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Financial Investment and Insurance –
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MATHEMATICAL RESERVES IN INSURANCE WITH EQUITY FUND VERSUS A REAL VALUE OF A REFERENCE PORTFOLIO

Summary: An insurance company, which wishes to secure its solvency, ought to have at its disposal a certain reserved amount referred to in life insurance as a mathematical reserve of premiums. Calculation methods of mathematical reserves in traditional insurance may be found in classical actuarial literature, according to which the reserve is calculated as actuarial value of accumulated future cash flows including death risk and risk of time value of money change, that is, the so-called actuarial risk. However, insurance companies, which offer complex insurance products such as insurance with equity fund (unit-linked insurance), in order to ensure their solvency in accordance with Solvency II, also ought to consider an additional aspect arising from financial risk. Benefits resulting from this type of policy are directly related to implementation of the reference portfolio and thus their stochastic nature ought to be included in valuation. In this article through combining a financial and actuarial approach, the reserves for the unit-linked insurance are determined as an appropriate conditional expected value including extended actuarial risk and the influence of established investment strategy of the insured on their value is investigated.

Keywords: Equity-linked insurance, value of the reference portfolio in ULIP, solvency, mathematical reserves.

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1. Introduction

It is characteristic of the equity-linked insurance (ELI) as the life insurance that a contract of insurance always terminates with benefit payment. Therefore, the insurance company, which wishes to secure its solvency in the future, ought to have at its disposal a certain reserved amount. Pursuant to the act on insurance activities [Ustawa z 22 maja 2003], an insurance carrier needs to have at its disposal a certain amount of reserves connected with insurance premium. This reserve is called a mathematical reserve of premiums and constitutes the amount accumulated to cover future liabilities. In this article the analysis of mathematical reserves required for the equity-linked insurance (ELI) that the insurance company ought to set including the aspects of Solvency II was carried out. A formula for mathematical reserve including actual risks connected with this type of insurance was modified and it was revealed that an insurer offering the ELI insurance additionally ought to take into consideration a financial risk and combine in valuation the actuarial approach with the financial one within the scope of option valuation (the European and American ones). So, mathematical reserves of premiums in the equity-linked insurance may be treated as a sum of two elements: a traditionally designated mathematical reserve of endowment insurance as the actuarial value of future flows and a risk surplus of the ELI reference portfolio risk determined as the price of a relevant call option.

2. Basic principles – Solvency II

Along with the development of insurance market and occurrence of new products and related risks, the existing requirements ceased to illustrate thoroughly all the risks that the insurance companies were exposed to. This mainly concerned financial risks, e.g. interest rate change risk. The outcome of action taken aiming at security of solvency was coming into existence of a new system called Solvency II [Dyrektywa Parlamentu Europejskiego i Rady 2009/138/WE, Art. 77]. Within this project, requirements of financing are determined and they are adjusted to actual risks that the insurance companies are exposed to.

Insurance activities due to their social and economic significance are subject to a specific supervision of a specialised state administration body. Many requirements assuring security of conducted insurance activities are imposed on insurance companies. A key aspect of regulatory frameworks concerns the necessity of determination of the so-called increased risk capital. In order to do that, the insurer who wishes to secure against fortuitous losses based on valuation of cash flows, ought to determine a relevant level of reserves that ought to counteract the risk borne by the insurer to assure financial security of the insured. The idea of Solvency II consists in a closer dependence of the amount of capital on size of risk taken by insurance companies. In other words, the capital ought to be enough to cover actual risk.

To achieve this aim, the assets of insurers are to be valued based on their actual market value, i.e. reserve valuations ought to be made based on their current disposal value, i.e. the value of reserves corresponds to the current amount that the undertaking would have to pay if it would made an immediate assignment of its contractual rights and obligations onto another undertaking [Wüthrich, Bühlmann, Furrer 2007]. The best assessment of reserves is a discounted value of all future cash flows.

