

DIDACTICS OF MATHEMATICS

9(13)



The Publishing House
of Wrocław University of Economics
Wrocław 2012

Reviewers cooperating with the journal
*Giovanna Carcano, Salvatore Federico, Marian Matłoka,
Włodzimierz Odyniec, Anatol Pilawski, Achille Vernizzi, Henryk Zawadzki*

Copy-editing
Elżbieta Macauley, Tim Macauley, Marcin Orszulak

Proof reading
Barbara Łopusiewicz

Typesetting
Elżbieta Szlachcic

Cover design
Robert Mazurczyk

Front cover painting: W. Tank, Sower
(private collection)

This publication is available at: www.journal.ue.wroc.pl and www.ibuk.pl

Abstracts of published papers are available in the international database
The Central European Journal of Social Sciences and Humanities
<http://cejsh.icm.edu.pl>

Information of submitting and reviewing paper is available
on the Publishing House's website www.wydawnictwo.ue.wroc.pl

All rights reserved. No part of this book may be reproduced in any form
or in any means without the prior written permission of the Publisher

© Copyright by Wrocław University of Economics
Wrocław 2012

PL ISSN 1733-7941

The original version: printed

Printing: Printing House TOTEM
Print run: 200 copies

Table of contents

Marek Biernacki, Katarzyna Czesak-Woytala	
Efficiency of mathematical education in Poland.....	5
Marek Biernacki, Wiktor Ejsmont	
Optimum class size. Testing Lazear’s theorem with intermediate mathematics scores in Polish secondary schools	15
Katarzyna Cegielka	
Composition of the European Parliament in the 2014-2019 term...	25
Piotr Dniestrzański	
Degressively proportional functions using the example of seat distribution in the European Parliament.....	35
Piotr Dniestrzański	
Systems of linear equations and reduced matrix in a linear algebra course for economics studies.....	43
Wojciech Rybicki	
Some reasons why we should teach matrices to students of economics.....	55
Wojciech Rybicki	
Further examples of the appearance of matrices (and the role they play) in the course of the economists’ education	75
OPINIONS	
Jacek Juzwiszyn	
6 th European Congress of Mathematics – report of the participation ..	91

COMPOSITION OF THE EUROPEAN PARLIAMENT IN THE 2014-2019 TERM

Katarzyna Cegielka

Abstract. Currently, representatives of 28 countries form the European Parliament. Their populations are characterized by a large disparity which leads to the need to seek methods of allocating the seats which are not based on proportional methods. They should have fulfilled the conditions of degressive proportionality by 2009. Although scientists have so far offered various solutions in line with the assumptions, MEPs did not take any of them. Furthermore, they changed the interpretation of the new rule in subsequent terms of office. At the same time, they have not determined a composition of the European Parliament which meets the conditions of degressive proportionality. In the paper, the author presents the degressive proportionality principle and analyzes the composition of the European Parliament in 2014-2019 term proposed by MEPs.

Keywords: European Union, European Parliament, degressive proportionality, fair division, indivisible goods.

1. Introduction

The large variation in the population of the Member States of the European Union prevents the use of proportional methods of distribution of seats in the European Parliament. A new principle was thus introduced – “the rule of degressive proportionality”, whereby members of Parliament from countries with a smaller population represent fewer citizens than the envoys from countries with a larger population. It appears, however, that the practical application of such a solution is not an easy task. Although MEPs declare that “the ideal alternative would be to agree on an undisputed mathematical formula of “degressive proportionality” that would ensure a solution not only for the present revision but for future enlargements or modifications due to demographic changes” (Report, 2007) they did not accept any of the developed solutions. A multitude of unknowns and the lack of a determined position of MEPs means that the problem of unification of the procedures for selecting the composition of the European Parliament is still not resolved.

Katarzyna Cegielka

Department of Mathematics and Cybernetics, Wrocław University of Economics, Komandorska Street 118/120, 53-345 Wrocław, Poland.

E-mail: katarzyna.cegielka@ue.wroc.pl

2. Introduction of degressive proportionality rule

According to Article 1 point 15 of the Lisbon Treaty, a new article is added:

15) An Article 9 A shall be inserted:

Article 9 A

1. The European Parliament shall, jointly with the Council, exercise legislative and budgetary functions. It shall exercise functions of political control and consultation as laid down in the Treaties. It shall elect the President of the Commission.

