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Financial Investment and Insurance – Global Trends and the Polish Market

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VOLATILITY AND RISK MODELS ON THE METAL MARKET

Summary: Steel industry is currently one of the most important parts in the structure of economy sectors both in developed and emerging countries. Therefore, it may be identified as a determinant of economic development of the country. Economic and financial crises have a significant influence on the economic activity of the emerging markets. Moreover, instability and fluctuations of GDP and other economic indicators have a significant impact on the demand of commodities, including ones related to the steel market. The aim of this article is to present some volatility models and risk analysis on the example of investments realized on the non-ferrous metal marker. The motivation to run this research is low popularity of empirical analysis in this field. Considering volatility analysis the GARCH models are presented (based on non-classical probability distributions). Within risk measures the value-at-risk approach is conducted. Initial results indicate that due to some features of time series of the metal market returns the use of classical models of volatility and risk measure is not very effective.

Keywords: Volatility, GARCH models, risk analysis, Value-at-Risk, metal market.

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1. Introduction

The construction of correct economic model which allows for describing the reality in a reliable and accurate way is the scientific problem in many areas of research. The special role of the use of statistics, mathematics and econometrics is presented in the area of risk analysis and risk management. Therefore, the term “risk” has to be considered as a result which can differs from the expected one. If defined that way, the corresponding model has to be constructed properly, especially because the differences between expected and real values can be negative or positive. Such deviations have a significant impact on investment decision making problems.

Considering the area of investment’s interest the common risk generating factor is the volatility observed both within prices and returns. Taking into account financial time series[1] (first of all time series of returns) it is possible to show some specific features which affect the construction of a selected model. Some of them are autocorrelation, leptokurtosis, fat tails, clustering, asymmetry (often positive), leverage and long-memory effects, etc. These characteristics do not allow for using classical models based on normal distribution of returns (e.g. GARCH model with normal distribution of residuals). Moreover, these features are correlated not only with a capital market, although in this area are usually analysed.

2. Modelling volatility in financial time series

The most popular models describing volatility phenomena are proposed by Robert Engle [Engle 1982] in 1982 – the AutoRegressive Conditional Heteroscedasticity Model (ARCH), and its generalization – presented by Tim Bollerslev [Bollerslev 1986] in 1986 – the Generalized AutoRegressive Conditional Heteroscedasticity Model (GARCH). The expected return and variance can be described by information available in the past. This can be written as:

$$\mu_t = E(r_t | I_{t-1}) \,,$$
$$\sigma_t^2 = \sigma^2(r_t | I_{t-1}),$$

where: μ_t and σ_t^2 define conditional expected return and conditional variance in time t; I_{t-1} defines the information set available in time $t-1$.

The GARCH(m, s) model of Bollerslev can be written as below [Tsay 2005]:

$$r_t - \mu_t = a_t = \sigma_t \varepsilon_t,$$
$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{m} \alpha_i a_{t-i}^2 + \sum_{j=1}^{s} \beta_j \sigma_{t-j}^2,$$

where $\alpha_0 \geq 0$, $\alpha_i \geq 0$ for $i > 0$, $\beta_j \geq 0$.

The simplest and most popular GARCH model is GARCH$(1,1)$:

$$a_t = \sigma_t \varepsilon_t,$$
$$\sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \beta_1 \sigma_{t-1}^2,$$

where $0 \leq \alpha_1, \beta_1 \leq 1$ and $(\alpha_1 + \beta_1) < 1$.

The practice in modelling financial time series assumes the need of simultaneous analysis of conditional mean, conditional variance and standardised residuals described by correct probability distribution. To describe residuals ε_t the most commonly used is conditional standarized normal distribution, *t*-Student distribution

[1] The term "financial" has to be considered in general, independently on overall financial market (capital market, exchange rates market, commodity market, etc.).

(symmetric and skewed) or Generalized Error Distribution (GED) [Piontek 2002]. The two last have gained popularity due to some features as leptokurtosis or fat tails (similar as alpha-stable distributions or distributions based on *Extreme Value Theory*). Conditional normal distribution, conditional *t*-Student distribution and conditional GED distribution for residuals can be described using formulas of distribution functions as per below:

$$f_{\text{Norm}}(\varepsilon_t,\sigma_t^2;\theta)=\frac{1}{\sigma_t\sqrt{2\pi}}\exp\left\{-\frac{\varepsilon_t^2}{2\sigma_t^2}\right\},$$

$$f_{t-\text{Stud}}(\varepsilon_t,\sigma_t^2;\theta)=\frac{\Gamma\left(\frac{v+1}{2}\right)}{\sigma_t\Gamma\left(\frac{v}{2}\right)\sqrt{\pi(v-2)}}\left(1+\frac{\varepsilon_t^2}{(v-2)\sigma_t^2}\right)^{\frac{v+1}{2}},$$

$$f_{\text{GED}}(\varepsilon_t,\sigma_t^2;\theta)=2^{-\frac{v+1}{v}}\frac{v}{\sigma_t\sqrt{\frac{\Gamma(v^{-1})}{\Gamma(3v^{-1})}2^{-\frac{2}{v}}\Gamma(v^{-1})}}\exp\left\{-\frac{1}{2}\left|\frac{\varepsilon_t}{\sigma_t\sqrt{\frac{\Gamma(v^{-1})}{\Gamma(3v^{-1})}2^{-\frac{2}{v}}}}\right|^{v}\right\},$$

where: $\{\varepsilon_t\}$ – the sequence of iid random variables; σ_t^2 – the conditional variance of the process; θ – the vector of estimated parameters; v – the number of degrees of freedom (which has to be estimated if *t*-Student distribution and GED distribution are considered); $\Gamma(k)=\int_0^{+\infty}x^{k-1}e^{-1}\,dx$ – the gamma function with parameter k.

