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Nr 14 (20)

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Spis treści

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Summaries

Aims and scope

Aims and scope of this journal were determined already in the period of the historical changes that took place in 1989 in the Europe, which had a great meaning for Poland, especially for the subsequent political and economic transformations. The introduction of the democratic system, and the transition from the state-controlled economy to the free market one were the driving forces behind the new Polish economy.

In the early 1990s, Poland made great progress towards achieving a fully democratic government and a market economy. In November 1990, Lech Wałęsa was elected President for a 5-year term. In 1991 were held the first free parliamentary elections. In the same year, 1991, the first issue of the journal was published under the title *Statistical Review of Lower and Opole Silesia*. In the foreword of that first issue it was stated what follows. "The changes in the socio-economic life of Lower Silesia and Opole region caused the Council of Wrocław Branch of Polish Statistical Society to publish Statistical Review of Lower and Opole Silesia, starting from the year 1991. This idea could come to life thanks to the generous help of directors of Voivodeship Statistical Offices in Jelenia Góra, Legnica, Wałbrzych and Wrocław, with a special involvement of the director of Statistical Office in Wrocław". The initial goal of the founders of the journal was to dedicate the journal to "ecological problems, demographic issues as well as social and economic well-being".

Starting in the year 2002 the journal has been published with a new layout and under a new title: *Silesian Statistical Review*. Together with *Statistical Review* (*Przegląd Statystyczny*) and *Statistical News* (*Wiadomości Statystyczne*), *Silesian Statistical Review* is now one of the three major journals in Poland dedicated to general statistical problems. Special attention has been focused on general methodological issues, as well as on the applications of various statistical methods in solving real social and economic problems. Papers concerning all topics of quality of life are published regularly. Historical essays are included on regular basis.

After 25 years of the existence, by entering in the next quarter of the century of its existence with the issue of 2016, the main scope of journal is amplified. This is again caused by changes which took place on the whole planet. In order to meet the challenge mounted by dramatic consequences of human dominance over the planet the scope of journal has been amplified to include any problems concerning the quality of human life, respecting all other forms of lives and not compromising the possibilities for future generations to live their ways of life.

Starting from the year 2016, *Silesian Statistical Review* is considered as a *Journal of Oikometrics*

The name, derived from Greek words *οικος* and *μετρω*, suggests that the journal focus is upon Nature's house (*oikos*), as a subject matter of a study, and the measurement, as a prevailing methodology of study. The journal is treated as an *interdisciplinary forum on a sustainable livelihood*. Contrary to the inscription on the door of Plato's Academy: *let no one ignorant of geometry enter here*, over the door to *Journal of Oikometrics* there is hanged the signboard with the inscription: *Everyone who cares about, and interested in any issue of sustainable livelihood is welcomed here*.

The Journal welcomes therefore papers from specialists in sustainability science, ecology, ecological economics and any other alternatives to neoclassical economics. It encompasses – but is not limited to – the following topics:

- actuarial methods and their applications,
- social justice, inequality, polarization, and stratification,
- quality of institutional performance,
- social metabolism, its measurement and analysis,
- statistical education,
- sustainable development,
- environmentalism.

As the official journal of the Polish Statistical Society, Branch in Wrocław, it is designed also to attract papers that have direct relation with the activity of the Society, particularly in the field of education, promotion and rising awareness of the statistics role in the civilization development.

Walenty Ostasiewicz

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23. SCIENTIFIC STATISTICAL SEMINAR "WROCŁAW-MARBURG", POTTENSTEIN-KIRCHENBIRKIG, 28.09.2015 – 1.10.2015. EXTENDED ABSTRACTS

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COMBINING MARIAGE REVERSE ANNUITY CONTRACT WITH DREAD DISEASE INSURANCES

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(Wrocław University of Economics)

Silver generation consists of people aged over 50. This population segment is not economically and socially homogenous and usually is divided into three subgroups: young-old (50–64 years of age), middle-old (65–79) and oldest-old (aged 80 and over). These groups have different levels and structures of consumption and income, which determines the specific needs and protection. Looking at the number of people belonging to each subgeneration, we can observe that it is growing in time (see Table 1).

Table 1. Population by age groups

	2004			2014		
	Germany	EU	Poland	Germany	EU	Poland
Young old	15 414 236	87 980 181	6 500 716	17 486 938	100 631 557	8 091 402
Middle old	11 411 632	61 617 873	4 039 325	12 464 656	67 910 434	4 179 586
Oldest-old	3 448 363	18 995 065	911 994	4 359 581	26 041 603	1 480 454
Silver generation	30 274 231	168 593 119	11 452 035	34 311 175	194 583 594	13 751 442

Moreover, the fraction of population belonging to the silver generation group is growing as well, which means that European population is getting older and older. This situation can also be observed when we consider separately males and female population as in Figures 1 and 2.

[1] The partial support of the grant scheme "Non-standard multilife insurance products with dependence between insured" 2013/09/B/HS4/00490 is gladly acknowledged.*

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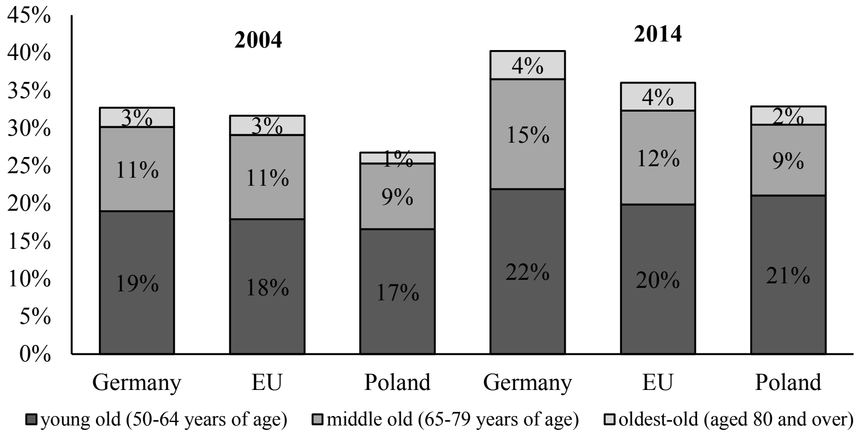

Figure 1. Males by age groups (% of total population)

Source: own elaboration.