2. The European Parliament shall be composed of representatives of the Union's citizens. They shall not exceed seven hundred and fifty in number, plus the President. Representation of citizens shall be degressively proportional, with a minimum threshold of six members per Member State. No Member State shall be allocated more than ninety-six seats. The European Council shall adopt by unanimity, on the initiative of the European Parliament and with its consent, a decision establishing the composition of the European Parliament, respecting the principles referred to in the first subparagraph.

3. The members of the European Parliament shall be elected for a term of five years by direct universal suffrage in a free and secret ballot.

4. The European Parliament shall elect its President and its officers from among its members (Treaty, 2007).

The Treaty explicitly indicates a degressively proportional form of representation of citizens. Nevertheless, it does not define the new principle – the added article only introduces a new idea. A further description of degressive proportionality was, for the first time, included in the Report of the Committee on Constitutional Affairs and the European Parliament Resolution, the draft of which is attached to the Report. According to Article 1 of Annex 1 of the European Parliament Resolution on the composition of the European Parliament (Report, 2007):

The principle of degressive proportionality provided for in Article [9a] of the Treaty on European Union shall be applied as follows:

– the minimum and maximum numbers set by the Treaty must be fully utilised to ensure that the allocation of seats in the European Parliament reflects as closely as possible the range of populations of the Member States;

– the larger the population of a country, the greater its entitlement to a large number of seats;

– *the larger the population of a country, the more inhabitants are represented by each of its Members of the European Parliament.*

The 2007 Report of the Committee on Constitutional Affairs contains additional rules describing degressive proportionality (Treaty, 2007):

- **The principle of efficiency:** the European Parliament cannot function with too many members, so it is necessary to limit the maximum number of deputies to 751.

- **The principle of national representation and motivation of voters:** to provide appropriate representation of national political trends and mobilize a country's citizens to vote and participate in the democratic processes of the European Union, each Member State receives a minimum number of seats.

- **The principle of European solidarity:** to allow better representation for less populated states, more populated states receive fewer seats than they would receive using a proportional allocation.

- **The principle of the relative proportionality:** the ratio of population to the number of seats is greater, the greater the state and respectively smaller, the smaller the state.

- **The principle of fair distribution:** no country will be given fewer seats than a less populated state and more seats than a state with a larger population.

- **The principle of the justified flexibility or of a flexible direct proportionality:** the number of granted seats can be modified if it levels off to the largest possible extent the differences between countries, and other principles are followed.

The rules mentioned in the Report and the European Parliament Resolution allow to specify the conditions of degressively proportional allocation of seats. For n being the number of Member States, l_i – population of the country i and m_i – the number of mandates of the country i one can write them as follows:

$$W1. \sum_{i=1}^n m_i = 751, 6 \leq m_i \leq 96.$$

$$W2. l_1 < l_2 < \dots < l_n \Rightarrow m_1 \leq m_2 \leq \dots \leq m_n.$$

$$W3. l_1 < l_2 < \dots < l_n \Rightarrow \frac{l_1}{m_1} < \frac{l_2}{m_2} < \dots < \frac{l_n}{m_n}.$$

3. The debate over the adoption of a particular method of division

In February 2011, at the meeting of the Committee on Constitutional Affairs, a group of mathematicians led by Professor Geoffrey Grimmett, who had been asked by members of the European Parliament to develop a solution to the problem of allocation of seats, presented a proposal to standardize the composition of the European Parliament. Scientists proposed a “base+prop” method which is also known as the “Cambridge Compromise”. According to the proposed solution, each state receives a certain number of seats (“base”) and then the remaining number of seats is divided by one of the classic methods of proportional allocation (“prop”). They inferred that the best choice is the base equal to five mandates and division of the Adams divisor method (assuming rounding fractions up to the nearest whole integer). This way each member receives a minimum of six seats guaranteed in the Treaty of Lisbon. The authors, in their considerations, went even further (Grimmett, 2011). They deliberated that - apart from the introduction of an algorithm developed by them – there should also be a change in the definition of degressive proportionality as proposed by A. Lamassoure and A. Severin in the Report of the Committee on Constitutional Affairs on the composition of the European Parliament from 2007: [The European Parliament] “[...] considers that the principle of degressive proportionality means that the ratio between the population and the number of seats of each Member State must vary in relation to their respective populations in such a way that each Member from a more populous Member State represents more citizens than each Member from a less populous Member State and conversely, but also that no less populous Member State has more seats than a more populous Member State” (Report, 2007).