3. Conception of the equity-linked insurance and related cash flows

The insurance with equity fund is an endowment contract between the insured party and the insurer, according to which the insured party pays premiums and the insurance company in return provides a benefit in amount equal to greater than the value of [Hardy 2003]:

– a guaranteed amount (denoted as G_{Π}),
– the sum resulting from the value of reference portfolio depending on price determination of the fund price (denoted as $b(S_t)$).

In the process, the ELI insurance differs fundamentally from classic endowment or life insurance, in that this is related to investment of means coming from premiums into separated funds. In Poland, the equity-linked contracts enable the insured party to accumulate savings within investment portfolio formed by him on an individual basis, consisting of funds conducted independently of the insurer by external investment fund associations. Investment funds differ in terms of risk and investment policy and because these insurance have an open structure and are transparent, they give the insured an opportunity to decide on portfolio composition within the insurance period. Therefore, this is the very party insured which bears responsibility for potential negative results of his decisions and is encumbered with financial risk.

By contrast with classic life insurance, where the insurance cost (represented in a premium paid) is uniform throughout the entire insurance period and does not arise from the size of risk in a particular year, but from the averaged risk of the entire insurance period, in the ELI insurance such cost depends on the age of the insured party and it changes depending on payments, interest, administration costs and burden connected not only with risk of death, but also with the additional financial risk dependent of the price of the fund units. In the equity-linked insurance, the insurer through combination of protective and investment nature pays the insured party at the moment of the occurred event covered by the contract, a higher from the values: the guaranteed amount and market value of portfolio. So, insurance reimbursement at the t-moment is a relevant function of accumulated investment dependent on the price of the fund units and it is equal [Ballotta, Haberman 2006]:

$$b(S_t) = \max\left\{G_\pi, X_t\right\} = G_\pi + \max\left\{0, X_t - G_\pi\right\} = G_\pi + \left(X_t - G_\pi\right)^+ .$$

Then one ought to bear in mind a change in money value in time and determine updated value of payment. In the thesis it is assumed that the process of monetary unit value is set by an equation [Jajuga, Jajuga 2006]:

$$B_t = e^{\delta t}.$$

So, the value of payment updated at the *t*-moment made at the *T*-moment by way of event covered by the contract is equal [Bacinello 2003]:

$$Z_t\left(b,T\right) = e^{-\delta(T-t)} \cdot b(S_T) = e^{-\delta(T-t)} \cdot \left(G_\pi + \left(X_T - G_\pi\right)^+\right) =$$

$$= e^{-\delta(T-t)} \cdot G_\pi + e^{-\delta(T-t)} \cdot \left(S_T \sum_{u=0}^{\tilde{n}_T - 1} \pi_u \cdot S_u^{-1} - \cdot G_\pi\right)^+ .$$

One ought to observe that in case of the ELI insurance, not only the moment of payment is random, but also is its amount.

4. Calculation of reserves for the equity-linked insurance

Pursuant to Solvency II, the best assessment of reserves is the probability-weighted average of future cash flows with allowance for a change in money value in time, with application of a relevant time structure of risk-free interest rates. By reason of this, mathematical reserves of endowment insurance (*EI*) and life insurance (*LI*) ought to be determined as an expected value [Bowers et al. 1997]:

$$V_t(B,\mathcal{F}) = E\left(\int e^{-\delta(\tau-t)} dB(\tau) \mid \mathcal{F}_t\right),$$

where: $\{\mathcal{F}_t\}_{0\le t\le T}$ – filtration defining the history of process at the *t*-moment.