2.1. Asymmetric Laplace Distribution as a conditional distribution of residuals

The conditional distributions of residuals presented previously are mostly used in practice. However, there are some others which cover specific features of financial time series as leptokurtosis, asymmetry or fat tails. In this paper to describe model residuals the Asymmetric Laplace Distribution (ALD) is considered. The probability function of AL distribution is as follow [Kozubowski, Podgórski 1999]:

$$f_{\text{ALD}}(x;\xi,\tau,\kappa)=\frac{\kappa}{\tau(1+\kappa^2)}\begin{cases}\exp\left\{-\frac{\kappa}{\tau}(x-\xi)\right\}, \text{for } x\geq\xi\\ \exp\left\{\frac{1}{\kappa\tau}(x-\xi)\right\}, \text{for } x<\xi\end{cases},$$

where: ξ – location parameter; τ – scale parameter; κ – asymmetry.

Additionally $\xi\in\mathcal{R}$, $\tau>0$ and $\kappa>0$.

The ALD distribution is leptokurtic and unimodal. Furthermore, due to the property of scale invariant parameter it is possible to generate distributions with selected level of asymmetry. The probability function of ALD distribution depends only on location parameter ξ and scale parameter τ. The scale invariant parameter is strictly correlated with location one.

If standardised version of ALD distribution is considered, the probability function has a form:

$$f_{\text{ALD}}(x;0{,}1{,}1) = \frac{1}{2}\left\{\exp\begin{Bmatrix} -x, \text{for } x \geq 0 \\ x, \text{for } x < 0 \end{Bmatrix}\right.,$$

The asymmetry in ALD distribution is determined by the parameter κ defined in literature [Kozubowski, Podgórski 1999] as a scale invariant parameter which satisfies properties:

$$\kappa = \frac{2\tau}{\left(\xi+\sqrt{4\tau^2+\xi^2}\right)},\ \frac{1}{\kappa} - \kappa = \frac{\xi}{\tau},$$

$$\frac{1}{\kappa} + \kappa = \sqrt{4+\left(\frac{\xi}{\tau}\right)^2},\ \frac{1}{\kappa^2} + \kappa^2 = 2 + \left(\frac{\xi}{\tau}\right)^2,$$

Applying ALD distribution to describe the conditional distribution of residuals ε_t in GARCH model, its probability function is of the form:

$$f_{\text{ALD}}(\varepsilon_t, \sigma_t^2; \theta) = \frac{\kappa}{\sigma_t^2(1+\kappa^2)}\begin{cases} \exp\left\{-\frac{\kappa}{\sigma_t^2}\varepsilon_t\right\}, \text{for } \varepsilon_t \geq 0 \\ \exp\left\{\frac{1}{\kappa\sigma_t^2}\varepsilon_t\right\}, \text{for } \varepsilon_t < 0 \end{cases},$$

The standarization feature of conditional distribution is maintained through the relation $\xi = \tau\left(\frac{1}{\kappa} - \kappa\right)$. The vector of parameters θ in all presented conditional distributions is obtained using Maximum Likelihood Method.

3. Risk measurement with Value-at-Risk approach

The Value-at-Risk (VaR) is one of the most popular investment risk measures. It was presented for the first time in 1994 by the bank J.P. Morgan in the document describing the system of risk management RiskMetrics™. VaR is a statistical measure of risk, which shows the potential loss of investment, portfolio, institution, which can occur within some time interval with arbitrarily determined probability level, called tolerance level. The formula defining VaR is as follow [Piontek 2002]:

$$P(W_t \leq W_{t-1} - VaR_\alpha),$$

where: W_t – the value of investment at the end of analysed period; W_{t-1} – the current investment value; α – the tolerance level.

VaR can be considered as well in terms of investment returns as an α-quantile of distribution:

$$P\left(r_t \leq F_{r_t,t}^{-1}(\alpha)\right) = \alpha,$$

where: r_t – the investment return; $F_{r_t,t}^{-1}(\alpha)$ – an α-quantile of the return distribution.

Finally, the VaR is described by:

$$VaR_\alpha = -r_t F_{r_t,t}^{-1}(\alpha).$$

In practice, there exist different methods of estimating VaR but all of them are strongly correlated with investment risk management. These methods are determined by some specific assumptions, as the form of probability distribution, relations between assets in created portfolio, etc. The most popular methods for estimating VaR are: variance-covariance method, historical simulation method, methods based on Extreme Value Theory, or estimating VaR using quantile of any fitted distribution (quantile based method), etc. In this paper only this last one method is considered.