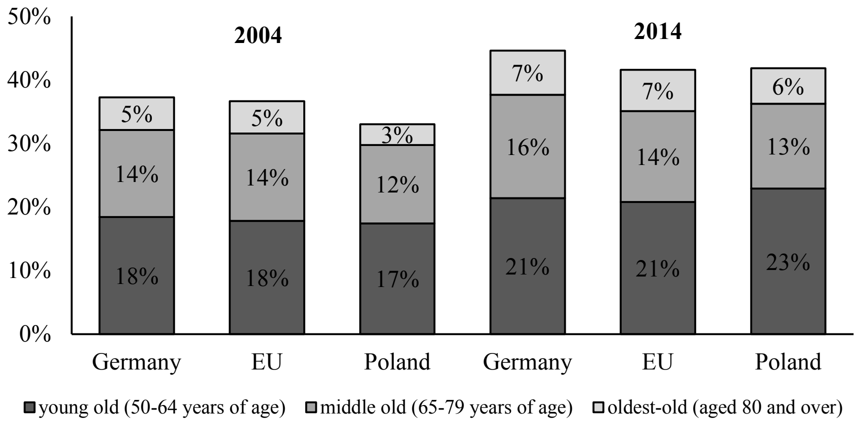

Figure 2. Female by age groups (%of total population)

Source: own elaboration.

Since the expected future lifetime is increasing, people are faced with at least two problems: reduced income declines living standards and the deteriorating state of health increases outlays for treatment. One of the possible solutions to the first problem is *marriage reverse annuity contract*, and to the second one – *dread disease insurance*.

Marriage reverse annuity contract is a financial product offered to elderly spouses. Owners receive annuity-due benefits in return for the transfer of the ownership onto the company (mortgage fund), owners

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have an ensured right to live in property until their death (e.g. [Dicson et al. 2009; Marciniuk 2014]).

Dread disease (or critical illness) *insurance* (offered to individuals – separately for wife and for husband) provides the policyholder with lump sum in case of dread disease included in the set of diseases specified by the policy conditions, such as heart attack, cancer or stoke. The benefit is paid on diagnosis. We analyse a stand-alone cover which means that the insurance policy ceases immediately after payment of the sum assured (e.g. [Pitacco 2014]).

We propose the joint product which consists of both the marriage reverse annuity contract and the dread disease insurance for a husband and for a wife (for one person such a product is proposed in [Dębicka et al. 2014]). This product could be addressed to elderly spouses since it takes advantage of both financial and insurance products and improves the living conditions of pensioners and provides additional funds in case of a critical illness. Moreover, the combined product reduces costs associated with maintenance expenses of three separate products (one company). The idea of the new product is that net premiums are cared for by customer's capital for the reverse annuity contract (it is the percentage of the value of property). The amount b of annual marriage reverse annuity is divided into three parts: $b\beta$ – an amount of annual marriage reverse annuity paid when at least one of the spouses is alive, and $0,5b\,(1-\beta)$ – a period premium for the dread disease insurance (the same amount for the wife and the husband separately), where $\beta \in [0, 1]$ is a reverse annuity parameter. $0,5b\,(1-\beta)$ is net premium and for wife and husband a dread disease lump sum benefits c_x and c_y are counted respectively.

We construct a multiple state model for the combined product and we estimate its probabilistic structure under assumptions that the future life time of husband and wife are independent and the benefit is paid until the death of the last surviving spouse (status of the last surviving). To make calculation easier we use matrix formulas for

- annual marriage reverse annuity paid when at least one of the spouses is alive [Dębicka, Marciniuk 2014].
- dread disease lump sum benefit [Dębicka, Zmyślona],
- net single premium [Dębicka 2013].

We made a numerical analysis for spouses who are aged between 65 and 85. We assume that the reverse annuity parameter is equal to 99% and the dread disease covers risk against the lung cancer. The fixed long-term interest rate 5.79% was estimated on the basis of the actual

Polish market related to the yield to maturity on fixed interest bonds and Treasury bills from 2008 in Nelson-Siegel model. It appears that the yearly value of marriage revers annuity contract (*b*) increases with age. For a younger woman and an older man the benefit is lower than for an older woman and a younger man. The woman’s age has a greater impact on the benefit. Moreover, the critical illness lump sum benefit is lower for men ($c_y < c_x$), which directly reflects the fact that lung cancer is more frequent in men population. Independently of woman’s age, the highest difference is for 70-year-old men and the lowest difference is for 85-year-old men. It is not surprising that the value of the benefits depends on the age of the spouses (and the length of the contract consequently), but the calculations show a strong influence of sex on the amount of dread disease benefits (up to 300%).

To sum up, the consideration of this product poses a new proposition of protection against the effects of longevity. This product is not offered on the market. Although the benefits obtained from the reverse annuity contract are a little bit lower than in the situation when the whole capital is used for calculation of the reverse annuity benefit, the illness benefits are considerably high. This gives the owner of a real estate an additional financial protection in a dread disease time. Thanks to reduction of the acquisition and maintenance costs charged to the customer, a combined product can be less expensive for spouses than independently purchasing each single component.

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VOLATILITY SPILLOVER FROM WIND ENERGY TO ELECTRICITY PRICE

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1. Introduction

Renewable energy supply becomes more and more meaningful, especially wind energy. However, wind energy is a stochastic source. Therefore, many studies examine the impact of wind production on electricity price volatility like for example in [Ketterer 2014]. But no one has study the impact of wind power volatility on price volatility. To remediate this deficit we use in our analysis a Two-Step-GARCH approach which we present here. Our main aim is to answer the question, if there is the so called "Volatility Spillover" or not. We use hourly wind energy feed in data of the Transaction System Operators in Germany[2] from 2006 to mid-2015 and convert them into day-base data. Electricity prices are calculated via an arithmetic average above hourly spot price of the EPEX[3] to estimate a daybased price [www.energinet.dk].