In case of traditional insurance (endowment insurance – *EI* and life insurance – *LI*) with calculation of mathematical reserves, filtration generated by portfolio of insurance is considered and it is based on the mortality process:

$$\mathcal{F}_t = \sigma\{\mathrm{I}(T_i \le t), 0 \le t \le T, i = 1,\dots,l_x\}.$$

So, taking into consideration the fact that the insured party pays premiums of the π fixed value and the insurer pays the benefit in the *b* amount, then classic reserves are determined in the following manner:

$$V_t^{EI\&LI}(B,\mathcal{F}) = E\left(\int_t^{n\vee T} e^{-\delta(\tau-t)} dB(\tau) \mid \{T_i > T\}\right) =$$

$$= b \cdot \left[e^{-\delta T}\,{}_T p_x + \int_t^{n\vee T} e^{-\delta(\tau-t)}\,{}_{\tau-t} p_{x+t}\,\mu(x+t)\,d\tau \right] - \pi \int_t^{n\vee T} e^{-\delta(\tau-t)}\,{}_{\tau-t} p_{x+t}\,d\tau.$$

A new system of solvency assessment consistent with Solvency II is to be adjusted to actual risks that the insurance companies are exposed to. In case of insurance institutions, potential risks are specific to types of contracts concluded. In case of insurance connected with the financial market, a stochastic variation of economic variables is a key factor deciding on future value of liabilities. So, in case of insurance with equity fund, the cash flows ought to be estimated taking into consideration a full risk that the insurer is exposed to [Graf, Kling, Ruß 2011]. Because of this, the history (filtration) may be divided into four groups presented in Figure 1, which ought to be taken into consideration during valuation.

So, valuation made on the basis of the best assessment of future cash flows with allowance for discounting ought to be based on market value of all the risks, i.e. with allowance of filtration determining full information available at the *t*-moment concerning the mortality process and price determination by the financial market, which indicates the necessity of allowance for filtration of the form:

$$\mathcal{F}_t = \mathcal{L}_t \wedge \mathcal{G}_t \wedge \mathcal{H}_t$$
$$= \mathcal{L}_t \wedge \sigma\{(B_t, S_t), 0 \le t \le T\} \wedge \sigma\{\mathrm{I}(T_i \le t), 0 \le t \le T, i = 1, \ldots, l_x\}$$

where: T_i – a future lifetime of the i-th insured party,
l_x – a number of people in the insurance portfolio.

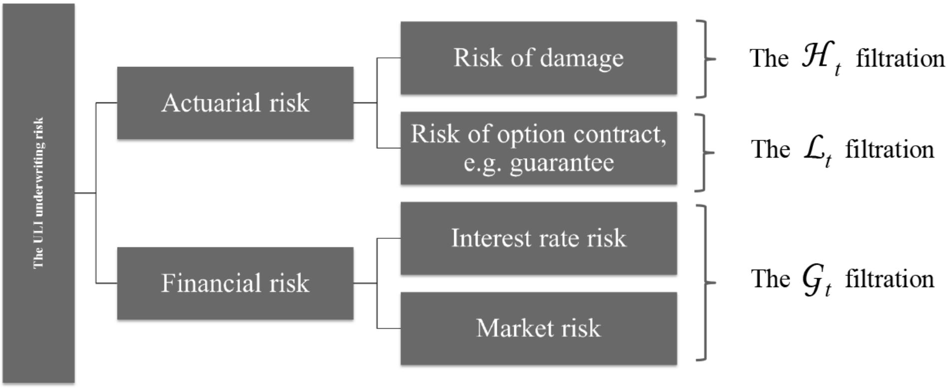

Figure 1. Classification of the UFK risk insurance along with attributed filtration

Source: own elaboration.

The $\mathcal{L}_t$ filtration interpreted as the risk of contract options determines a type of cash flows resulting from a concluded insurance policy. Furthermore, it is assumed that the financial market is ideal and everybody has the same knowledge about such market and information is obtained solely from the S_t observation of price process and the B_t value process of a monetary unit. Then about the σ-body $\mathcal{G}_t$ is interpreted as knowledge acquired until the t-moment, we assume that: $\mathcal{G}_t = \mathcal{G}_t^{S \wedge B}$. While the σ-body $\mathcal{H}_t$ is the knowledge concerning mortality process obtained until the t-moment and information concerning such process determines a future lifetime of the insured party, therefore also it is assumed that: $\mathcal{H}_t = \mathcal{H}_t^T$ So, the $\mathcal{F}_t$ filtration determined full information concerning the mortality process and price determination available at the t-moment. Taking into account the filtration above and thus extended actuarial risk, one ought to determine reserves of equlity-linked insurance as the following conditional expected value of future cash flows:

$$V_t^{ELI}(B, \mathcal{F}) = E\left(\int_t^{n \vee T} e^{-\delta(\tau - t)} dB(\tau) \mid \mathcal{L}_t \wedge \mathcal{G}_t^{S \wedge B} \wedge \mathcal{H}_t^T \right).$$