The quantile based method is non-parametric approach for estimating VaR, because does not assume an analytical form of probability distribution function. This method relies on using historical data to estimate parameters of a distribution, which is the best fitted to the data represented by empirical distribution. In the next step, using estimated probability function, the α-quantile of and Value-at-Risk are calculated. Using this method the most important is to fit the proper theoretical distribution, especially if financial time series are of interest. If selected, it is obliged to check the goodness of fit using corresponding statistical tests [Krężołek 2014].

However VaR, as an investment risk measure, is not perfect. Its value answers the question what is the minimum loss from the investment in α possibles cases. But representing some threshold, this measure does not take into account possibilities of occurring losses exceeding its level. Therefore an alternative risk measure is the Conditional Value-at-Risk (CVaR) defined as:

$$\text{CVaR}_\alpha = E(r_t - \text{VaR}_\alpha | r_t > \text{VaR}_\alpha).$$

The advantage of VaR over the other risk measures is significant in terms of coherency of the risk measure. VaR do not satisfy one of the properties of coherent risk measure – the sub-additivity. This results that if a portfolio investment is take into account the overall risk of the portfolio in not higher than the sum of the risks of its components. This property is satisfied if CVaR is considered.

The accuracy of VaR models is assessed by the method called "back testing". To assess the effectivenes of estimating VaR the series of failures is used, with the form presented herein [Ganczarek 2007]:

$$[I_{t+1}(\alpha)]_{t=1}^{t=T} = \begin{cases} 1, r_t \leq -\text{VaR}_\alpha \\ 0, r_t > -\text{VaR}_\alpha \end{cases}.$$

The most popular test used in practice is the Proportion of Failures Test (POF) proposed by Kupiec [1995]:

$$LR_{\mathrm{POF}} = -2\ln\left\{\frac{(1-\alpha)^{T-N}\alpha^{N}}{\left[\left(1-\frac{N}{T}\right)^{T-N}\right]\left(\frac{N}{T}\right)^{N}}\right\},$$

where: N – the number of observation exceeding VaR for the series of length T.

Under the null hypothesis the LR_{POF} test has χ^2 distribution with 1 degree of freedom: $LR_{\mathrm{POF}} \sim \chi^2(1)$.

4. Empirical analysis on the metal market

Financial and economic crises observed within the first decade of the 21st century have led to search for other possibilities to invest capital, ones which despite the generally observed decline would generate positive returns [Krężołek 2012]. In commodity market we can find relations between supply and demand of some specific products characterized by standarized level of quality. Among commodities the most popular are electricity, fuel, agricultural products, precious stones and metals [Hammoudeh et al. 2011], etc. In this paper only non-ferrous metals are investigated.

Metals market is strongly related to world economic situation. There are many factors which determine the level of prices and volatility of price returns. The growth of GDP increases investment expenditures, which is directly related to demand of metal products. Thus, in times of economic crises there is a need for reliable risk assessment. In the literature there is a lot of research related to Value-at-Risk measurement on metals market, but mostly referred to the precious metals (e.g. Füss et al. 2010; He et al. 2012]). The research results show the possibility to allocate capital in an alternative way if compared to stock market.

The purpose of this article is to present some volatility and risk models applied to time series observed on the metal market. This investment area can be considered as an alternative to capital market. The main hypotheses verified in this paper are:

I. Steel market can be considered as an attractive alternative for investing if compared to the capital market (especially in terms of volatility and risk measurement).

II. The use of fat-tailed and asymmetric probability distributions allows for more accurate volatility and risk assessment and the assessment of Value-at-Risk and Conditional Value-at-Risk using GARCH models with not normal probability distributions of residuals provides the estimates of risk in more accurate way than using the normal one.

The analysis is based on daily log-returns from London Metal Exchange (LME) within the period January 2004 – April 2014. The study focuses on a set of six commodities: COPPER, ALUMINIUM, ZINC, THIN, LEAD and NICKEL. The analysis of volatility is based on an $\mathrm{AR}(1) - \mathrm{GARCH}(1,1) - \{\eta_{\mathrm{dist}}\}$ models, where $\{\eta_{\mathrm{dist}}\}$ is conditional standarized distribution of residuals, according to approach presented previously. Therefore the $\mathrm{AR}(1) - \mathrm{GARCH}(1,1) - \{\eta_{\mathrm{dist}}\}$ has a form:

$$r_t = \mu + \varphi_1 r_{t-1} + \varepsilon_t,$$
$$\varepsilon_t = \sqrt{\sigma_t^2}\eta_t,$$
$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2,$$
$$\eta_t \sim IID \left\{ \text{dist} = \begin{bmatrix} \text{Norm} \\ t - \text{Stud} \\ \text{GED} \\ \text{ALD} \end{bmatrix} \right\}.$$

Figure 1 shows time series of returns of analysed metals.

