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NUMERICAL MODEL OF HEAT TRANSFER IN THREE PHASES OF THE POROELASTIC MEDIUM

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Abstract: In this paper, the results of numerical analysis of the thermal consolidation of a two phase medium, under the assumption of independent heat transfer in fluid and the solid phase of the medium, are presented. Three cases of pore fluid were considered: liquid, represented by water, and gas, represented by air and carbon dioxide. The mathematical model was derived from irreversible thermodynamics, with the assumption of a constant heat transfer between the phases. In the case of the accepted geometry of the classical dimensions of the soil sample and boundary conditions, the process leads to equalization of temperatures of the skeleton on the pore fluid. Heat transfer is associated with the fluid flow in the pores of the medium. In the case of gas as the pore fluid, a non-linear mathematical model of gas filtration through the pores of the medium was accepted. For the computing process, relationships between viscosity or density and temperature proposed by other authors were taken into account. Despite accepting mechanical constants of the solid phase that do not depend on temperature, the obtained model is non-linear and develops the classical Biot–Darcy model.

Key words: *poroelasticity, Biot model, thermal consolidation, GAS*

1. INTRODUCTION

In the classical approach to modelling soil consolidation, pores of the medium are assumed to be filled with water, and the process is considered to be isothermal. Such an assumption cannot be fulfilled in areas such as soil over the water table or zones in which phase transition occurs due to elevated temperature, such as UCG (Underground Coal Gasification) sites, for which, one can assume that pores are filled with gas. Moreover, in such zones, temperature variations cannot be neglected. Apart from the calculations of strain and fluid flow, heat flow needs to be calculated. The present model of thermal consolidation concerns a porous medium consisting of an elastic skeleton; pores are filled with liquid or gas (2 phases simultaneously are considered). The model is based on the Biot–Darcy consolidation theory, presented by Biot [1], and Biot and Willis [2]. The constitutive equations and fluid flow can be written as

$$\begin{gathered}\sigma_{ij} = 2N\varepsilon_{ij} + (A\varepsilon + Q\theta)\delta_{ij}, \\ \sigma - \sigma_a = Q\varepsilon + R\theta, \\ \nabla^2\sigma = \frac{\dot{\theta} - \dot{\varepsilon}}{k_w},\end{gathered} \tag{1}$$

where σ_{ij} is the skeleton stress tensor, ε_{ij} is the strain tensor, ε is the volumetric strain of the skeleton, θ is the volumetric strain of the pore fluid, N is the shear modulus of the skeleton, consonant with Lame's constant μ, A is the constant characterizing volumetric strain of the poroelastic medium, consonant with Lame's constant λ of elastic medium, Q is the coefficient describing impact of the volumetric strain on the fluid or stress in the skeleton (and vice versa), R characterizes the change of the volume of fluid under impact of fluid stress σ, k_w is a coefficient depending on the filtration coefficient.

The results of the numerical simulation of thermal consolidation for the two phase medium, with pores filled alternatively with water, air or carbon dioxide, under the assumption of heat transfer taking place in each phase separately, are presented below.

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In the thermal consolidation problem, the unknowns are – apart from strain in 3 dimensions – stress in the pore fluid and temperature. Researchers who have conducted studies in the thermodynamic approach to mathematical modelling of the heat transfer in poroelastic medium are, among others, Coussy [3], Derski (Kisiel, Derski et al. [5]), and Strzelecki (Strzelecki et al. [7]). Taking into account the fact that the heat transfer occurs in both phases of the medium with simultaneous heat transfer between them, analysis of the mutual impact of the heat transfer in the skeleton and the pore fluid seems to be important.