Assuming the independence of the insurance and financial markets, which generate the history of process, we further obtain:

$$V_t^{ELI}(B,\mathcal{F}) = E\left(\int_t^{n\vee T} e^{-\delta(\tau-t)} dB(\tau) \mid \mathcal{G}_t^{S\wedge B} \wedge \sigma\{\mathrm{I}(T_i \le t), 0 \le t \le T, i = 1,\dots,l_x\} \right)$$
$$= E\left(e^{-\delta(T-t)} b(S_T) \mid \mathcal{G}_t^{S\wedge B}\right) \cdot E\left(\mathrm{I}_{\{T_i>t\}} \mid \{T_i > t\}\right) +$$
$$+ \int_t^{T\vee n} E\left(e^{-\delta(\tau-t)} \cdot b(S_\tau) \mid \mathcal{G}_t^{S\wedge B}\right) \cdot E\left(d\mathrm{I}\{\tau \le T\} \mid \{T_i > t\}\right)$$
$$+ \int_t^{T\vee n} E\left(e^{-\delta(\tau-t)} \cdot \pi(\tau) \mid \mathcal{G}_t^{S\wedge B}\right) \cdot E\left(d\mathrm{I}\{\tau > T\} \mid \{T_i > t\}\right).$$

Through using a functional form of payment for the ELI insurance and probability of death and survival, the formula above adopts the form:

$$V_t^{ELI}(B,\mathcal{F}) = {}_{T-t}p_{x+t} \cdot E\left(e^{-\delta(T-t)} \max\{G_\pi, X_T\} \mid \mathcal{G}_t^{S\wedge B}\right) +$$
$$+ \int_t^{T\vee n} {}_{\tau-t}p_{x+t} \cdot \mu(x+\tau) \cdot E\left(e^{-\delta(\tau-t)} \cdot \max\{G_\pi, X_\tau\} \mid \mathcal{G}_t^{S\wedge B}\right) \cdot d\tau$$
$$+ \int_t^{T\vee n} {}_{\tau-t}p_{x+t} \cdot E\left(e^{-\delta(\tau-t)} \cdot \pi(\tau) \mid \mathcal{G}_t^{S\wedge B}\right) \cdot d\tau.$$

One ought to note that conditional expected value mentioned in the formula above is, by definition, arbitrage price of derivatives, according to which, the arbitrage price of the $h(S_t)$ instrument of the European type at the $t < T$ moment, amounting to assets of price described by the $\{S_t\}_{t\ge 0}$ process and the T maturity date is called the value [Jakubowski 2011]:

$$E\left[e^{-\delta(T-t)} \cdot h(S_T) \mid \mathcal{G}_t\right]$$

Therefore, assuming the notation:

$$E\left[e^{-\delta(T-t)} \cdot (X_T - G_\pi)^+ \mid \mathcal{G}_t\right] = C_t(X_T, G_\Pi)$$

and making further ones, the final formula for mathematical reserve of insurance with equity fund is obtained:

$$V_t^{ELI}(B,\mathcal{F}) = \underbrace{G_\pi \left[e^{-\delta(T-t)} {}_{T-t}p_{x+t} + \int_t^{n\vee T} e^{-\delta(\tau-t)} {}_{\tau-t}p_{x+t}\mu(x+\tau)d\tau \right]}_{V_{EI\&LI}} -$$

$$\underbrace{-\int_t^{n\vee T} e^{-\delta(\tau-t)} {}_{\tau-t}p_{x+t}\pi(\tau)d\tau}_{V_{EI\&LI}} +$$

$$\underbrace{+C_t(X_T, G_\Pi)_{T-t}p_{x+t} + \int_t^{n\vee T} C_t(X_\tau, G_\Pi)_{\tau-t}p_{x+t}\mu(x+\tau)d\tau}_{\text{surlplus } ELI\text{-}V_{(X-G)^+}}$$

