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PROSPECTS AND FRONTIERS OF MULTIDIMENSIONAL PANEL SPATIAL AUTOREGRESSIVE MODELS*

Summary: This paper considers some innovative techniques which allow for the investigation of certain issues concerning regional development in the EU. Namely for accessing the space-time process we suggest a *Multidimensional Panel Spatial Autoregressive Model* – MPSAR. It is assumed that the multidimensional perspective allows for a better description of the spatial dependence structure, especially in the context of regional science. The paper discusses the advantages and limitations of the presented approach. We point out that the failure to recognise these multidimensional effects may lead to an incorrect inference and therefore to biased conclusions. Therefore this approach constitutes an innovation in the spatial econometric studies providing additional information, and hence a deeper analysis of the investigated problem.

Keywords: spatial econometrics, space-time process, panel models.

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1. Introduction

One of the most important problems in spatial econometrics is the correct assessment of the dependence structure among cross-sectional observations. A classic approach is to assume that for each location there is a set of spatial neighbours defined. This can be used for the specification of a spatial stochastic process where the covariance structure is modelled indirectly by the relation of membership in a set of neighbours [Anselin and Bera 1998, pp. 237-289].

According to the field of research, different concepts of distances or neighbourhood are considered since different factors result in spatial dependences or directly affect them. For example, in some cases the researcher must take into account isolated units without any direct geographical connection with other regions. Sometimes it may seem to be appropriate to diversify the strength of the influence of the neighbours. For instance, the spatial dependence of commercial activities

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between a municipality and its surrounding rural area appear to be asymmetric. As a result, in certain circumstances a spatial structure introduced in the model should represent not only geographical relations but also socio-economic interactions such as commuting and trade flows or even ethnic linkages and many others. As the spatial weight matrix is, in fact, meant to approximate the true spatial relations (regardless of its nature) it seems that it is justified to allow for even more flexibility in the representation of the spatial structure.

The aim of this paper is to propose a conceptual framework for the analysis of multidimensional spatial dependence structure in a spatial, temporal, and spatio-temporal context as well as to discuss its possibilities and limitations. In this study we propose the idea of *multidimensional spatial weight matrix* and *multidimensional spatial coefficient matrix* which give the opportunity for a better description of the complete pattern of the dependence structure [cf. Olejnik 2012a]. Enhancing the model with a time dimension allows for a more detailed analysis of spatial relations in terms of stability of space, time and even investigation of time lag in the spatial structure. Thus in this paper we present the idea of *Multidimensional Panel Spatial Autoregressive Model* (MPSAR).

The remainder of this paper is structured as follows. Section 2 introduces the general idea of the *multidimensional spatial structure* with a strategy for the incorporation of a multidimensional matrix into a spatial econometric model. In Section 3 we suggest an approach to include a time dimension into the multidimensional spatial structure. Finally, in Section 4 we present an example of multidimensional space-time structure in order to test some of the hypotheses of the New Economic Geography. Section 5 provides a summary and some concluding remarks.

2. Multidimensional spatial structure

Let us consider a classic spatial autoregressive SAR model for cross-sectional observations with normal disturbances:

$$\mathbf{y} = \boldsymbol{\rho}\mathbf{W}\mathbf{y} + \mathbf{X}\boldsymbol{\beta} + \mathbf{u}, \quad \mathbf{u} \sim N\left(0, \sigma^2\mathbf{I}\right) \tag{1}$$

where $\mathbf{y}$ ($N\times1$) is a spatially lagged endogenous variable and $\mathbf{X}$ ($N\times K$) denotes the matrix of observations on K exogenous variables. Typically, matrix $\mathbf{W}$ is a given *a priori* spatial weight matrix. The elements w_{ij} of $\mathbf{W}$ are ones ($w_{ij} = 1$) if locations i and j are neighbours and all other (in particular the diagonal elements) are zero. Therefore, the neighbourhood structure is represented by the spatial weight matrix $\mathbf{W}$.