2. Two-step-GARCH approach

First of all we estimated a GARCH model for wind power using trend, seasonal and autoregressive terms in the mean equation. Besides the standard terms in the variance equation we include trend and season terms as well. To our knowledge Campbell and Diebold [2005] were the first who controlled for seasonal effects in the variance equation. Furthermore to get a smoother conditional variance with less spikes, we took the squared root of wind energy supply as dependent variable. Therefore our model looks as follows.

$$Wind_t = Trend_t + Season_t + \sum_{l=1}^{8} \rho_{t-l} \cdot Wind_{t-l} + \sigma_t \cdot Z_t,$$

where $Z_t \overset{i.i.d}{\sim} NV(0, 1)$ with

$Trend_t = \tau_w^m \cdot t;\ Season_t = \sum_{p=1}^{3} \left(s_p^c \cdot \cos\left(\frac{2\pi pt}{365.25}\right) + s_p^s \cdot \sin\left(\frac{2\pi pt}{365.25}\right)\right);$

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[2] In particular Tennet, 50Hertz, Amprion.
[3] Electricity exchange for delivery in Germany and Austria.

$$\sigma_{w,t}^2 = w + \alpha \cdot u_{t-1}^2 + \beta \cdot \sigma_{t-1}^2 + \tau_w^v \cdot t + \sum_{q=1}^{3} \left(\gamma_q^c \cdot \cos\left(\frac{2\pi pt}{365.25}\right) + \gamma_q^s \cdot \sin\left(\frac{2\pi pt}{365.25}\right) \right).$$

The estimated conditional variance $\hat{\sigma}_{w,t}^2$ can be found as time series in Figure 1. We define it as wind volatility and use it in the second step of our approach. Therefore, we estimated another AR(8)-GARCH(1,1) model with electricity price as dependent variable.

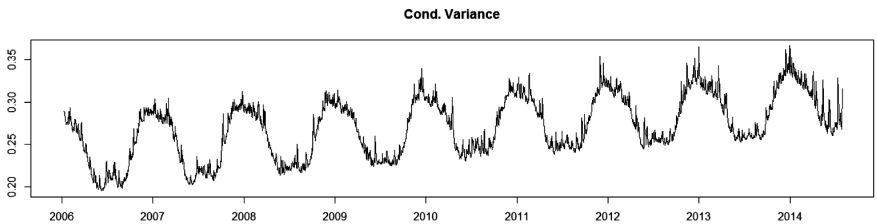

Figure 1. Conditional variance of *Wind*

Source: own calculations.

Our second model with electricity price as dependent variable looks as follows.

$$Price_t = Trend_t + Season_t + \sum_{d=1}^{6} \delta_d \cdot Day_d + \sum_{l=1}^{8} \rho_l \cdot Price_{t-l} + \sigma_t \cdot Z_t,$$

where $Z_t \overset{i.i.d}{\sim} NV(0, 1)$ with

$Trend_t = \tau_m^p \cdot t;\ Season_t = \sum_{p=1}^{3} \left(\eta_p^c \cdot \cos\left(\frac{2\pi pt}{365.25}\right) + \eta_p^s \cdot \sin\left(\frac{2\pi pt}{365.25}\right) \right);$

$\sigma_t^2 = w_p + \alpha_p \cdot u_{t-1}^2 + \beta_p \cdot \sigma_{t-1}^2 + \epsilon \cdot \hat{\sigma}_{w,t-1}^2 + \tau_v^v \cdot t + \sum_{d=1}^{6} \delta_d \cdot Day_d.$

In addition to the wind model we include weekday dummy variables to control for weekly seasonality in price. Besides these dummies we insert in the variance equation the GARCH(1,1) terms as well as a linear trend variable and of course the wind volatility $\hat{\sigma}_{w,t-1}^2$ from the first model. The aim is to look for a significant influence of $\hat{\sigma}_{w,t-1}^2$. Actually there seems to be a significant influence with an estimated coefficient in the amount of 0.218 followed by a p-value of 0.0309. To test whether this result is robust, we expand the same

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model with a series of Fourier-polynomials in the variance equation to control for seasonality. The extended variance equation is written down below.

$$\sigma_t^2 = w_p + \alpha_p \cdot u_{t-1}^2 + \beta_p \cdot \sigma_{t-1}^2 + \epsilon \cdot \hat{\sigma}_{w,t-1}^2 + \sum_{d=1}^{6} \delta_d \cdot Day_d + \sum_{p=1}^{P} \left(\theta_p^c \cdot \cos\left(\frac{2\pi pt}{365.25}\right) + \theta_p^s \cdot \sin\left(\frac{2\pi pt}{365.25}\right) \right).$$

Furthermore you find the final estimating output at the following page. As you can see the wind volatility turns out to be insignificant with change in sign. Anyway, the coefficients of the Fourier-polynomials seem to be significant. Additionally you find the time series of the conditional variance (volatility) of price for the two models (without and with Fourier-controlled variance) below the estimating output.

Table 1. Mean equation (model 2)

	Dependent variable: spot price		
	Coefficient	Standard Error	Probability
CONST	–0.383852	0.041832	0.0000
TREND	–0.002839	0.002684	0.2901
AR(1)	0.654716	0.022070	0.0000
AR(2)	–0.010383	0.026259	0.6925
AR(3)	0.053425	0.023830	0.0250
AR(4)	0.036261	0.023513	0.1230
AR(5)	0.022132	0.023225	0.3406
AR(6)	0.111848	0.024581	0.0000
AR(7)	0.031114	0.021802	0.1535
AR(8)	0.049040	0.018467	0.0079
Mo	1.117222	0.035309	0.0000
Tue	0.636272	0.038954	0.0000
Wed	0.629319	0.035247	0.0000
Thu	0.600400	0.033733	0.0000
Fri	0.549095	0.036431	0.0000
Sat	0.218098	0.036090	0.0000
$\cos_1$	0.020608	0.009919	0.0377
$\sin_1$	–0.022418	0.008788	0.0107
$\cos_2$	0.009681	0.009227	0.2941
$\sin_2$	–0.000537	0.008736	0.9510
$\cos_3$	0.002753	0.008954	0.7585
$\sin_3$	0.009551	0.008809	0.2783