The formula above is a generalization of the formula for mathematical reserve in traditional life insurance (endowment insurance & life insurance) and indicates that the insurance reserve with equity fund may be presented as a sum of two elements:

- reserve of traditional insurance,
- surplus of the reference portfolio risk.

The first part of the formula determines the amount of a reserve in traditional insurance (classic endowment insurance or life insurance) with the G_Π amount insured and premiums in the amount of the $\pi(t)$, which is defined as the actuarial value of future cash flows. The second part is the additional part of reserves, that is, a surplus resulting from the financial risk of reference portfolio within the *UFK* insurance. When making its valuation, one ought to combine the actuarial perspective with the financial one within the scope of tools applied in valuation of call option (the European or American ones depending on the endowment insurance or life insurance).

5. Results of simulation

The unit-linked insurance endowment term contract was analysed as the example, pursuant to which the insurer undertakes to pay benefits by way of survival to the end of the insurance period (endowment insurance EI) and also if the insured party dies during the period of its duration (life insurance LI). The insurer shall pay the insured party a free cover limit plus a surplus resulting from the value of reference portfolio at the moment of payment. It has been assumed that the insured party pays premiums in the $\pi(t_k)$ amount at the t_k moments; furthermore a continuous capitalisation is considered. To determine probability of survival and death, the mortality tables are used and they are based on the Gompertz-Makeham law, in accordance to which [Dahl 2004]:

$$l_x = 1000401{,}71 \cdot 0{,}99949255^x \cdot 0{,}99959845^{1{,}10291509^2}.$$

In case of valuation of the equity-linked insurance contracts and in order to determine the best assessment apart from actuarial approach, the issue of a proper valuation of a financial instrument, which is the option, becomes significant. Depending on the investment variants, this is the European or American call option respectively. The European option may be done solely at the time of the moment of the *T*-expiration (concerns the *EI*-type insurance), while the American one may be done at any time (the *LI*-type insurance), therefore the thesis is focused on simulation techniques. In a classical approach, that is, the Black-Scholes model the price of the European option is represented by an analytical formula, while the valuation of the American option is not simpler and thus finally the Monte Carlo method (MC) is used. The simplest model of price evolution is assumed as a model of financial market which is, a geometric Brownian motion, so the price of the S_t share fund unit is described by the geometric Brownian motion, with a relevant drift coefficient, which may be done by using the Euler scheme. With the use of such mathematical apparatus, one obtains a formula simulating a future value of a base instrument (option price) [Jakubowski 2011]:

$$S_{t_k}^i = S_{t_{k-1}}^i \exp\left[\left(r - \frac{\sigma^2}{2}\right)(t_k - t_{k-1}) + \sigma\sqrt{t_k - t_{k-1}}\,\varepsilon_k^i\right],$$

where: ε_k^i – independent values generated from a normal distribution; r – a risk-free interest rate; σ – determines fluctuation of the instrument prices.

We simulate the process of share prices in the t_k finite number of time points.

The results of calculations of mathematical reserves made concern an insured man at the age of 30, with a free cover limit which equals 1000 monetary units and a 5% risk-free interest rate, and the insured party invests in the best capital funds offered on the Polish market in the four basic groups:

- equity portfolio,
- balanced portfolio,
- stable growth portfolio,
- debt securities portfolio.

The mentioned insurance portfolios, which are constructed based on the funds from the groups above, differ in investment policy and thereby in financial risk, which will allow assessing the impact of financial risk on the amount of surplus of the reserves required. The level of mathematical reserves determined for the equity-linked insurance of the *EI*- and *LI*-types in classic version and with allowance for a surplus of the reference portfolio risk is presented in Figures 2 and 3.