The mathematicians proposed the following changes: [The European Parliament] “[...] considers that the principle of degressive proportionality means that the ratio between the population and the number of seats of each Member State **before rounding to whole numbers** must vary in relation to their respective populations in such a way that each Member from a more populous Member State represents more citizens than each Member from a less populous Member State and conversely, but also that no less populous Member State has more seats than a more populous Member State” (Grimmett, 2011).

Members admit that the “fix-prop” method guarantees respect of the degressive proportionality rule but, on the other hand, they criticize it for returning a division in which medium-sized and small member states lose,

and larger ones gain, too many mandates – in other words “its implementation would trigger a traumatic reallocation of seats” (Report, 2013). Therefore the algorithm proposed by the scientists was not accepted. Part of their work, however, has gained the acceptance of MEPs. The definition of degressive proportionality referring to the ratio between the population and the number of seats has been weakened. The report on the composition of the European Parliament with a view to the 2014 elections contains a changed interpretation of the discussed principle (Report, 2013):

In the application of the principle of degressive proportionality provided for in the first subparagraph of Article 14(2) TEU, the following principles shall apply:

- *the allocation of seats in the European Parliament shall fully utilise the minimum and maximum numbers set by the Treaty in order to reflect as closely as possible the sizes of the respective populations of Member States;*
- *the ratio between the population and the number of seats of each Member State, before rounding to whole numbers, shall vary in relation to their respective populations in such a way that each Member of the European Parliament from a more populous Member State represents more citizens than each Member from a less populous Member State and, conversely, that the larger the population of a Member State, the greater its entitlement to a large number of seats.*

The new conditions of degressive proportionality may be written as follows:

$$V1. \sum_{i=1}^n m_i = 751, 6 \leq m_i \leq 96 \text{ for } i = 2, \dots, n-1, m_1 = 6, m_n = 96.$$

$$V2. l_1 < l_2 < \dots < l_n \Rightarrow m_1 \leq m_2 \leq \dots \leq m_n.$$

$$V3. l_1 < l_2 < \dots < l_n \Rightarrow \frac{l_1}{A(m_1)} < \frac{l_2}{A(m_2)} < \dots < \frac{l_n}{A(m_n)}.$$

Where $A(x)$ is a function assigning the number of seats to the number of citizens of a country.

4. Distribution of seats in the 2014-2019 term

Currently there are 754 members in Parliament – 736 elected in 2009 and 18 appointed under the Lisbon Treaty. After the Accession Treaty came into force, Croatia obtained 12 seats. As a result the total number of mandates is 766, therefore 15 seats need to be reduced. This reduction is to be

made in accordance with the principle that “nobody gains and nobody loses more than one” mandate (Report, 2013).

Table 1. Incompatibility of step 1 of the pragmatic solution with the third condition of degressive proportionality

Member States	Population	Seats (step 1)	Ratio pop./seats (step 1)	Member States	Population	Seats (step 1)	Ratio pop./seats (step 1)
Germany	81843743	96	852539	Austria	8443018	19	444369
France	65397912	78	838435	Bulgaria	7327224	17	431013
United Kingdom	62989550	76	828810	Denmark	5580516	13	429270
Italy	60820764	74	821902	Slovakia	5404322	13	415717
Spain	46196276	57	810461	Finland	5401267	13	415482
Poland	38538447	51	755656	Ireland	4582769	11	416615
Romania	21355849	31	688898	Croatia	4398150	11	399832
Netherlands	16730348	26	643475	Lithuania	3007758	9	334195
Greece	11290935	20	564547	Slovenia	2055496	7	293642
Belgium	11041266	20	552063	Latvia	2041763	7	291680
Portugal	10541840	20	527092	Estonia	1339662	6	223277
Czech Republic	10505445	20	525272	Cyprus	862011	6	143669
Hungary	9957731	19	524091	Luxembourg	524853	6	87476
Sweden	9482855	19	499098	Malta	416110	6	69352
TOTAL						751	

Source: (Report, 2013).

Such an approach was introduced by rapporteurs of the 2013 Report – Roberto Gualtieri and Rafał Trzaskowski – who proposed an allocation which, as they indicate, is not degressively proportional but to the slightest degree deviates from the existing distribution. In the same way they argue the rejection of the Cambridge Compromise and V. Ramirez-Gonzalez’s Parabolic method¹ - in their opinion divisions obtained by these methods are largely different from the present composition. Instead, the rapporteurs propose a “pragmatic solution” that can be achieved through a two-step approach. As we may read in the 2013 Report, “the first step is a reallocation fully in line with the three principles of degressive proportionality and,

¹ For the details see (Ramirez-Gonzalez, 2007).

at the same time, involving as little change as possible in the number of seats” (Report, 2013). Notwithstanding that, this statement is not true. Firstly, the Finland–Ireland pair does not satisfy the third condition of degressive proportionality (see Table 1).