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Table 2. Variance EQUATION

	Dependent variable: σ_{t-1}^2		
	Coefficient	Standard Error	Probability
CONST	0.104724	0.031362	0.0008
TREND	0.000788	0.001533	0.6070
u_{t-1}^2	0.231389	0.050775	0.0000
σ_{t-1}^2	0.621496	0.052108	0.0000
Cond.Var.	–0.561788	0.391984	0.1518
Mo	–0.014261	0.031544	0.6512
Tue	–0.054609	0.025072	0.0294
Wed	–0.054837	0.021462	0.0106
Thu	–0.063220	0.018846	0.0008
Fri	–0.036755	0.019680	0.0618
Sat	–0.036129	0.024147	0.1346
$\cos_1$	0.019437	0.008775	0.0268
$\sin_1$	–0.000136	0.002735	0.9604
cos2	0.004118	0.002508	0.1006
sin2	–0.010540	0.002852	0.0002
R^2	0.783671	Mean dependent var	2.647534
adj. R^2	0.781139	S.D. dependent var	0.999770
S.E. of regression	0.467718	Akaike info criterion	0.921168
Sum squared resid.	672.9062	Schwarz criterion	0.992997
Log likelihood	–1396.797	Hannan–Quinn criter.	0.946956
F-statistic	309.5300	Durbin–Watson stat	1.986308

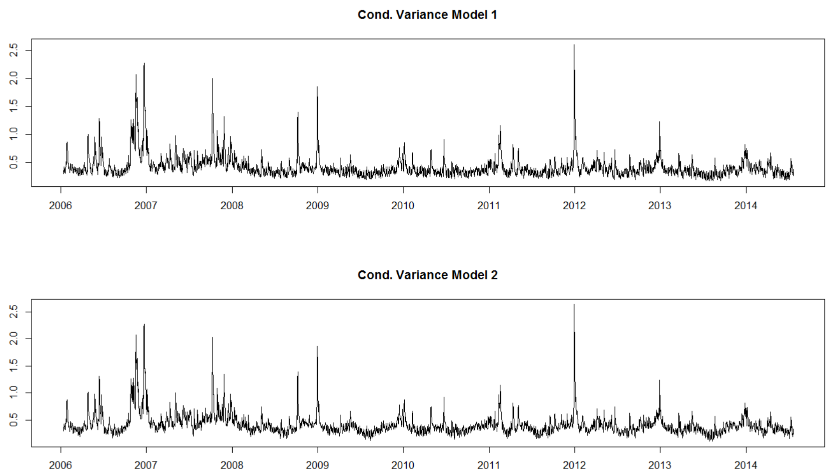

Figure 2. Time series conditional variance of electricity price

Source: own calculations.

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3. Conclusion

If we control for seasonal components in the variance equation, we do not find any significant influence of the wind power volatility. Hence, it seems more to be a positive correlation within the seasons of both volatilities than a spillover from wind volatility to price volatility and the season of price volatility could be caused by fluctuating demand. However, our analysis is unfinished. Future work has to be done and more (complex) models have to be analyzed.

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STOCHASTIC CASH FLOWS OF MARRIAGE INSURANCE WITH DEPENDENCE BETWEEN INSURED PRODUCTS[4]

Agnieszka Marciniuk (Wrocław University of Economics)

Non-standard multilife insurance products are considered. We focused on marriage products for example: life insurance, life annuity and reverse annuity contract. The aim of presentation was to determine the value of marriage insurance product benefit. In contrast to the simplified approach which assumes that future lifetimes of the wife and the husband are independent, dependence of lifetimes between the spouses is assumed. Moreover, typically, for simplicity, it is assumed that the interest rate is fixed and the same for all years. However, the interest rate that will be used in future years is of course neither known nor constant. Therefore, the stochastic interest rate is applied (cf. [Jakubowski et al. 2003]). Valuation of actuarial benefits is made by the use of a stochastic payment streams. The probability of life expectancy is determined using a Markov model on the basis of The Central Statistical Office data for Lower Silesia (cf. [Denuit et al. 2001; Heilpern 2014]).

[4] Research funded by 2013/09/B/HS4/00490.

The stochastic cash flows and the discounted value of the cash flows of the above-mentioned products are given. Subsequently the net single premium, that means the expected value of the discounted value of the cash flows, and the standard deviation of the discounted value of cash flows are determined. Let us consider The Common Life Status. This status is then, when the benefit is paid only until the death of one spouse. The first and second moment of the discounted value of cash flows are given by the following theorem.

Theorem

Let us denote *m* as a part of the year (*m* > 0). The net single premium of marriage life insurance is given by the following formulae

$$A_u = E^{\mathbf{Q}}\left(\Lambda^{-1}_{\frac{K_u^{(m)}+1}{m}} \middle| F_0 \right) = \sum_{t=0}^{\infty} P_{0,\frac{t+1}{m}} \cdot \mathbf{P}\left(K_u^{(m)} = t\right), \tag{1}$$

where $P_{0,t}$ is the price of zero-coupon bond at the moment 0 with maturity *t* and moreover

$$\mathbf{P}\left(K_u^{(m)} = t\right) = {}_{\frac{t}{m}}p_u \cdot {}_{\frac{1}{m}}q_{u+\left(\frac{t}{m}\right)} = \frac{1}{m} \cdot {}_{\left[\frac{t}{m}\right]}p_u \cdot q_{u+\left[\frac{t}{m}\right]}. \tag{2}$$

The probability (2) is calculated under the assumption of the uniform distribution of death during the year.

The second moment of the discounted cash flows of marriage life annuity is determined as follows

$$^2A_u = E^{\mathbf{Q}}\left(\Lambda^{-2}_{\frac{K_u^{(m)}+1}{m}} \middle| F_0 \right) = \sum_{t=0}^{\infty} E^{\mathbf{Q}}\left(\Lambda^{-2}_{\frac{t+1}{m}} \middle| F_0 \right) \cdot \mathbf{P}\left(K_u^{(m)} = t\right), \tag{3}$$

where

$\Lambda_{0,t}^{-2} \equiv \Lambda_t^{-2} = \exp\left(-2\int_0^t r_s ds\right)$ is the second moment of the discounting process and $\{r_t\}_{t\geq 0}$ – a stochastic process of short-term rate, defined on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$, and $\mathbf{P}\left(K_u^{(m)} = t\right)$ is calculated by the use formulae (3).