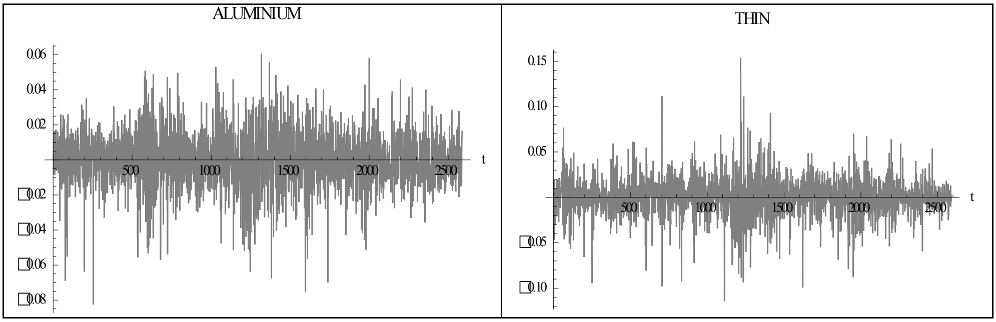

Figure 1. Time series representing returns of metal prices

Source: own calculations.

Descriptive statistics for returns are presented in Table 1.

Table 1. Descriptive statistics – returns

Descriptive statistics	COPPER	ALUMINIUM	THIN	ZINC	LEAD	NICKEL
Mean	0.00040	0.00003	0.00048	0.00025	0.00039	–0.00003
Median	0.00046	0.00018	0.00120	0.00065	0.00138	–0.00049
Variance	0.00040	0.00025	0.00042	0.00050	0.00059	0.00067
Standard deviation	0.02009	0.01566	0.02047	0.02225	0.02434	0.02596
Skewness	–0.14807	–0.29344	–0.17469	–0.22263	–0.21563	–0.13975
Kurtosis	6.08854	4.76734	7.50634	4.76472	5.26633	6.20264

Source: own calculations.

To summarize results presented in Table 1: it was found that only investments in nickel generate negative returns (in average). The other metals generate small profits (the highest for THIN and COPPER). The returns have similar level of volatility.

Coefficients of skewness and kurtosis indicate a lack of normality assumption for empirical distributions. Table 2 presents the results of some selected normality tests: Jarque-Bera (JB), Anderson-Darling (AD) and Kolmogorov-Smirnov (KS).

Table 2. Normality tests

Metal	JB		AD		KS	
	statistics	p-value	statistics	p-value	statistics	p-value
COPPER	1045.64	0.0000	18.35	0.0000	0.0584	0.0000
ALUMINIUM	376.89	0.0000	6.98	0.0003	0.0336	0.0056
THIN	2217.97	0.0000	31.64	0.0000	0.0822	0.0000
ZINC	360.09	0.0000	10.83	0.0000	0.0423	0.0002
LEAD	578.32	0.0000	12.03	0.0000	0.0447	0.0001
NICKEL	1122.52	0.0000	14.47	0.0000	0.0523	0.0000

Source: own calculations.

All results show discrepancy with normal distribution at the 0.05 significance level.

AR(1) model describes conditional expected value for returns of COPPER, ALUMINIUM and LEAD. In other cases higher time-lags result statistically significant. Despite this only AR(1) model is taken into account. In the next stage the parameters of $\mathrm{AR}(1) - \mathrm{GARCH}(1,1) - \{\eta_{\mathrm{dist}}\}$ model have been estimated, using one of the following conditional distributions of residuals:

- $\mathrm{AR}(1) - \mathrm{GARCH}(1,1) - \{\eta_{\mathrm{norm}}\}$
- $\mathrm{AR}(1) - \mathrm{GARCH}(1) - \{\eta_{t-\mathrm{Stud}}\}$
- $\mathrm{AR}(1) - \mathrm{GARCH}(1,1) - \{\eta_{\mathrm{GED}}\}$
- $\mathrm{AR}(1) - \mathrm{GARCH}(1,1) - \{\eta_{\mathrm{ALD}}\}$

Tables 3–6 present values of estimated parameters, log-likelihood functions and information criteria for presented models (Akaike (AIC), Schwarz (BIC) and Hannan-Quinn (HQC)).

The values of information criteria are calculated using formulas:

$$\mathrm{AIC} = -2\ln\left[\mathrm{LLF}\left(\hat{\theta}\right)\right] + 2k,$$

$$\mathrm{BIC} = -2\ln\left[\mathrm{LLF}\left(\hat{\theta}\right)\right] + k\ln(n),$$

$$\mathrm{HQC} = -2\ln\left[\mathrm{LLF}\left(\hat{\theta}\right)\right] + 2k\ln[\ln(n)],$$

where: $\mathrm{LLF}\left(\hat{\theta}\right)$ – the log-likelihood function of the parameters vector $\hat{\theta}$; k – the number of estimated parameters; n – the number of observations.

The selection of model is based on minimization of information criteria.