The system of thermal consolidation equation takes the following form:

- Equilibrium equation

$$(\sigma_{ij} - \sigma\delta_{ij})_{,j} - (\gamma x_j \delta_{i3})_{,j} = 0\,, \tag{2}$$

- Filtration equation

$$\nabla^2\sigma = \frac{\dot{\theta} - \dot{\varepsilon}}{k_w}\,, \tag{3}$$

- Heat transfer equations, derived by the authors from irreversible thermodynamics

$$\begin{aligned} \lambda^s(1-n)\nabla^2\vartheta^s &= T^s 3K\alpha^s\dot{\varepsilon} + T^s 3Q\alpha^s\dot{\theta} \\ &+ \frac{T^s}{T_0}(\rho_1 c_v^s\dot{\vartheta}^s + \rho_v^{ls}\dot{\vartheta}^l), \\ \lambda^l n\nabla^2\vartheta^l &= T^l Q\alpha^l\dot{\varepsilon} + T^l R\alpha^l\dot{\theta} \\ &+ \frac{T^l}{T_0}(\rho c_v^{ls}\dot{\vartheta}^s + \rho_2 c_v^l\dot{\vartheta}^l), \end{aligned} \tag{4}$$

where indexes i and s stand for the fluid and the skeleton, respectively, index ls stands for parameters that determine mutual influence of both phases, λ is the heat transfer coefficient, n is porosity, T is the absolute temperature, T_0 is the initial temperature, $\vartheta = T - T_0$, c_v is the specific heat, K is the bulk modulus of the skeleton, α is the coefficient of thermal expansion (linear for the skeleton and volumetric for the fluid), ρ is the density: $\rho_1 = \rho^s(1-n)$, $\rho_2 = \rho^l n$, $\rho = \rho_1 + \rho_2$.

Equations (4) have been derived from the equations of entropy, which were obtained from the free energy balance $\sigma_{ij}\dot{\varepsilon}_{ij} + \sigma\dot{\theta} - s_1\dot{T}^s - s_2\dot{T}^l - \dot{\chi} = 0$, where $\dot{\chi}$ is a change of the Helmholtz free energy for the two-phase medium

$$-s_1 = \frac{\partial\chi}{\partial T^s}, \quad -s_2 = \frac{\partial\chi}{\partial T^l}.$$

The parameters of equations have been obtained by assuming that strain is caused only by the temperature for parameters standing by $\dot{\varepsilon}$ and $\dot{\theta}$ (equations of entropy are coupled with the constitutive equations), and constant strain for parameters standing by ϑ^l and ϑ^s.

In the case of pores of the medium filled with water, there apply Biot's constitutive equations modified by the authors for non-isothermal issue with the assumption of heat transfer in both phases separately

$$\begin{aligned} \sigma_{ij} &= 2N\varepsilon_{ij} + (A\varepsilon + Q\theta - 3\mathrm{K}\alpha^s\vartheta^s - Q\alpha^l\vartheta^l)\delta_{ij}, \\ \sigma - \sigma_a &= Q\varepsilon + R\theta + 3Q\alpha^s\theta^s + R\alpha^l\theta^l. \end{aligned} \tag{5}$$

In the case of gas as the pore fluid, the volume of which strongly depends on pressure and temperature, equations with the assumption of uncompressible fluid cannot be applied. In such cases, Strzelecki et al. [6] proposed relations resulting from Boyle–Mariotte's law. A slightly modified equation for volumetric strain in the fluid (atmospheric pressure assumed as a reference point) takes the form

$$\theta = \frac{\sigma_a}{\sigma - Q\varepsilon} - 1. \tag{6}$$

Hence the constitutive equation based on (5) for pore fluid stress in the case of gas as the pore fluid takes the form

$$\sigma - \sigma_a = Q\varepsilon - \frac{\sigma_a}{1+\theta}\theta + 3Q\alpha^s\vartheta^s + \frac{\sigma_a}{1+\theta}\alpha^l\vartheta^l. \tag{7}$$

2. NUMERICAL MODELLING

2.1. THE GEOMETRY OF THE MODEL AND THE BOUNDARY CONDITIONS

Numerical FEM analysis has been conducted using the professional software FlexPDE v6.37. The cylindrical soil sample has a height of 76 mm and a diameter of 40 mm. It is only loaded with dead weight; its consolidation is influenced by the temperature gradient. It is placed in an impermeable adiabatic shell. The initial temperature equals 10 °C; after 100 seconds it starts to be heated to a temperature of 100 °C. The computing time is 300 seconds. After this period, the process turns into a steady state, so further changes do not appear.