The numerical examples are made by the use of own programs written in MATLAB. There are calculated the net single premium,

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the standard deviation in the case when the future lifetimes of spouses are independent or dependent random variables. The interest rate is modelled by using the Svensson model of short-term rate (cf. [Marciniuk 2014; Dębicka, Marciniuk 2014]). The parameters of this function are estimated by the use of least square method on the basis of actual Polish market data, related to the yield to maturity on fixed interest bonds and Treasury bills [www.money.pl/pieniadze/bony/przetargi/ and http://bossa.pl/notowania/stopy/rentownosc_obligacji/].

On the following graph we can see that the percentage differences between premiums for independence and dependence future spouses lifetimes in the case when husband and wife are of the same age.

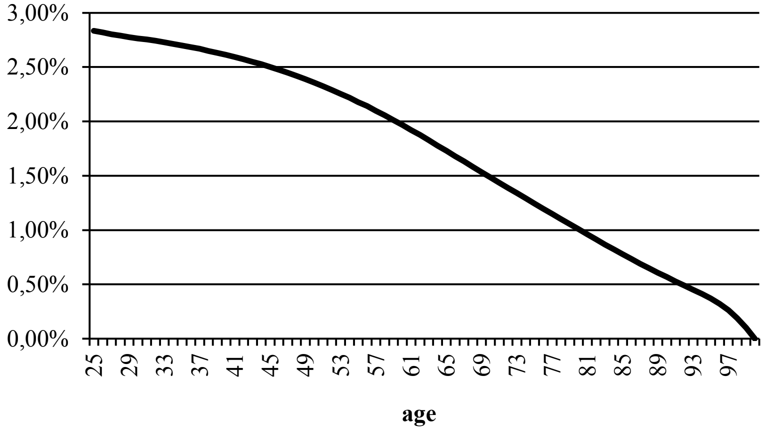

Figure 1. The the percentage differences between premiums for independence and dependence future spouses lifetimes in the case when husband and wife are of the same age

Source: own calculations.

In the first case of independent random variables the premium is higher but the differences decrease with rise of insured age.

On the following graph there are presented the percentage differences between standard deviation of the discounted future cash flows for independence and dependence future spouses lifetimes in the case when husband and wife are at the same age (Figure 2).

In the case of the standard deviation the situation is not so clear. The standard deviation is higher for independence random variables but only to the age of 53 years. Then the standard deviation is higher for dependence random variables. The first the differences increase and the decrease to the age of 100 years.

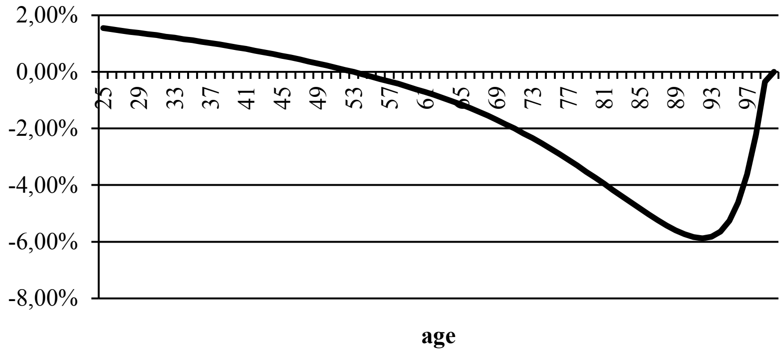

Figure 2. The percentage differences between standard deviation of the discounted future cash flows for independence and dependence future spouses lifetimes in the case when husband and wife are of the same age

Source: own calculations.

The same calculations are made for the marriage reverse annuity contract. The reverse annuity is the benefit which an owner can receive in exchange for surrender-ring his real estate to a company (mortgage fund), created especially for this purpose. The owner is guaranteed the right to live in the property until his death by a notarial act. The married reverse annuity contract is an agreement in which property owners are husband and wife. This marriage annuity is not currently offered in Poland. It is considered The Common Life Status.

There are calculated benefits, when annuity is paid m-th ($m = 1, 2, 4, 6, 12$) times a year. The benefit increases with the rise of the spouses’ age. A greater impact on the amount of benefit has the husband’s age. The differences between yearly value of annuities when the future lifetimes of spouses are independent and dependent random variables are higher in the first case The differences are between 2 and 5%. The yearly benefits in the case, when annuity is paid monthly are much higher, when benefit is paid only once a year. However the partition of the year into more than 12 parts does not cause a significant increase in premium.

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ON THE MONITORING OF CAPM PORTFOLIO BETAS

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1. Introduction

Despite substantial criticism, variants of the Capital Asset Pricing Model (CAPM) remain still the primary statistical tools for portfolio managers to assess the performance of financial assets. In the CAPM the risk of an asset is expressed through its correlation with the market, widely known as the beta. There is now a general consensus among economists that these portfolio betas are time-varying and that, consequently, any appropriate analysis has to take this variability into account. Moreover, recent advances in data acquisition and processing techniques have led to an increased research output concerning high-frequency models.

Within this framework, we first briefly discuss a modified functional CAPM, introduced in Aue et al. [2012], that incorporates microstructure noise, as well as sequential monitoring procedures to test for the constancy of the portfolio betas in this setting. The main result in Section 2 provides some large-sample properties of this procedure.

In Section 3, we present recent results of Chochola et al. [2013, 2014] on more robust procedures for the monitoring of CAPM portfolio betas. Some asymptotic sequential tests are discussed for the (ordinary) CAPM in discrete time as well as some extensions of the procedures to a functional version of the CAPM are provided. A few simulation results indicate that our methods perform well in finite samples.