In a classical spatial autoregressive model the spatial parameter ρ is a scalar multiplied by the spatial weight matrix $\mathbf{W}$. Then it is impossible for various degrees of different types of spatial influence to be estimated in the model. The general concept is to let the spatial weight matrix be a matrix with more than two dimensions and, as a result, better describe the spatial interactions of the cross-sectional units. The natural consequence of this is that the spatial autoregressive coefficient ρ becomes at least a vector of parameters. Apparently, such an idea is a straightforward generalization of the standard concept and enables a more complex analysis of the spatial relations.

Let us propose the following notation [cf. Olejnik 2012a, 2012b; Suchecki 2012]:

$$\mathbf{y} = \boldsymbol{\rho}\mathbf{D}\mathbf{y} + \mathbf{X}\boldsymbol{\beta} + \mathbf{u} \quad \mathbf{u} \sim N\left(0, \sigma^2 \mathbf{I}\right), \tag{2}$$

where the spatial structure of the process is represented by the general expression **ρDy**. The matrix **D** denotes a multidimensional spatial weight matrix of dimension N×P1×P2×…×PR×N, while **ρ** is a multidimensional spatial coefficient matrix. The form of **D** and ρ depends on the subject of interest of the empirical study. In the expression **ρDy** we use a multidimensional multiplication which is determined by the dimensions of **D** and **ρ** [cf. Olejnik 2012a, 2012b]. Hence the model incorporating a multidimensional spatial weight matrix **D** of the form (1) is called the *Multidimensional Spatial Autoregressive Model* (MSAR) [cf. Olejnik 2012a].

3. Multidimensional space-time structure

The motivation for a multidimensional line of thinking has been the need to solve some issues concerning regional development in the EU. To conceptualize the idea, we focused on some of the problems regarding NUTSII regions. In a classic approach it is assumed that geographically neighbouring regions do interact with each other and these interactions are the same within and outside countries. However this does not necessary reflect the true spatial structure. Similarly, the assumption of the same power of spatial interaction within and outside the Eurozone seem be too restrictive. Correspondingly, the same disputable issues arise for old and new EU countries, as well as inside and outside the Schengen Area and other historically or culturally close or distant regions. Therefore, it seems to be justified to test whether all of the regions of interest influence their neighbours in the same way.

Let us notice that the time dimension gives new ‘spatial’ possibilities to account for spatial dependence in terms of more possible spatial parameters to estimate. Employing the time dimension also raises the question of whether the power of interaction remains stable over time. Finally, is there any time lag in the spatial interaction? Therefore in this section we propose a methodological solution to deal with the above problems by presenting the *Multidimensional Spatial Panel Model* – MPSAR.

Following Paelinck [2012] *spatial dependence* should be adopted to the specific region since every region has its own level of economic activity. In particular, as the transport flows are different in urban and rural regions the *spillover effect* may vary from region to region. Let us assume that the *spatial coefficient* is diversified across some groups of regions and is constant within each club. Let $P=\{\rho_k\}$ denote the set of all possible spatial coefficients. Therefore with N spatial observations the maximal cardinality of the set is also N. In the classic spatial setting the estimation of such an econometric model is not feasible. In the space-time case, however, theoretically one can consider the estimation of the parameters over a time dimension obtaining different spatial coefficients for each or almost each location.[1] If the number of time observations is relatively small, one can still consider a coefficient for just a few specific locations. Henceforth we will call the former the *local spatial effect* ρ_i and the latter the *group specific coefficient* ρ_k^{gr} which we will defined more precisely below.

Therefore in the first case the *local spatial effect* $\rho_{i,t}$ measures the power of spatial interaction for a specific location i with its neighbours $N(i)$ over time. In particular it indicates the strength of influence of region i on its neighbours or how the neighbours affect region i. Alternatively, we could exclude some locations and focus (within one slice of the multidimensional weight matrix) on only a few regions of interest. It should be emphasised that in our model the set of neighbours $N(i)$ is allowed to change over time as the spatial weight matrix can change over time (e.g. different $\mathbf{W}^t$ for the old and new Schengen Area). In the same way, the power of interaction with its neighbours may change over time.

