 2$ ), for the efficiency score of DMU<sub>o</sub> at the time period  $t$  measured by the frontier technology  $s$ .

Using this notation, the Catch-up effect can be expressed by the following formula.

$$\text{Catch-up} = \frac{\delta^2((x_o, y_o)^2)}{\delta^1((x_o, y_o)^1)} \quad (1)$$

(Catch-up)>1 indicates progress in relative efficiency from period 1 to 2, while (Catch-up)=1 and (Catch-up)<1 indicate the status quo and regress in efficiency, respectively. In addition to the catch-up term, we must take account the frontier-shift effect in order to totally evaluate the efficiency change of the DMU, since the catch-up is determined by the efficiencies as measured by the distances from the respective frontiers.

The Frontier shift effect is described as

$$\text{Frontier shift} = \left[ \frac{\delta^1((x_o, y_o)^1)}{\delta^2((x_o, y_o)^1)} \times \frac{\delta^1((x_o, y_o)^2)}{\delta^2((x_o, y_o)^2)} \right]^{1/2} \quad (2)$$

As the product of Catch-up and Frontier-shift, we obtain the following formula for the computation of MI, the Malmquist index.

$$\text{MI} = \left[ \frac{\delta^1((x_o, y_o)^2)}{\delta^1((x_o, y_o)^1)} \times \frac{\delta^2((x_o, y_o)^2)}{\delta^2((x_o, y_o)^1)} \right]^{1/2} \quad (3)$$

This last expression gives another interpretation of MI as the geometric mean of the two efficiency ratios: the one being the efficiency change measured by the period 1 technology and the other the efficiency change measured by the period 2 technology. As can be seen from these formulas, MI consists of four terms:  $\delta^1((x_o, y_o)^1)$ ,  $\delta^2((x_o, y_o)^2)$ ,  $\delta^1((x_o, y_o)^2)$  and  $\delta^1((x_o, y_o)^2)$ . The first two are related to the measurements within the same time period with  $t = 1$  or  $t = 2$ , while the last two are for intertemporal comparison.  $\text{MI} > 1$  indicates progress in the total factor productivity of the  $\text{DMU}_o$  from period 1 to 2, while  $\text{MI} = 1$  and  $\text{MI} < 1$  respectively indicate the status quo and deterioration in the total factor productivity.

We utilized a non-radial and non-oriented model. The models in this category deal with input and output slacks. The [SBM] and [Super-SBM] models used for computing  $\delta^s((x_o, y_o)^t)$  are represented by the following fractional programs:

[SBM]

$$\delta^s((x_o, y_o)^t) = \min_{\lambda, s^-, s^+} \left( 1 - \frac{1}{m} \sum_{i=1}^m s_i^- / x_{io}^t \right) / \left( 1 + \frac{1}{q} \sum_{i=1}^q s_i^+ / y_{io}^t \right) \quad (4)$$

subject to

$$x_o^t = X^s \lambda + s^-,$$

$$y_o^t = Y^s \lambda - s^+,$$

$$L \leq e\lambda \leq U,$$

$$\lambda \geq 0, s^- \geq 0, s^+ \geq 0.$$

Where  $X^s = (x_1^s, \dots, x_n^s)$  and  $Y^s = (y_1^s, \dots, y_n^s)$  are respectively input and output matrices of observed data for period  $s$ , the vectors  $s^+$  and  $s^- \in R^m$  denotes output and input-slacks. We solve this program for  $s = 1$  and 2. It holds that  $\delta^s((x_o, y_o)^t) \leq 1$ , and  $\delta^s((x_o, y_o)^s) = 1$  indicates  $(x_o, y_o)^s$  being on the technically efficient frontiers of  $(X, Y)^s$ .

[Super-SBM]

$$\delta^s((x_o, y_o)^t) = \min_{\lambda, s^-, s^+} \left( \frac{1}{m} \sum_{i=1}^m \bar{x}_i / x_{io}^s \right) / \left( \frac{1}{q} \sum_{i=1}^q \bar{y}_i / y_{io}^s \right) \quad (5)$$

subject to

$$\begin{aligned} \bar{x} &\geq X^s \lambda, \\ \bar{y} &\leq Y^s \lambda, \\ \bar{x} &\geq x_o^t, \bar{y} \leq y_o^t \\ L &\leq e\lambda \leq U, \\ \bar{y} &\geq 0, \lambda \geq 0. \end{aligned}$$

We solve this program for the pairs  $(s, t) = (1, 2)$  and  $(2, 1)$ . If  $(x_o, y_o)^t$  is not enveloped by the technology of period  $s$ , the score  $\delta^s((x_o, y_o)^t)$ , if it exists, results in a value greater than 1.

These fractional programs can be transformed into LPs. See Tone (2002) for detailed method of doing so. This model under the exclusive scheme evaluates the four components of MI:  $\delta^1((x_o, y_o)^1)$ ,  $\delta^2((x_o, y_o)^2)$ ,  $\delta^1((x_o, y_o)^2)$  and  $\delta^2((x_o, y_o)^1)$  using [SBM], and, if the corresponding LP is found infeasible, we then apply [Super-SBM].

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