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2. An L_2-monitoring procedure

In Aue et al. [2012], we discuss a monitoring procedure for detecting a “change in the portfolio betas” in the following functional CAPM:

$$\boldsymbol{r}_i(s) = \boldsymbol{\alpha}_i + \boldsymbol{\beta}_i r_{M,i}(s) + \boldsymbol{\varepsilon}_i(s), s \in [0,1], i = 1,2, \ldots,$$

where $\{\boldsymbol{r}_i\}$ are vectors of functional daily log-returns of d risky assets in a portfolio, $\{r_{M,i}\}$ are the corresponding log-returns of the observable market portfolio, and $\{\boldsymbol{\varepsilon}_i\}$ are the unobservable, centered random error functions (on day i). The d-dimensional vectors $\boldsymbol{\beta}_i$ contain the portfolio betas which quantize the assets’ market risks and the $\boldsymbol{\alpha}_i$’s are related to the expected returns on a risk-free asset. Note that the model incorporates the possibility of high-frequency data, since the argument s is used to describe an intra-day behaviour, while i is understood, e.g., as a daily index. Dependence in the functional CAPM can thus have two sources, i.e., serial dependence across trading days $i = 1, 2, \ldots$ and intra-day dependence across $s \in [0, 1]$. For a more detailed discussion we refer to Aue et al. [2012].

It is assumed that the betas are piecewise constant (say per day) and that there is no change in the betas of a “training sample” of size m, i.e., that $\boldsymbol{\beta}_i = \boldsymbol{\beta}_1$, $i = 1, \ldots, m$. We are interested in constructing an appropriate stopping rule for testing the null hypothesis

$$H_0: \boldsymbol{\beta}_1 = \cdots = \boldsymbol{\beta}_m = \boldsymbol{\beta}_{m+1} = \cdots,$$

i.e., constancy of the portfolio betas over time, against the alternative

$$H_A: \exists\ k^* \geq 1 \text{ such that } \boldsymbol{\beta}_1 = \cdots = \boldsymbol{\beta}_{m+k^*-1} \neq \boldsymbol{\beta}^* = \boldsymbol{\beta}_{m+k^*} = \boldsymbol{\beta}_{m+k^*+1} = \cdots,$$

that is, there is a structural break (change-point) at $m + k^*$.

The random functions $\boldsymbol{r}_i$ and $r_{M,i}$ are observed at n equidistant intra-day time points s_ν, $\nu = 1, \ldots, n$, and the sequential procedure can roughly be described as follows:

Step 1: Compute the least squares estimator $\widehat{\boldsymbol{\beta}}_m$ for the common portfolio betas from the training period.

Step 2: Sequentially compute the least squares estimators $\widetilde{\boldsymbol{\beta}}_k$ based on the new observations $(\boldsymbol{r}_{m+1}, r_{M,m+1}), \ldots, (\boldsymbol{r}_{m+k}, r_{M,m+k})$ in the monitoring period.

Step 3: Compare $\widehat{\boldsymbol{\beta}}_m$ to $\widetilde{\boldsymbol{\beta}}_k$ and reject H_0 in favour of H_A at time lag k if the estimators are significantly different.

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More precisely, we construct a detector statistic V_k, which is a suitably normalized quadratic form of the difference $\tilde{\boldsymbol{\beta}}_k - \hat{\boldsymbol{\beta}}_m$, and compare the latter to a threshold function w. Then we reject H_0 at the first time k such that the detector V_k exceeds the value of the scaled threshold function w, that is, our stopping rule is as follows:

$$\tau_m = \tau_m(c, w) = \min\left\{1 \leq k \leq [mT]: V_k > cw\left(\frac{k}{m}\right)\right\},$$

with $T > 0$ fixed, c being a suitable critical constant and min $\emptyset := \infty$. Depending on the application, the threshold function w can be chosen as a more sensitive one to be able to detect changes quickly, which typically inherits higher false discovery rates, or in a less sensitive way which then will take longer to detect a change, but therefore is more stable and reliable. The constant $c = c_\alpha$ is determined such that

$$\lim_{n\to\infty} P(\tau_m < \infty) = \alpha \text{ under } H_0,$$

where $\alpha \in (0, 1)$ denotes the asymptotic significance level.

The critical constant c can be determined via the following null asymptotic, which provides the main result of Aue et al. [2012, Theorem 3.1]:

Theorem 1. Under suitable dependence and regularity assumptions, we have under H_0, for any $T > 0$,

$$\lim_{n\to\infty} P(\tau_m < \infty) = P\left(\sup_{0\leq t\leq T} \frac{\Gamma_d(t)}{w(t)} > c\right),$$

where, for $t \in [0, 1]$, $\Gamma_d(t) = \Sigma_{1\leq j\leq d} B_j^2(t)$, with $B_1, \ldots, B_d$ denoting independent, identically distributed Gaussian processes such that $E[B_j(t)] = 0$ and $E[B_j(s)B_j(t)] = \min(s, t) + st$.

In Aue et al. [2012, Theorem 3.2], we also show that the above monitoring procedure is consistent (has asymptotic power 1) under a one-change alternative, even if we allow $\boldsymbol{\beta}_1$ and $\boldsymbol{\beta}^*$, the pre- and postbreak betas, to depend on m. Let _$\Delta_m = |\boldsymbol{\beta}_1 - \boldsymbol{\beta}^*|$ measure the break size.

Theorem 2. Under the same model assumptions as in Theorem 1, we have under H_A, with $m < k^* = [mT^*]$ for some $0 < T^* < T$,

$$\lim_{n\to\infty} P(\tau_m < \infty) = 1,$$

provided that $m\Delta_m^2 \to \infty$ as $m \to \infty$.

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Remark 1. In Aue et al. [2012], we present a number of examples of function-valued stochastic processes satisfying the model assumptions required for the proofs of Theorems 1 and 2. In particular, it is shown that functional AR(1) processes and functional ARCH(1) processes are covered by this framework. Moreover, it is demonstrated by a real data example, comparing five stocks (BA, BAC, MSFT, T, XOM) versus the OEX index, that the procedure is able to detect the shock effect of the 9/11 terrorist attack.

3. Robust monitoring

Since least squares estimators are known to be sensitive with respect to outliers in the data, the L_2-procedure described in Section 2 may sometimes be overrejecting, that is, it may signal alarm although no structural change has occured yet. This was our main motivation in Chochola et al. [2013, 2014] to consider some alternative detection rules which are based on more robust estimators than the least squares ones used above.