Figures 2 and 3 present diagrams of functions of mathematical reserves that the insurer ought to possess if it offers a traditional life insurance or a traditional pure endowment insurance and when it offers the ELI life insurance or the ELI pure endowment insurance with selected funds.

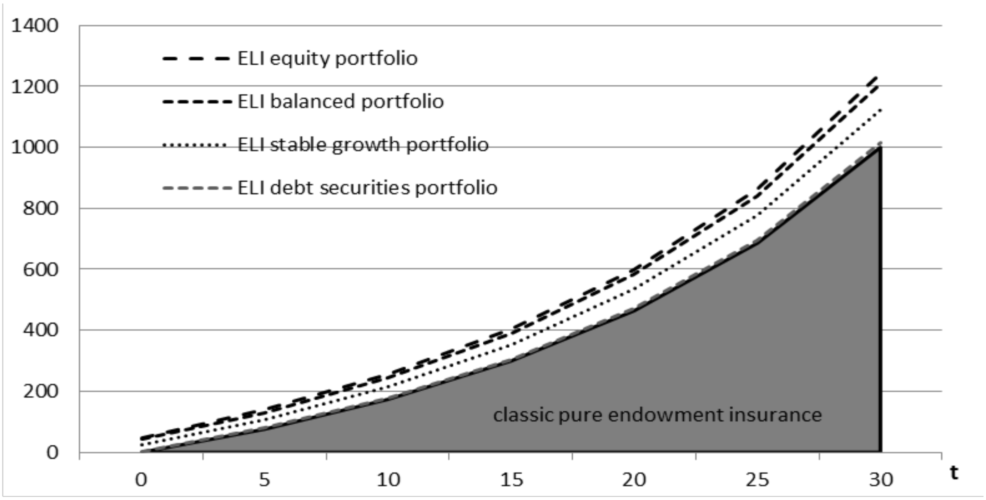

Figure 2. A division of the mathematical reserve of the ELI pure endowment insurance as the sum of a traditional reserve and a risk surplus depending on the reference portfolio

Source: own elaboration.

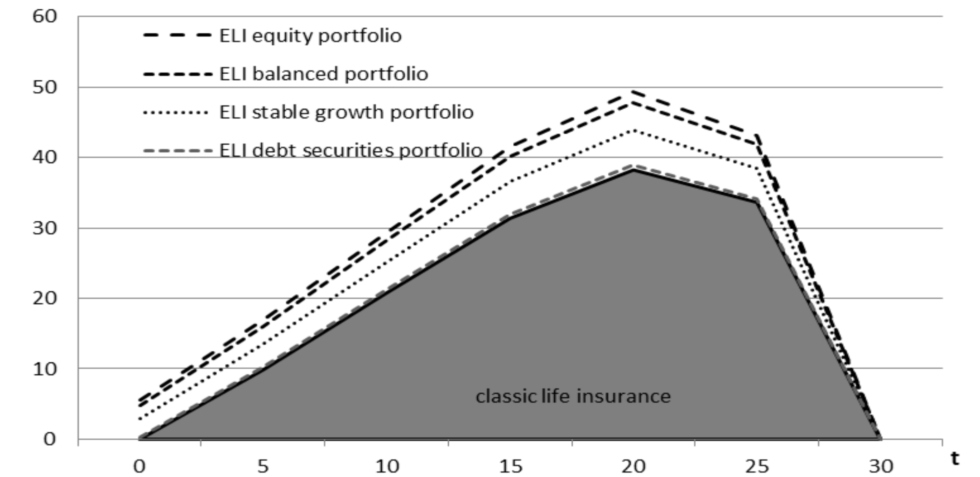

Figure 3. A division of the mathematical reserve of the ELI life insurance as the sum of a traditional reserve and a risk surplus depending on the reference portfolio

Source: own elaboration.