The *group spatial coefficient* $(\rho^{gr})_k$ averages spatial effect within a group of interest. For example, considering the EU regions, the subject of interest might be the spatial spillover effect separately among new-member and old-member countries. The group spatial coefficient for the first cluster accounts for spatial interaction specific for new member countries, while the second one explains spatial dependence within the group of old member countries. Alternatively we could consider clusters for countries with similar convergence objectives. In this case the group spatial coefficient embodies the spatial spillover effect among regions of countries with similar convergence objectives. Another example of the use of this would be the group spatial coefficient as a differentiation of spatial effect for strong and catching-up economies, or any other in cases where we expect significantly dissimilar spatial dependences between groups, but similar within the group. Analogically to the *local spatial effect,* the *group spatial effect* may vary over time – $(\rho^{gr})_{k,t}$. It is noteworthy that theoretically each group can be a singleton with only one region making the *local spatial effect* a special case of the *group specific coefficient*.

The *dynamic spatial effect* δ^d_{it} reflects the power of spatial interaction for a specific location i in moment t with its neighbours $N(i)$ at the moment t+1, t+2,… . Hence it can embody an economic spatial interaction which is spread with a certain delay (e.g. economic growth). Let us notice that in some cases the *dynamic spatial coefficient* may appear even more appropriate than the classic one.

[1] Assuming that the number of time observations T is sufficiently large.

In the case of *Multidimensional Spatial Autoregressive Model* – MSAR the spatial structure is represented by the **W** or **D** matrix. The typical weight matrix **W** considers only neighbourhood relations and omits the power of interactions. The multidimensional spatial matrix **D** is built a priori to incorporate the varied strength of interactions between neighbours. In effect, in the MSAR model the information about the spatial structure must be put into the model. On the other hand, in the *Multidimensional Spatial Panel Model* (MPSAR) the information on the strength and significance of the spatial local interactions is given by the model. Therefore, the MPSAR *model* takes the following form:

$$\mathbf{y}^{(it)} = \Upsilon \mathbf{y}^{(it)} + \mathbf{X}^{(it)}\boldsymbol{\beta} + \mathbf{u} \quad \mathbf{u} \sim N\left(\mathbf{0}, \sigma^2 \mathbf{I}\right), \tag{3}$$

with **y** as dependent variable ($TN\times 1$), $i=1,\ldots,N$, t=1, …, T, **X** – ($TN\times K$) – matrix of explanatory variables and $\boldsymbol{\Upsilon}$ – *spatio-temporal structure* ($TN\times TN$).

In the above model the *multidimensional spatio-temporal structure* is represented by $\boldsymbol{\Upsilon}$ term:

$$\Upsilon_{ij}^{(TN\times TN)} = \sum_{\eta=1}^{E} \mathbf{P}_{ij}^{\eta} \mathbf{D}_{ij}^{\eta}, \tag{4}$$

where **P** refers to the *multidimensional spatial coefficient matrix* reflecting the power of spatial interactions and **D** is the *multidimensional spatial weight matrix* which maps geographical proximity or indicates which locations do interact. Letters *i,j* refer to locations and index η denotes the economic dimension.

The spatial coefficients in **P** matrix are allowed to vary for both: different regions, group of regions, time and economic dimension. Therefore a matrix of spatial coefficients in general combines the local, group and dynamic effect as well as economic factors. Thus the dimension of matrix **P** is ($TN\times TN\times E$) where E denotes the economic dimension. Hence for fixed index η, matrix **P** is given by the formula:

$$\mathbf{P}^{\eta} = \begin{bmatrix} \boldsymbol{\Phi}^{\eta,1} & 0 & \ldots & 0 & 0 \\ \boldsymbol{\Delta}_1^{\eta,2} & \boldsymbol{\Phi}^{\eta,2} & \ldots & 0 & 0 \\ \boldsymbol{\Delta}_2^{\eta,3} & \boldsymbol{\Delta}_1^{\eta,3} & \ddots & 0 & 0 \\ \vdots & \vdots & & \boldsymbol{\Phi}^{\eta,T-1} & 0 \\ \boldsymbol{\Delta}_{T-1}^{\eta,T} & \boldsymbol{\Delta}_{T-2}^{\eta,T} & \ldots & \boldsymbol{\Delta}_1^{\eta,T} & \boldsymbol{\Phi}^{\eta,T} \end{bmatrix}_{(TN\times TN)}, \tag{5}$$