Table 3. Results of parameter estimation – AR(1) − GARCH(1,1), normal distribution of residuals

	COPPER	ALUMINIUM	THIN	ZINC	LEAD	NICKEL
μ	0.00040	0.00005	0.00049	0.00027	0.00040	0.00003
p-value	0.27710	0.85910	0.22570	0.53470	0.43040	0.95190
φ_1	–0.06729*	–0.04558*	0.01932	–0.01747	0.06982*	0.01137
p-value	0.00060	0.01910	0.31770	0.35790	0.00030	0.54880
α_0	0.0000037*	0.0000037*	0.0000243*	0.0000006	0.0000002	0.0000045*
p-value	0.00100	0.00350	0.00001	0.24910	0.71490	0.01410
α_1	0.06785*	0.04013*	0.11461*	0.03440*	0.03028*	0.05967*
p-value	0.00000	0.00000	0.00000	0.00001	0.00000	0.00000
β_1	0.92359*	0.94472*	0.82837*	0.96474*	0.96968*	0.93415*
p-value	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
LFF	6806.71	7236.21	6635.51	6493.23	6266.26	6067.36
AIC	–13605.42	–14464.42	–13263.01	–12978.45	–12524.52	–12126.72
BIC	–13581.96	–14440.94	–13239.55	–12955.00	–12501.06	–12103.27
HQC	–13596.92	–14455.92	–13254.51	–12969.96	–12516.02	–12118.22

* Significant at the level 0.05.

Source: own calculations.

Table 4. Results of parameter estimation – AR(1) − GARCH(1,1), t-Student distribution of residuals

	COPPER	ALUMINIUM	THIN	ZINC	LEAD	NICKEL
μ	0.00040	0.00005	0.00049	0.00027	0.00040	0.00003
p-value	0.27710	0.85910	0.22570	0.53470	0.43040	0.95190
φ_1	–0.06729*	–0.04558*	0.01932	–0.01747	0.06982*	0.01137
p-value	0.00060	0.01910	0.31770	0.35790	0.00030	0.54880
α_0	0.0000025*	0.0000058*	0.0000281*	0.0000017	0.0000021	0.0000033
p-value	0.00004	0.00121	0.00000	0.10263	0.55345	0.05345
α_1	0.04359*	0.05201*	0.12212*	0.02638*	0.03218*	0.06234*
p-value	0.00000	0.00000	0.00000	0.00001	0.00000	0.00000
β_1	0.91785*	0.94335*	0.81426*	0.95432*	0.96881*	0.92871*
p-value	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
LFF	7396.61	7615.86	7303.99	6754.61	6544.11	6454.73
AIC	–14783.22	–15221.72	–14597.98	–13499.22	–13078.22	–12899.46
BIC	–14729.18	–15299.31	–14558.64	–13458.73	–13011.69	–12841.45
HQC	–14744.32	–15275.11	–14585.22	–13490.45	–13049.19	–12866.11

* Significant at the level 0.05.

Source: own calculations.

Table 5. Results of parameter estimation – $AR(1) - GARCH(1,1)$, GED distribution of residuals

	COPPER	ALUMINIUM	THIN	ZINC	LEAD	NICKEL
μ	0.00040	0.00005	0.00049	0.00027	0.00040	0.00003
p-value	0.27710	0.85910	0.22570	0.53470	0.43040	0.95190
φ_1	–0.06729*	–0.04558*	0.01932	–0.01747	0.06982*	0.01137
p-value	0.00060	0.01910	0.31770	0.35790	0.00030	0.54880
α_0	0.0000011*	0.0000023*	0.0000311*	0.0000011	0.0000021	0.0000012*
p-value	0.00002	0.00021	0.00000	0.11235	0.21387	0.00567
α_1	0.05322*	0.05278*	0.08546*	0.04563*	0.02897*	0.06234*
p-value	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
β_1	0.91224*	0.92376*	0.89887*	0.92876*	0.94877*	0.91209*
p-value	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
LFF	7776.83	7510.99	7998.99	7112.88	7002.34	6776.64
AIC	–15543.66	–15011.98	–15987.99	–14215.76	–13994.67	–13543.27
BIC	–15521.95	–14972.89	–15934.21	–14191.87	–13944.71	–13511.98
HQC	–15533.53	–15003.13	–15967.91	–14208.43	–13971.17	–13531.73

* Significant at the level 0.05.

Source: own calculations.

Table 6. Results of parameter estimation – $AR(1) - GARCH(1,1)$, ALD of residuals

	COPPER	ALUMINIUM	THIN	ZINC	LEAD	NICKEL
μ	0.00040	0.00005	0.00049	0.00027	0.00040	0.00003
p-value	0.27710	0.85910	0.22570	0.53470	0.43040	0.95190
φ_1	–0.06729*	–0.04558*	0.01932	–0.01747	0.06982*	0.01137
p-value	0.00060	0.01910	0.31770	0.35790	0.00030	0.54880
α_0	0.0000034*	0.0000039*	0.0000257*	0.0000011	0.0000007*	0.0000121
p-value	0.00054	0.00214	0.00000	0.12135	0.01326	0.06213
α_1	0.05467*	0.04123*	0.09482*	0.02556*	0.05643*	0.07735*
p-value	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
β_1	0.93469*	0.93578*	0.90034*	0.93259*	0.91879*	0.92412*
p-value	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
LFF	7781.59	7554.69	8042.57	7416.12	6969.59	7054.57
AIC	–15553.17	–15099.38	–16075.13	–14822.24	–13929.17	–14099.13
BIC	–15519.21	–15023.14	–16012.66	–14792.85	–13914.22	–14012.55
HQC	–15535.72	–15043.19	–16059.99	–14801.18	–13921.09	–14077.61

* Significant at the level 0.05.