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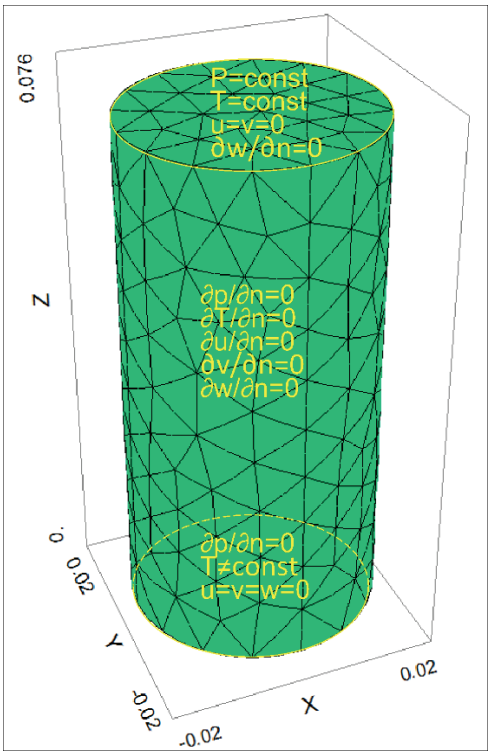

Fig. 1. Model geometry and boundary conditions

2.2. PARAMETERS OF THE MODEL

Two gases, air and carbon dioxide, we accepted as pore fluids, as well as liquid water. The physical parameters of these fluids are presented in Table 1.

Table 1. Physical parameters of fluids

	Air	Carbon dioxide	Water
Thermal expansion coefficient $[\alpha] = 1/K$	1/T	1/T	69e-6
Individual gas constant $[r]$ = J/Kg.K	287.05	188.9	–
Dynamic viscosity at a temperature of 0 °C $[\mu_0]$ = Pa.s	$17.21 \cdot 10^{-6}$	$13.75 \cdot 10^{-6}$	$1.8 \cdot 10^{-3}$
Heat transfer coefficient at a temperature of °C $[\lambda_0]$ = W/m·K	0.0244	0.0147	0.5
Sutherland's constant C_s	122	250	–
Specific heat $[c_v]$ = J/kgK	1005	844	4190

The viscosity and heat transfer coefficient for gases have been modelled depending on the temperature, using the Sutherland formula

$$\mu = \mu_0 \frac{273 + C_s}{T + C_s} \left(\frac{T}{273} \right)^{1.5}, \tag{8}$$

$$\lambda = \lambda_0 \frac{273 + C_s}{T + C_s} \left(\frac{T}{273} \right)^{1.5}. \tag{9}$$

The viscosity of water has been calculated using the Thorpe–Rodger formula

$$\mu = \frac{1.79 \cdot 10^{-3}}{1 + 3.37 \cdot 10^{-2} \cdot (T - 273) + 2.2 \cdot 10^{-4} \cdot (T - 273)^2}. \tag{10}$$

The density of water and gases is described by the formulas

$$\rho_w = \frac{1000}{1 + \alpha^w (T - T_p)}, \qquad \rho_g = -\frac{\sigma}{f_o \cdot r \cdot T}, \tag{11}$$

where r is the individual gas constant.

For the skeleton, the following parameters were accepted:

- Linear coefficient of thermal expansion: $\alpha_s = 5 \cdot 10^{-6} / K$,
- Specific heat: Cv = 500 J/kgK,
- Heat transfer coefficient: λ_s = 2 W/m·K.