Consider again the functional CAPM of Section 2, but in the following reparametrized setting:

$$r_{i,j}(s) = \alpha_j^0 + \beta_j^0 r_{M,i}(s) + (\alpha_j^1 + \beta_j^1 r_{M,i}(s))\delta_m I\{i > m + k^*\} + \varepsilon_{\boldsymbol{i,j}}(s), s \in [0,1], i = 1,2, \ldots,$$

where k^* denotes the change-point and $\alpha_j^0, \beta_j^0, \alpha_j^1, \beta_j^1, \delta_m$ $(j = 1, \ldots, d)$ are unknown parameters. So, as in Section 2, we assume a training period of size m in which no parameter change occurs and we want to monitor a change in the betas, i.e., construct a sequential procedure for testing H_0 versus H_A.

The robust monitoring procedure makes use of M-estimators $\hat{\alpha}_{jm}, \hat{\beta}_{jm}$ for α_j^0, β_j^0 based on the training period, i.e., uses the minimizers of

$$\Sigma_{i=1}^m \Sigma_{\nu=1}^n \rho_j(r_{i,j}(s_\nu) - a_j - b_j r_{M,i}(s_\nu)),$$

where ρ_j are convex loss functions with derivatives $\psi_j (j = 1, \ldots, d)$ and $s_\nu = \nu/n$ $(\nu = 1, \ldots, n)$. We define M-residuals

$$\boldsymbol{\psi}(\hat{\boldsymbol{\varepsilon}}_i(s_\nu)) = \left(\psi_1\left(\hat{\varepsilon}_{i,1}(s_\nu)\right), \ldots, \psi_d\left(\hat{\varepsilon}_{i,d}(s_\nu)\right)\right)^T \text{ with } \hat{\varepsilon}_{i,j}(s_\nu) = r_{i,j}(s_\nu) - \hat{\alpha}_{jm} - \hat{\beta}_{jm} r_{M,i}(s_\nu)$$

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and set

$$\hat{\boldsymbol{z}}_i = \frac{1}{n} \Sigma_{i=1}^{n} r_{M,i}\, \boldsymbol{\psi}(\hat{\boldsymbol{\varepsilon}}_i(s_\nu)).$$

Then the null hypothesis is rejected as soon as

$$\hat{Q}(k,m)/q_\gamma(k/m) \geq c_\alpha,$$

where

$$\hat{Q}(k,m) = \left(\frac{1}{\sqrt{m}} \Sigma_{i=m+1}^{m+k} \hat{\boldsymbol{z}}_i\right)^T \hat{\boldsymbol{\Sigma}}_m^{-1} \left(\frac{1}{\sqrt{m}} \Sigma_{i=m+1}^{m+k} \hat{\boldsymbol{z}}_i\right),$$

the matrix $\hat{\boldsymbol{\Sigma}}_m$ is an estimator of the asymptotic covariance matrix

$$\boldsymbol{\Sigma} = \lim_{m\to\infty} \operatorname{cov}\left\{\frac{1}{\sqrt{m}} \Sigma_{i=1}^{m} \int_0^1 r_{M,i}(s) \boldsymbol{\psi}\big(\boldsymbol{\varepsilon}_i(s)\big)\, ds\right\},$$

and $q_\gamma(t) = (1+t)^2\, (t/(t+1))^{2\gamma}$, $t \in (0,\infty), \gamma \in [0,1/2)$, is a typical threshold function and c_α is a critical value. So, we have the following stopping rule:

$$\tau_m = \tau_m(c,q) = \min\left\{1 \leq k \leq [mT]\colon \hat{Q}(k,m) > c\, q(k/m)\right\},$$

with $T > 0$ fixed and $\min \emptyset \coloneqq \infty$. The critical value $c = c_\alpha$ is again chosen such that

$$\lim_{n\to\infty} P(\tau_m < \infty) = \alpha \text{ under } H_0,$$

and

$$\lim_{n\to\infty} P(\tau_m < \infty) = 1 \text{ under } H_A.$$

Typical score functions $\psi(x) = \rho'(x)$ to be used for the M-residuals are, e.g., $\psi(x) = x$ (L_2-residuals), $\psi(x) = \operatorname{sign} x$ (L_1-residuals), or the Huber (1981) function $\psi(x) = x\ (|x| \leq K), = K \operatorname{sign} x\ (|x| > K)$, with some $K > 0$.

In a non-functional framework (cf. Chochola et al. [2013, Theorem 2.1]) as well as under a high-frequency setting (cf. Chochola et al. [2014, Theorem 2.1]), the critical constant $c = c_\alpha$ for the procedure can be chosen via the following asymptotic:

Theorem 3. Under suitable dependence and regularity assumptions and assuming that $\hat{\boldsymbol{\Sigma}}_m - \boldsymbol{\Sigma} = o_P(1)$ as $m \to \infty$, we have under H_0, for any $T > 0$,

$$\lim_{n\to\infty} P(\tau_m < \infty) = P\left(\sup_{0\leq t\leq T/(T+1)} \frac{\Sigma_{j=1}^{d} W_j^2(t)}{t^{2\gamma}} > c\right),$$

where $\{W_j(t), t \in (0,1)\}$, $j = 1, \ldots, d$, are independent Brownian motions.

Remark 2. In Chochola et al. [2014, Theorem 2.3] we suggest a Bartlett type estimator $\widehat{\mathbf{\Sigma}}_m$ satisfying the required assumption in Theorem 3.

Some asymptotics under local alternatives can also be derived (cf. Chochola et al. [2013, Theorem 2.2] and Chochola et al. [2014, Theorem 2.2], which, in particular show that the robust monitoring procedure is consistent, i.e., has asymptotic power 1.

Theorem 4. If $\delta_m \to 0$, $|\delta_m|\sqrt{m} \to \infty$, $\beta_j^1 \neq 0$ $(\exists\, j = 1, \dots, d)$ and $k^* = [m\theta]$, with some $0 < \theta < 1$, then, as $m \to \infty$,

$$\max_{1 \leq k \leq [mT]} \frac{\hat{Q}(k,m)}{q_\gamma\left(\frac{k}{m}\right)} \xrightarrow{P} \infty,$$

that is,

$$\lim_{n \to \infty} P(\tau_m < \infty) = 1.$$

Remark 3. In a small simulation study in Chochola et al. [2014, Section 3], the asymptotic results of Theorems 3 and 4 have been investigated concerning empirical level and detection delays, under various scenarios and for L_1-, L_2- and Huber residuals. Conclusions from this empirical study are that

- the L_2-procedure is often overrejecting, but sometimes may even have larger detection delays,
- the Huber procedure provides a good balance between robust and sensitive monitoring, and that
- the tuning constant $\gamma = 1/4$ is an appropriate choice for the threshold function q, if no prior knowledge about the change-point k^* is available.