Source: own calculations.

All analysed models show statistical significance of estimated parameters α_1 and β_1 regardless of the form of conditional distribution of residuals. Using information criteria, it was noted that comparing models which allow for leptokurtosis, clustering and fat tails, the models based on Gaussian distribution of residuals have to be rejected. Similar conclusions are obtained for log-likelihood functions. Figure 2 shows QQ-plots of residuals for COPPER.

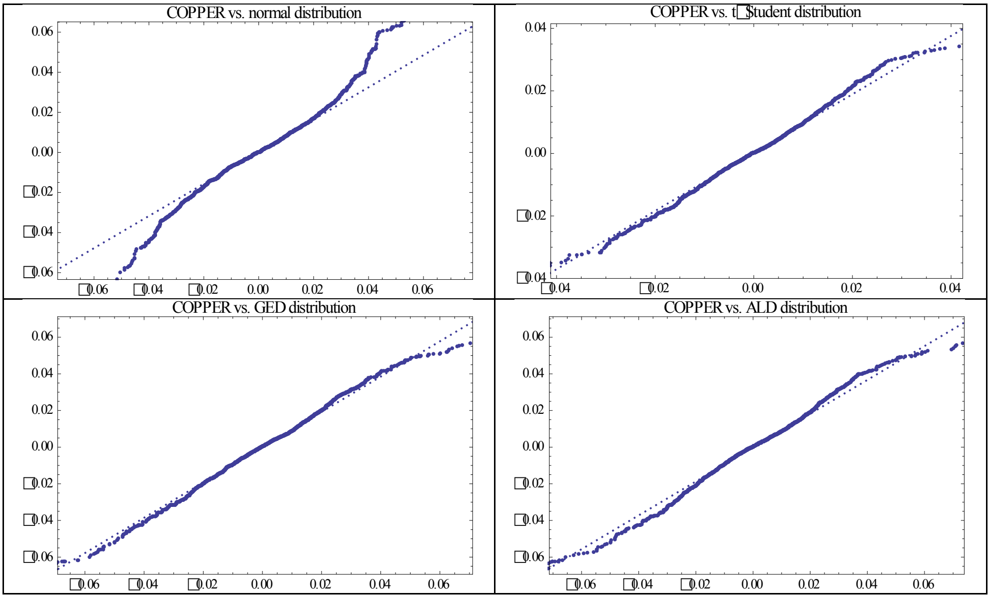

Figure 2. QQ-plots of residuals – COPPER

Source: own calculations.

QQ-plots clearly confirm discrepancy with normal distribution. It results directly from leptokurtosis, significant asymmetry and fat tails of empirical distributions. Similar conclusions were drawn for other metal returns.

In the next stage the estimated models were used to calculate VaR. The estimation method for calculating VaR (CVaR) is this which uses quantile of any fitted distribution. Analysed quantiles are 0.01 and 0.05 with one day forecast horizon. All results for empirical ($\mathrm{VaR_{emp}}$), normal ($\mathrm{VaR_{norm}}$), *t*-Student ($\mathrm{VaR}_{t-\mathrm{Stud}}$), GED ($\mathrm{VaR_{GED}}$) and ALD distributions ($\mathrm{VaR_{ALD}}$) are presented in Tables 7 and 8.

Table 7. VaR estimates for one day forecast horizon

	$\mathrm{VaR_{emp}}$		$\mathrm{VaR_{norm}}$		$\mathrm{VaR_{t\text{-}Stud}}$		$\mathrm{VaR_{GED}}$		$\mathrm{VaR_{ALD}}$	
	0.01	0.05	0.01	0.05	0.01	0.05	0.01	0.05	0.01	0.05
COPPER	–0.0576	–0.0312	–0.0462	–0.0326	–0.0492	–0.0332	–0.0512	–0.0355	–0.0533	–0.0317
ALUMINIUM	–0.0448	–0.0247	–0.0363	–0.0257	–0.0399	–0.0274	–0.0422	–0.0263	–0.0404	–0.0262
THIN	–0.0623	–0.03555	–0.05137	–0.03624	–0.05743	–0.03528	–0.05824	–0.03678	–0.05953	–0.03487
ZINC	–0.0589	–0.03413	–0.04699	–0.03308	–0.05386	–0.03511	–0.05572	–0.03561	–0.05311	–0.03512
LEAD	–0.0668	–0.04096	–0.05609	–0.03954	–0.06053	–0.04023	–0.06197	–0.04173	–0.06676	–0.04299
NICKEL	–0.0701	–0.04144	–0.06023	–0.04258	–0.06321	–0.04173	–0.06419	–0.04241	–0.07012	–0.04257

Source: own calculations.