Secondly, the proposed division is supposed to involve as little change as possible in the number of mandates. The authors do not explain how they measure the distance between the new and current division. It seems natural to use one of the distance measures: Euclidean, Manhattan or Chebyshev. It turns out that, interpreting the distance in one of the mentioned ways, the allocation proposed in step 1 is not the nearest one (see Table 2).

Table 2. Distances between current and proposed divisions

Member States	Seats (current division)	Step 1 GT2013	Manhattan	Euclidean	Chebyshev	Nearer division	Manhattan	Euclidean	Chebyshev
Germany	99	96	3	9	3	96	3	9	3
France	74	78	4	16	4	77	3	9	3
United K.	73	76	3	9	3	75	2	4	2
Italy	73	74	1	1	1	73	0	0	0
Spain	54	57	3	9	3	57	3	9	3
Poland	51	51	0	0	0	50	1	1	1
Romania	33	31	2	4	2	31	2	4	2
Netherlands	26	26	0	0	0	25	1	1	1
Greece	22	20	2	4	2	20	2	4	2
Belgium	22	20	2	4	2	20	2	4	2
Portugal	22	20	2	4	2	20	2	4	2
Czech R.	22	20	2	4	2	20	2	4	2
Hungary	22	19	3	9	3	20	2	4	2
Sweden	20	19	1	1	1	20	0	0	0
Austria	19	19	0	0	0	18	1	1	1
Bulgaria	18	17	1	1	1	17	1	1	1
Denmark	13	13	0	0	0	13	0	0	0
Slovakia	13	13	0	0	0	13	0	0	0
Finland	13	13	0	0	0	13	0	0	0
Ireland	12	11	1	1	1	12	0	0	0
Croatia	12	11	1	1	1	12	0	0	0
Lithuania	12	9	3	9	3	9	3	9	3
Slovenia	8	7	1	1	1	8	0	0	0
Latvia	9	7	2	4	2	8	1	1	1
Estonia	6	6	0	0	0	6	0	0	0
Cyprus	6	6	0	0	0	6	0	0	0
Luxembourg	6	6	0	0	0	6	0	0	0
Malta	6	6	0	0	0	6	0	0	0
TOTAL	766	751	37	91	4	751	31	69	3

Source: own elaboration.

In the second step, Member States like Germany lose 3 seats and 12 of 13 countries which have lost mandates in step 1 lose one mandate (Romania, Greece, Belgium, Portugal, the Czech Republic, Hungary, Sweden, Bulgaria, Ireland, Croatia, Lithuania, Latvia,). Slovenia, being more populated than Latvia, does not lose any mandates (see Table 3).

Table 3. Pragmatic solution proposed in 2013 Report

Member States	Population	Seats (step 1)	Difference	Ratio pop./seats (step 1)	Seats (step 2)	Difference	Ratio pop./seats (step 2)
Germany	81843743	96	minus 3	852539	96	minus 3	852539
France	65397912	78	plus 4	838435	74		883756
United Kingdom	62989550	76	plus 3	828810	73		862871
Italy	60820764	74	plus 1	821902	73		833161
Spain	46196276	57	plus 3	810461	54		855487
Poland	38538447	51		755656	51		755656
Romania	21355849	31	minus 2	688898	32	minus 1	667370
Netherlands	16730348	26		643475	26		643475
Greece	11290935	20	minus 2	564547	21	minus 1	537664
Belgium	11041266	20	minus 2	552063	21	minus 1	525775
Portugal	10541840	20	minus 2	527092	21	minus 1	501992
Czech Republic	10505445	20	minus 2	525272	21	minus 1	500259
Hungary	9957731	19	minus 3	524091	21	minus 1	474178
Sweden	9482855	19	minus 1	499098	19	minus 1	499098
Austria	8443018	19		444369	19		444369
Bulgaria	7327224	17	minus 1	431013	17	minus 1	431013
Denmark	5580516	13		429270	13		429270
Slovakia	5404322	13		415717	13		415717
Finland	5401267	13		415482	13		415482
Ireland	4582769	11	minus 1	416615	11	minus 1	416615
Croatia	4398150	11	minus 1	399832	11	minus 1	399832
Lithuania	3007758	9	minus 3	334195	11	minus 1	273433
Slovenia	2055496	7	minus 1	293642	8		256937
Latvia	2041763	7	minus 2	291680	8	minus 1	255220
Estonia	1339662	6		223277	6		223277
Cyprus	862011	6		143669	6		143669
Luxembourg	524853	6		87476	6		87476
Malta	416110	6		69352	6		69352
TOTAL		751			751		

Source: (Report, 2013).