On the basis of graphs of mathematical reserves above for the ELI insurance of the *EI*- and *LI*-type, one may find that financial risk of portfolio does not influence the functional form of mathematical reserves; also it does not change its structure

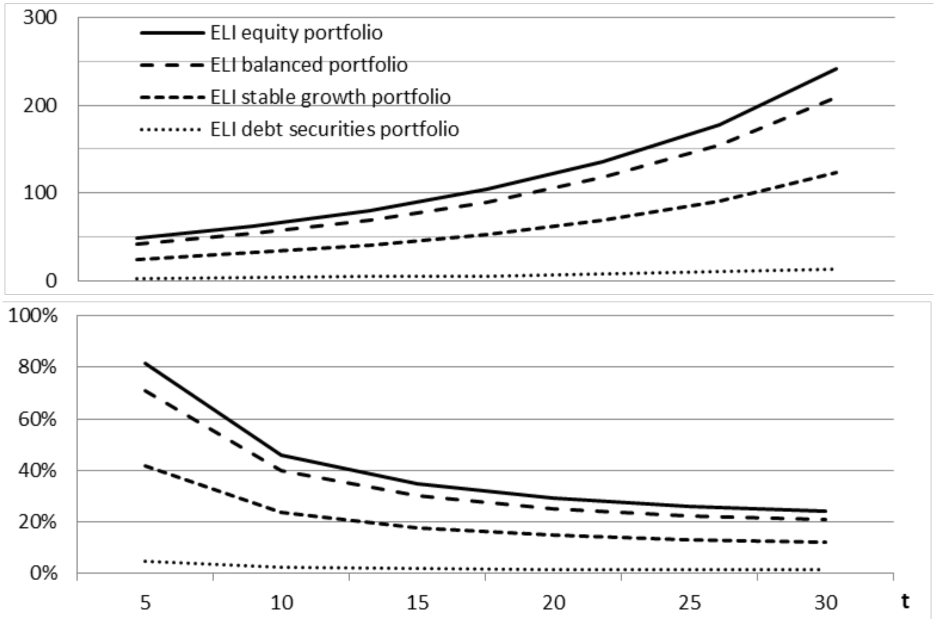

Figure 4. The amount of a surplus of reserves resulting from the additional risk of the ELI pure endowment insurance and its participation in a traditional reserve during the insurance period

Source: own elaboration.

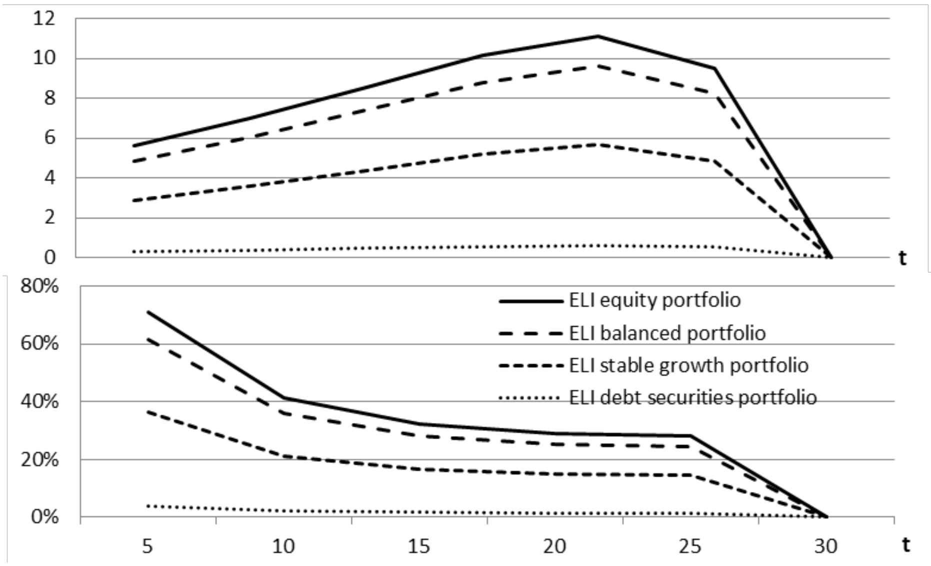

Figure 5. The amount of a surplus of reserves resulting from the additional risk of the ELI life insurance and its participation in a traditional reserve during the insurance period