Moreover, the real data set from Aue et al. [2012] has also been checked in Chochola et al. [2014] under robust monitoring. The conclusions are similar as in the simulations described above, in particular, the Huber procedure with $\gamma = 1/4$ turns out to be a good compromise between sensitive and non-sensitive monitoring.

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MODELLING EXTRA MORTALITY FOR PATIENTS SUFFERING FROM CRITICAL ILLNESS

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The problem of estimating mortality of terminally ill patients is useful in many issues related to health economics, for example in health policy issues and in health insurance. Extra mortality is defined as some specific part of mortality due to critical illness. Estimation of extra mortality relies on creation of numerical tables or parametric mortality models. The second possibility is an adjustment of standard age-pattern of mortality. Extra mortality should depend on the expected lifetime of a patient which is equivalent to dependency on age and gender, it should also depend on the stage and duration of illness. In the paper, the malignant tumour is chosen as an example of a critical illness. In developed counties, cancer is the second most frequent cause of death. Because age is an important risk factor, the incidence of cancer is growing due to the phenomenon of aging of population. Despite advances in medicine, certain types of cancer are still very difficult to treat. The effectiveness of treatment depends on the type of cancer and the stage of illness at the time of detection. Some types of cancer are specified as cancers with worse prognosis due the fact that only a small number of patients are cured [Pitacco 2014].

Modelling extra mortality is connected with the problem of estimation of fatality rate for a patient with lung cancer. Residents of Lower Silesia region, who in the years 2006–2011 suffered from lung cancer, has been chosen as a study population. Due to the fact that morbidity and mortality are different for men and women populations, we analyze these two populations separately. Three data sets are used for the analysis, life tables, data from the national cancer registry for Lower Silesia [Wojciechowska, Didkowska 2014] and data concerning the hospitalizations histories from Lower Silesia Department of the National Health Fund which is the public payer in Poland. Data for the period from 2006 to 2011 was included in the analysis. The year 2008, as one of the middle periods, was established as the reference year. The choice of the middle period allows for considering the histories of hospitalization of these patients in the time period from 2006 to 2011 (NHF 2014).

We distinguish two states of illness, the milder without metastases and critical with diagnosed metastases [Dębicka, Zmyślona 2016]. We take into account the possibility of deterioration of health state. The probability of death for a patient in age x suffering from lung cancer is defined using some epidemiological and demographic rates in the following way

$$d_x = (1-\beta_x)\left[(1-\rho_x)q_x + \rho_x p_x\right] + \beta_x p_x,$$

where: β_x – percentage of patients aged x with metastases (in a critical state); ρ_x – probability of diagnosis of metastases within one year after the first diagnosis; p_x – probability of patient's death in a critical state (with diagnosed metastases); q_x – probability of death of a person aged x on the basis of life tables (contains the mortality rate due to lung cancer in population ζ_x).

The values of probability d_x are shown in Figure 1.

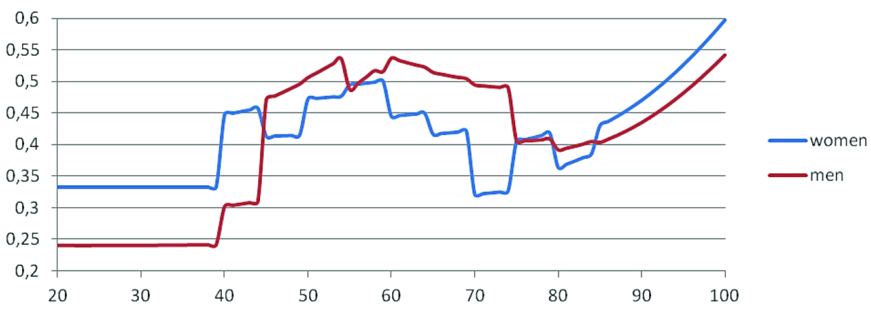

Figure 1. The values of probability d_x

Source: own calculations.

The probability of death for a patient with lung cancer can be determined using mortality rate for a whole population q_x and parameter η_x which measures extra mortality for patient suffering from lung cancer. Parameter η_x can be expressed by the following formula

$$d_x = q_x(1+\eta_x).$$

We defined extra mortality due to lung cancer as

$$\eta_x = (\rho_x + \beta_x - \rho_x\beta_x)\left(\frac{p_x}{q_x} - 1\right).$$

The values of parameter ηx depending on age for men and women population inhabiting the Lower Silesia region are presented in Figure 2.

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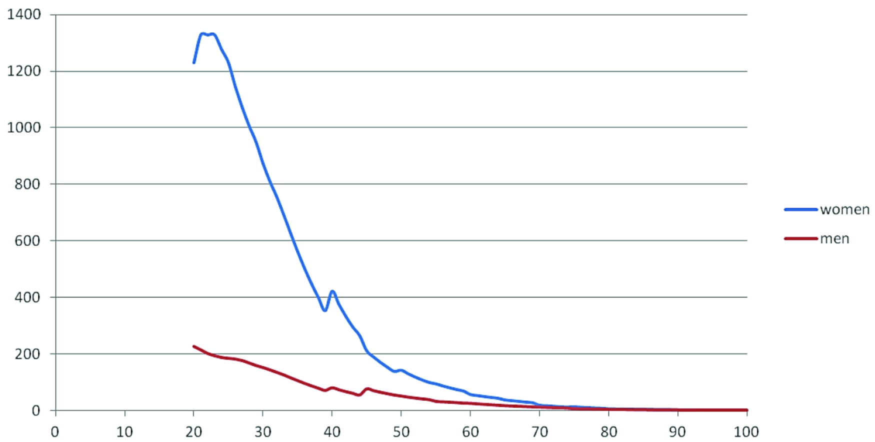

Figure 2. The values of parameter η_x

Source: own calculations.

The huge gap between the values of extra mortality for young women and young men results from the differences between probabilities of death in population of men and women in younger groups.

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