where:

$$\mathbf{\Phi}_{(N\times N)}{}^{\eta,t}=\boldsymbol{\iota}\otimes\left[\rho_1^{\eta,t}\quad\dots\quad\rho_N^{\eta,t}\right],\ \mathbf{\Delta}_d^{\eta,t}=\boldsymbol{\iota}\otimes\left[\delta_1^{\eta,t,d}\quad\dots\quad\delta_N^{\eta,t,d}\right],\text{ with }\boldsymbol{\iota}=\begin{bmatrix}1\\ \vdots\\ 1\end{bmatrix}$$

as vector of ones and $\otimes$ – Kronecker product.

The corresponding *multidimensional spatial weight matrix* **D** takes the form:

$$\mathbf{D}^{\eta}=\begin{bmatrix}\mathbf{W}^{\eta,1} & 0 & \dots & 0 & 0\\ \mathbf{\Omega}_1^{\eta,2} & \mathbf{W}^{\eta,2} & \dots & 0 & 0\\ \mathbf{\Omega}_2^{\eta,3} & \mathbf{\Omega}_1^{\eta,3} & \ddots & 0 & 0\\ \vdots & \vdots & & \mathbf{W}^{\eta,T-1} & 0\\ \mathbf{\Omega}_{T-1}^{\eta,T} & \mathbf{\Omega}_{T-2}^{\eta,T} & \dots & \mathbf{\Omega}_1^{\eta,T} & \mathbf{W}^{\eta,T}\end{bmatrix}_{(TN\times TN)},\quad \eta\le E, \tag{6}$$

where E is an economic dimension, **W** refers to the spatial structure which is allowed to change with time and Ω to matrices of spatio-dynamic interactions.

As the *multidimensional spatio-temporal structure* is represented by the $(TN\times TN)$ $\mathbf{\Upsilon}$ term, its introduction into the spatio-temporal econometric model does not require a special method of estimation. Let us also notice that small sample research findings might not reflect the true spatial structure.

4. Theoretical example

From the viewpoint of policy-making, the analyses of economic processes at NUTSII level seem to be attractive as they can provide detailed knowledge about local conditions. In some applications of the New Economic Geography theory and regional development it might be essential to apply the *multidimensional spatio-temporal structure* [Fingleton 2006, pp. 501–530]. Let us notice that both regio-specific as well as Schengen-group-specific spatial coefficients seem to be important in the context of the EU improvement of the cross-border infrastructure within the EU. In this setting we provide a theoretical example which might turn out to be helpful in the context of EU policy making.

In this section we propose a theoretical example – to be more precise – the suggestion of matrix **P**. Let us consider five economic layers (E=5) and an arbitrary but sufficiently large T. Notice that for the simplification of the notation, the order of the regions is changed with every $\eta \le E$ (the economic dimension).[2]

It is argued that distinguishing the strength of influence between the core and peripheral regions is of great importance [see Fujita et al., 1999]. In the case of EU

[2] All the data used in this study are taken from the regional database of Eurostat: http://epp.eurostat.ec.europa.eu

NUTS II regions, from a set of all 271 regions we can choose a subset of regions being local centres. As a *core region* we understand regions with a high level of innovation, technology, employment and with a high potential for innovation and growth (e.g. metropolitan regions). Therefore it seems to be justified to assume that core regions have a greater impact on their neighbours than peripheral ones. Furthermore, they interact within a greater radius as they affect more distant regions due to the extensive trade and labour market. Thus, the suggested coefficient matrix $\mathbf{\Phi}^{1,t}$ would take the following form:

$$\mathbf{\Phi}^{1,t} = diag\left[\rho^{1,t}_{\text{core}},\ldots,\rho^{1,t}_{\text{core}},0,\ldots,0\right] \tag{7}$$

with $\rho^{1,t}_{\text{core}}$ denoting the spatial coefficient for core regions in a certain *t*-moment in time.