Source: own elaboration.

during the insurance period. While, undoubtedly pursuant to Solvency II, the reference portfolio risk, in a significant manner, determines the amount of required levels of reserves. In case of the debt securities fund (the ELI portfolio of the lowest risk) the reserve for equity-linked insurance is slightly higher than the reserve that the insurer ought to accumulate in case of traditional insurance of the *EI*- and *LI*-type respectively, so in actuarial calculations a risk surplus of such portfolio may be omitted. Simulations made reveal that along with the increase of reference portfolio risk, the level of required reserves increases meaning that the insurer ought not to avoid such risk in calculations and valuations made.

This is confirmed by the graphs on Figures 4 and 5 showing the amount of a surplus of the ELI financial risk reserves for particular portfolios and a percentage increase of mathematical reserves as a result of the assumed additional financial risk connected with insurance.

On the basis of the graphs above, one may find that in case of pure endowment insurance, a surplus of the ELI risk reserves clearly increases during the insurance period within all the reference portfolios connected with the additional financial risk, whereas in case of the debt securities portfolio it remains at the level close to zero. Simultaneously, a decline in participation of such surplus is observed during the insurance period in a reserve in general. The same tendencies characterise equity-linked life insurance.

6. Conclusions

The results obtained confirm that the employment of a standard approach applied in traditional life insurance and traditional pure endowment insurance does not properly illustrate the actual risk connected with the equity-linked insurance, thus it may lead to shortfalls within the scope of provident funds. It is demonstrated that through making valuation of the ELI insurance cash flows in order to calculate mathematical reserves, one ought to combine the actuarial approach with the financial one. The first approach concerns determination of actuarial value of the future payments connected with insurance, whereas the financial one concerns the tools applied in valuation of call option (the European or American one respectively depending on the type of insurance). This solution will allow obtaining assessment of mathematical reserves in accordance with Solvency II.

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REZERWY MATEMATYCZNE SKŁADEK UFK A RZECZYWISTA WARTOŚĆ PORTFELA REFERENCYJNEGO

Streszczenie: Firma ubezpieczeniowa, chcąc zabezpieczyć swą wypłacalność, powinna dysponować pewną zarezerwowaną kwotą nazywaną w ubezpieczeniach życiowych rezerwą matematyczną składek. Metody obliczania rezerw matematycznych w tradycyjnych ubezpieczeniach można znaleźć w klasycznej literaturze aktuarialnej, według której rezerwę oblicza się jako wartość aktuarialną zakumulowanych przyszłych przepływów pieniężnych z uwzględnieniem ryzyka śmierci i zmiany wartości pieniądza w czasie, czyli tzw. ryzyka aktuarialnego. Jednak firmy ubezpieczeniowe oferujące złożone produkty ubezpieczeniowe, jakimi są ubezpieczenia z funduszem kapitałowym (UFK), powinny w celu zapewnienia swojej wypłacalności zgodnie z Solvency II uwzględniać również dodatkowy aspekt wynikający z ryzyka finansowego. Świadczenia wynikające z tego typu polisy są bowiem bezpośrednio związane z realizacją portfela referencyjnego, a tym samym należy uwzględnić w wycenie ich stochastyczny charakter. W artykule, łącząc podejścia finansowe i ubezpieczeniowe, wyznaczono rezerwy dla ubezpieczenia UFK jako odpowiednią warunkową wartość oczekiwaną z uwzględnieniem rozszerzonego ryzyka aktuarialnego oraz zbadano wpływ przyjętej strategii inwestycyjnej ubezpieczonego na ich wysokość.

Słowa kluczowe: ubezpieczenie z funduszem kapitałowym (UFK), wartość portfela referencyjnego UFK, wypłacalność, rezerwy matematyczne składek, solvency.