Table 4. Pragmatic solution with the usage of “nearer division”

Member States	Population	Seats (step 1)	Difference	Ratio pop./seats (step 1)	Seats (step 2)	Difference	Ratio pop./seats (step 2)
Germany	81843743	96	minus 3	852539	96	minus 3	852539
France	65397912	77	plus 3	849324	74		883756
United Kingdom	62989550	75	plus 2	839861	73		862871
Italy	60820764	73		833161	73		833161
Spain	46196276	57	plus 3	810461	54		855487
Poland	38538447	50	minus 1	770769	50	minus 1	770769
Romania	21355849	31	minus 2	688898	32	minus 1	667370
Netherlands	16730348	25	minus 1	669214	25	minus 1	669214
Greece	11290935	20	minus 2	564547	21	minus 1	537664
Belgium	11041266	20	minus 2	552063	21	minus 1	525775
Portugal	10541840	20	minus 2	527092	21	minus 1	501992
Czech Republic	10505445	20	minus 2	525272	21	minus 1	500259
Hungary	9957731	20	minus 3	497887	21	minus 1	474178
Sweden	9482855	20		474143	20		474143
Austria	8443018	18	minus 1	469057	18	minus 1	469057
Bulgaria	7327224	17	minus 1	431013	17	minus 1	431013
Denmark	5580516	13		429270	13		429270
Slovakia	5404322	13		415717	13		415717
Finland	5401267	13		415482	13		415482
Ireland	4582769	12		381897	12		381897
Croatia	4398150	12		366513	12		366513
Lithuania	3007758	9	minus 3	334195	11	minus 1	273433
Slovenia	2055496	8		256937	8		256937
Latvia	2041763	8	minus 1	255220	8	minus 1	255220
Estonia	1339662	6		223277	6		223277
Cyprus	862011	6		143669	6		143669
Luxembourg	524853	6		87476	6		87476
Malta	416110	6		69352	6		69352
TOTAL		751			751		

Source: own elaboration.

The pragmatic solution based on the rule that nobody gains and nobody loses more than one seat, forces an allocation where the most populated countries do not receive any more seats, so the third condition of degressive proportionality in relation to Germany, France, the United Kingdom and

Spain cannot be obtained.² However, following the procedure of step 2, using the “nearer division” would lead to a distribution which, apart from the listed biggest Member States, would not be consistent with degressive proportionality for only one pair of countries³ (see Table 4).

5. Conclusions

The lack of a specific algorithm on the basis of which the composition of the European Parliament could be determined, has caused many difficulties. Since the degressive proportionality rule was introduced any allocation of seats has not met its conditions. Members do not accept any of the methods developed by the scientists, while their own proposals are based on questionable grounds. They issued an assurance that their “decision shall be revised sufficiently far in advance of the beginning of the 2019-2024 parliamentary term with the aim of establishing a system which in future will make it possible, before each fresh election to the European Parliament, to allocate the seats between Member States in an objective, fair, durable and transparent way, based on the principle of degressive proportionality” (Report, 2013). To date, the division of mandates remains inconsistent with the new principle and the unclear rules for its determining preclude an analysis of its correctness.

References

- Grimmett G.R. (2011). *European apportionment via the Cambridge Compromise*. Mathematical Social Sciences.
- Ramirez Gonzalez V. (2007). *The parabolic method for the allotment of seats in the European Parliament among Member States of the European Union*. Real Instituto Elcano. Area Europe – ARI 63/2007.
- Report (2007). *Report on the composition of the European Parliament*. A6-0351/2007.
- Report (2013). *Report on the composition of the European Parliament with a view to the 2014 elections*. A7-0041/2013.
- Treaty (2007). *Treaty of Lisbon*. Official Journal of the European Union. C 306. Volume 50. 17 December 2007.

² If Germany has 96 seats, France needs to have at least 77, the United Kingdom – 75 and Spain – 56 seats.

³ As it is easy to verify, there is no allocation consistent with “nobody gains and nobody loses more than one seat” rule that does not meet the third condition of degressive proportionality for fewer than three pairs of Member States.