Moreover, let us notice that access to information and mobile technology as well as developed communication infrastructure can influence the spatial interaction between core regions. Hence, the geographical distance between them becomes less important. We propose six spatial coefficients for core EU regions: ρ_{L} – for London (UKI region of NUTS I level), ρ_{IL} – for Île-de-France, ρ_{BX} – for the strongest local economies of Benelux (Belgium NUTS II regions: BE21, 24 and 10 which comprise Antwerpen, Vlaams-Brabant and Region de Bruxelles-Capitale; Denmark; Nederland NUTS II regions: NL11, 31, 32, 33 and 41, which consist of Groningen, West Nederland and Noord-Brabant), for Hamburg and Bremen – ρ_{HH}, ρ_{AL} for Åland and finally ρ_{BL} for Bratislavský kraj.

$$\mathbf{\Phi}^{2,t} = diag\left[\rho^{2,t}_{\text{L}},\rho^{2,t}_{\text{IL}},\rho^{2,t}_{\text{BX}},\rho^{2,t}_{\text{HH}},\rho^{2,t}_{\text{AL}},\rho^{2,t}_{\text{BL}},0,\ldots,0\right] \tag{8}$$

Apart from the core regions impact, the subject of interest is also a *spatial slipover effect* among new and old member countries. Then the *group spatial coefficient* ρ_{new} – would account for spatial interaction specific for the Czech Republic, Estonia, Hungary, Latvia, Lithuania, Poland, Slovakia, and Slovenia, Malta, Cyprus, Bulgaria and Romania. The second coefficient – ρ_{old} – explains the spatial dependence within the group of EU member states before 2004.

$$\mathbf{\Phi}^{3,t} = diag\left[\rho^{3,t}_{\text{new}},\rho^{3,t}_{\text{old}},0,\ldots,0\right] \tag{9}$$

Furthermore, it seems to be justified to assume that each country has its own spillover level. This effect constitutes the *country-specific* component and *cross border effect* due to language and cultural barriers as well as institutional limitations. Therefore we propose 27 state-specific spatial coefficients for each member country:

$$\mathbf{\Phi}^{4,t}=diag\begin{bmatrix}\rho_{\mathrm{AT}}^{4,t},\rho_{\mathrm{BE}}^{4,t},\rho_{\mathrm{BG}}^{4,t},\rho_{\mathrm{CY}}^{4,t},\rho_{\mathrm{CZ}}^{4,t},\rho_{\mathrm{DK}}^{4,t},\rho_{\mathrm{EE}}^{4,t},\rho_{\mathrm{FI}}^{4,t},\rho_{\mathrm{BE}}^{4,t},\rho_{\mathrm{FR}}^{4,t},\ \rho_{\mathrm{DE}}^{4,t},\ \rho_{\mathrm{GR}}^{4,t},\ \rho_{\mathrm{HU}}^{4,t},\\ \rho_{\mathrm{FR}}^{4,t},\ \rho_{\mathrm{IE}}^{4,t},\ \rho_{\mathrm{IT}}^{4,t},\rho_{\mathrm{LV}}^{4,t},\ \rho_{\mathrm{LT}}^{4,t},\ \rho_{\mathrm{LU}}^{4,t},\ \rho_{\mathrm{MT}}^{4,t},\ \rho_{\mathrm{NL}}^{4,t},\ \rho_{\mathrm{PL}}^{4,t},\ \rho_{\mathrm{PT}}^{4,t},\ \rho_{\mathrm{RO}}^{4,t},\ \rho_{\mathrm{SK}}^{4,t},\\ \rho_{\mathrm{SI}}^{4,t},\ \rho_{\mathrm{ES}}^{4,t},\ \rho_{\mathrm{SE}}^{4,t},\ \rho_{\mathrm{GB}}^{4,t},0,\ldots,0\end{bmatrix} \quad 10$$