Table 8. CVaR estimates for one day forecast horizon of VaR

	$CVaR_{emp}$		$CVaR_{norm}$		$CVaR_{t\text{-Stud}}$		$CVaR_{GED}$		$CVaR_{ALD}$	
	0.01	0.05	0.01	0.05	0.01	0.05	0.01	0.05	0.01	0.05
COPPER	–0.0737	–0.0472	–0.0582	–0.0495	–0.0642	–0.0483	–0.0692	–0.0468	–0.0654	–0.0463
ALUMINIUM	–0.0562	–0.0364	–0.0471	–0.0313	–0.0522	–0.0322	–0.0533	–0.0344	–0.0542	–0.0354
THIN	–0.0789	–0.0518	–0.0612	–0.0422	–0.0643	–0.0473	–0.0711	–0.048	–0.0713	–0.0499
ZINC	–0.0769	–0.0499	–0.0623	–0.0442	–0.0662	–0.0455	–0.0702	–0.0462	–0.0722	–0.0483
LEAD	–0.0833	–0.0571	–0.0715	–0.0498	–0.0761	–0.0522	–0.0789	–0.0532	–0.0790	–0.0529
NICKEL	–0.0920	–0.0594	–0.0794	–0.0610	–0.0852	–0.0621	–0.0903	–0.0603	–0.0912	–0.0610

Source: own calculations.

Value-at-Risk estimated under the assumption of normally distributed residuals shows higher absolute differences comparing to empirical VaR than if other distributions considered: *t*-Student, GED or ALD. The most similar estimates are obtained for GED and ALD, regardless of chosen quantile. Similar conclusions are drawn if CVaR is taken into account.

The final stage of the study is related to hypothesis which states that the number of observation exceeding VaR (CVaR) complies with expected one at the significance level of 0.05. The results of back testing VaR using POF test are shown in tables 9 and 10 and those for CVaR in Tables 11 and 12 respectively.

Table 9. Back testing – $VaR_{0.01}$

	N	LR_{POF}	N	LR_{POF}	N	LR_{POF}	N	LR_{POF}
	VaR_{norm}		$VaR_{t\text{-Stud}}$		VaR_{GED}		VaR_{ALD}	
COPPER	49	16.257*	39	5.662*	37	4.130*	33	1.738
ALUMINIUM	47	13.776*	40	6.506*	34	2.248	38	4.869*
THIN	44	10.380*	27	0.036	27	0.036	24	0.164
ZINC	61	34.435*	38	2.248	34	2.248	40	6.506*
LEAD	49	16.257*	38	4.869*	36	3.446	26	0.000
NICKEL	46	12.599*	40	6.506*	38	4.869*	25	0.042

N – number of observation exceeding VaR; * significant at the level 0.05.

Source: own calculations.

Table 10. Back testing – $VaR_{0.05}$

	N	LR_{POF}	N	LR_{POF}	N	LR_{POF}	N	LR_{POF}
	VaR_{norm}		$VaR_{t\text{-Stud}}$		VaR_{GED}		VaR_{ALD}	
COPPER	116	1.678	108	4.202*	99	8.524*	127	0.081
ALUMINIUM	116	1.678	99	8.524*	108	4.202*	110	3.458*
THIN	112	2.791	123	0.421	110	3.458	125	0.217
ZINC	153	4.007*	135	0.011	129	0.011	135	0.188
LEAD	138	0.489	134	0.119	119	1.034	108	4.202*
NICKEL	116	1.678	125	0.217	119	1.034	116	1.678

N – number of observation exceeding VaR; * significant at the level 0.05.

Source: own calculations.

Table 11. Back testing – $CVaR_{0.01}$

	N	LR_{POF}	N	LR_{POF}	N	LR_{POF}	N	LR_{POF}
	$CVaR_{norm}$		$CVaR_{t\text{-}Stud}$		$CVaR_{GED}$		$CVaR_{ALD}$	
COPPER	25	0.042	20	1.533	16	4.526*	19	2.116
ALUMINIUM	23	0.371	14	6.751*	13	8.074*	11	11.198*
THIN	22	0.665	19	2.116	14	6.751*	14	6.751*
ZINC	25	0.042	23	0.371	21	1.051	21	1.051
LEAD	17	3.606	13	8.074*	13	8.074*	13	8.074*
NICKEL	14	6.751*	11	11.198*	10	13.026*	10	13.026*

N – number of observation exceeding CVaR, * significant at the level 0.05

Source: own calculations.

Table 12. Back testing – $CVaR_{0.05}$

	N	LR_{POF}	N	LR_{POF}	N	LR_{POF}	N	LR_{POF}
	$CVaR_{norm}$		$CVaR_{t\text{-}Stud}$		$CVaR_{GED}$		$CVaR_{ALD}$	
COPPER	38	94.128*	39	91.619*	46	75.447*	49	69.201*
ALUMINIUM	67	38.924*	64	43.199*	54	59.613*	48	71.241*
THIN	75	28.841*	51	65.247*	50	67.204*	46	75.447*
ZINC	77	26.602*	68	37.561*	64	43.199*	60	49.350*
LEAD	76	27.708*	66	40.318*	62	46.209*	63	44.688*
NICKEL	41	86.757*	41	86.757*	45	77.617*	41	86.757*

N – number of observation exceeding CVaR, * significant at the level 0.05.