For the spatio-dynamic relation we consider the core-peripheral relation with a one year delay. These coefficients for each core regions reflect the speed of the spread of influence on their neighbouring regions:

$$\mathbf{\Delta}_1^{5,t}=diag\left[\delta_{\mathrm{L}}^{5,t},\delta_{\mathrm{IL}}^{5,t},\delta_{\mathrm{BX}}^{5,t},\delta_{\mathrm{HH}}^{5,t},\delta_{\mathrm{AL}}^{5,t},\delta_{\mathrm{BL}}^{5,t},0,\ldots,0\right] \tag{11}$$

In this case the *multidimensional spatial coefficient matrix* **P** consists of 37 separate theoretical parameters. The corresponding multidimensional spatial weight matrix **D** is of the dimensions: *TN*x*TN*x5. Therefore, the first slice of matrix **D** represents geographical proximity, whereas the second reflects dependence only between core regions. The third slice regards separate spatial interactions for new and old member countries. The next layer represents the neighbourhood within the EU countries as it is assumed that regions interact differently within and outside the country. The last one distinguishes old and new EU countries. The matrix of spatio-dynamic interactions **Ω** represent only the delayed influence of core regions on the neighbouring ones.

The example above does not exhaust the topic of multidimensional spatial structures in this context, but we believe that it facilitates the understanding of the general idea and reveals the possibilities of its application. Moreover, it should be considered more as an inspiration for future studies than a ready formula.

5. Conclusion

This paper is fundamentally based on the concept of a spatial autoregressive model. However, this approach differs from the classical spatial approach. In our work the importance of the assessment of the *multidimensional space-time structure* is primarily emphasised. It has also been argued that taking into account the time dimension makes it possible to fully describe the pattern of spatial dependence structure. This paper explains the advantage of the use of the MPSAR model in which the information on the strength and significance of the spatial local interactions is given by the model. It has been stressed that the failure to recognise these multidimensional effects may lead to an incorrect inference and therefore to biased conclusions. In the multidimensional spatio-temporal analysis, different forms of spatial interactions across different spatial objects are allowed for to successfully model the complex structure of economic processes. The advantages and limitations of the presented approach have also been discussed.

A number of important restrictions to the present study need to be noted. Firstly, for a small sample research, caution must be applied as the findings might not reflect the true spatial structure. Secondly, the current study has only presented some aspects of applications in regional science, and did not evaluate the use of our techniques in other economic fields. Thirdly, those presentations are limited to an illustration with a theoretical example.

Further work needs to be done to create a proper technical tool and to make use of our findings in the designed experiment. The need to design a test for the significance of the spatial coefficients is recognized. Also a method of employment of the *multidimensional spatio-temporal structure* to the matrix of explanatory variables for a description of possible sectorial effects will be presented in a future paper.

Concluding, it is believed that this idea of an MPSAR model will find a wide range of applications in spatial econometrics. We hope that our contribution will draw the attention of researchers to the interesting topic of *multidimensional spatio-temporal structure* and will encourage the consideration and employment of these ideas in empirical studies in regional science.

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PERSPEKTYWY I OGRANICZENIA PANELOWEGO WIELOWYMIAROWEGO AUTOREGRESYJNEGO MODELU PRZESTRZENNEGO

Streszczenie: Przedmiotem referatu jest ocena procesu przestrzenno-czasowego z zastosowaniem wielowymiarowej macierzy wag przestrzennych. W szczególności zakłada się, że podejście wielowymiarowe pozwala na lepszy opis struktury zależności przestrzennych. Praca ma przybliżyć nowo opracowaną metodologię dotyczącą wielowymiarowego autoregresyjnego modelu przestrzennego WAMP z uwzględnieniem wymiaru czasowego. W opracowaniu omówiono wady oraz zalety prezentowanej metodologii. Zwrócono uwagę na konieczność prawidłowej oceny struktury przestrzennej z uwzględnieniem efektów wielowymiarowych, których pominięcie może prowadzić do błędnego wnioskowania. Zatem całość rozważań stanowi nowy element ekonometrii przestrzennej, a poprzez włączenie dodatkowej informacji na temat badanego zjawiska umożliwia wnikliwszą jego analizę.

Słowa kluczowe: ekonometria przestrzenna, procesy przestrzenno-czasowe, modele panelowe.