Source: own calculations.

Critical value allowing verify the number of observations exceeding VaR is $\chi^2_{0.05}(1) = 3.841$. Looking at the results in Tables 9 and 10 it was found that for quantile 0.01 the AR(1)-GARCH(1,1)-$\{\epsilon_{\mathrm{norm}}\}$ model is not correct. For all risk models the values of POF tests resulted statistically significant. The study shows that the best model in this case is with residuals described by GED or ALD: $\mathrm{AR}(1) - \mathrm{GARCH}(1,1) - \{\epsilon_{\mathrm{GED}}\}$ or $\mathrm{AR}(1) - \mathrm{GARCH}(1,1) - \{\epsilon_{\mathrm{ALD}}\}$. For quantile 0.05 all estimated models show similar effectiveness in risk measurement. Commenting results for conditional VaR only for normal distribution the null hypothesis is not rejected.

5. Conclusions

This paper presents results of the application of some models and risk measures to describe volatility observed in non-ferrous metal market. The assets selected for this study are: COPPER, ALUMINIUM, THIN, ZINC, LEAD and NICKEL. The selection of these metals is not accidental as they play significant role in steel industry. These metals are added to steel to increase its quality. Steel is one of the most important products used in many economic areas: automotive, building industry, medicine, aerospace, etc. That is why the analysis of volatility observed in steel prices is so important, especially in terms of investment.

The results presented in this paper show that it is possible to use effectively some methods of risk measurement commonly applied to risk assessment in stock market [Krężołek 2013a, b] to other markets. Time series of metals prices and returns (commodity market) exhibit similar characteristics as those from capital market (e.g. asymmetry, fat tails, leptokurtosis, etc.).

To describe volatility, the class of $\mathrm{AR}(1) - \mathrm{GARCH}(1{,}1) - \{\epsilon_{\mathrm{dist}}\}$ models is used, with the assumption that the distribution of residuals belong to the one of proposed conditional distributions: normal distribution, t-Student distribution, GED distribution or ALD distribution. The first three distributions are commonly used to model conditional variance. The last one – asymmetric Laplace distribution – has been proposed as their extension. If describing shortly, the ALD is leptokurtic, fat tailed and possess very important property – the existence of all moments of all orders. The asymmetry of distribution is verified by so called scale invariant parameter, independent of scale parameter.

The time lags in $\mathrm{AR}(p) - \mathrm{GARCH}(m, s)$ models are arbitrary focusing on the analysis of conditional distribution of residuals. Using information criteria indicated that better estimation of results has been obtained for the fat-tailed distributions. The same conclusion was drawn for log-likelihood function. Thus the class of $\mathrm{AR}(1) - \mathrm{GARCH}(1{,}1)$ models was proposed.

In the second part of the article the risk analysis based on VaR approach has been investigated. Using presented models it has been found that the normal distribution is not correct to estimate VaR, similarly as t-Student distribution, regardless of the fat-tails property. The most accurate estimates for VaR have been obtained for GED and ALD distributions (the same results have been obtained for CVaR).

In this paper only the most popular volatility models were presented – GARCH models with low levels of time-lags. The next stage is to introduce more complex models of conditional variance and additional assumptions for the conditional distribution of residuals, assuming statistical significance of higher moments of random variables represented by returns observed on the metal market [Orhan, Koksal 2012; Polański, Stoja 2010].

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WYBRANE MODELE ZMIENNOŚCI I RYZYKA NA PRZYKŁADZIE RYNKU METALI

Streszczenie: Przemysł stalowy jest obecnie jednym z ważniejszych segmentów w strukturze gałęzi gospodarki krajów zarówno rozwiniętych, jak i stojących u progu rozwoju. Tym samym może być identyfikowany jako wyznacznik rozwoju gospodarczego danego kraju. Kryzysy ekonomiczne oraz finansowe w istotny sposób wpływają na aktywność ekonomiczną rynków wschodzących, natomiast wahania w poziomie PKB oraz innych wskaźników ekonomicznych mają odzwierciedlenie w zapotrzebowaniu na różnego rodzaju towary, w tym także produkty rynku stalowego. Przedmiotami artykułu są prezentacja wybranych modeli zmienności oraz analiza ryzyka w przypadku inwestycji w walory rynku metali nieżelaznych. Motywacją ku prowadzonym badaniom jest mała popularność badań empirycznych w obszarze eksplorowanego rynku. W przypadku badania zmienności zaproponowano modele klasy garch, podejmując próbę ich konstrukcji przy założeniu nieklasycznych rozkładów prawdopodobieństwa. Analizę ryzyka przeprowadzono w oparciu o metodologię wartości zagrożonej. Wstępne wyniki analizy wskazują, że ze względu na własności szeregów stóp zwrotu cen badanych metali wykorzystywanie klasycznych modeli pomiaru zmienności i ryzyka jest mało efektywne.

Słowa kluczowe: zmienność, modele GARCH, analiza ryzyka, *Value-at-Risk*, rynek metali.