Moreover, the following parameters were accepted for the medium:

- Porosity: n = 0.25,
- Filtration coefficient: $k = \dfrac{k_0}{n \rho_f g}$, $k_0 = 10^{-5}$ m/s,
- Mechanical properties of the medium:
  - Shear modulus: $N = 2.5 \cdot 10^8$ Pa,
  - Bulk modlus for medium filled with gas: $A_g = 5 \cdot 10^8$ Pa,
  - Coefficient determining the change in volume of gas under the impact of fluid stress: $R_g = 1 \cdot 10^5$ Pa,
  - Coefficient determining the impact of the volumetric strain in gas on the stress in the skeleton (and vice versa): $3 \cdot 10^5$ Pa,
  - Coeficcient determining the change of the volume of water under the impact of fluid stress: $R_w = 1 \cdot 10^7$ Pa,
  - Bulk modulus for medium filled with water: $A_w = 5 \cdot 10^8$ Pa,
  - Coefficient determining the impact of the volumetric strain in water on the stress in the skeleton (and vice versa): $Q_w = 3 \cdot 10^7$ Pa.

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3. RESULTS OF THE ANALYSIS

A view of the finite elements mesh after computing time $t = 300$ s (exaggerated) is presented in Fig. 2. Thermal strain is evident in the bottom part of the sample where the temperature is highest.

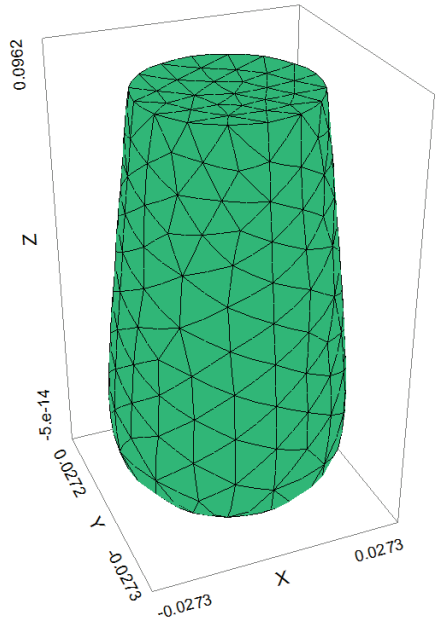


Fig. 2. Deformed mesh
(exaggerated scale, deformations × 1000)

The results for separate heat transfer in the fluid and the skeleton for 3 types of pore fluids are presented in Table 3. The locations of the points for which the plots were generated are shown in Fig. 3.

The plots showing the temperature difference between the phases of the medium are presented in Table 2. It can be read that for gas as a pore fluid, during the transitional period of temperature in the solid phase, readings are higher than in the fluid and they then equalize. The medium filled with fluid (in this case water) acts inversely. The temperature of water is higher than the temperature of the skeleton before they equalize. The process of temperature equalization in both phases is short. The longest period of equalization was observed in the case of carbon dioxide as the pore fluid (about 200 s).

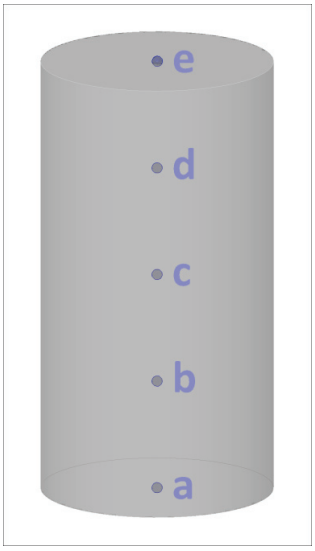
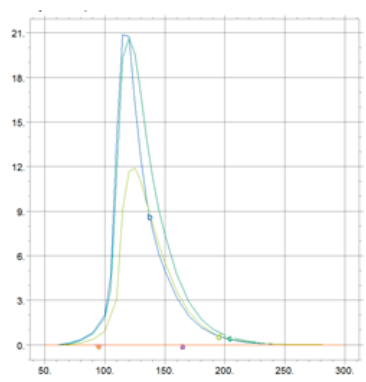
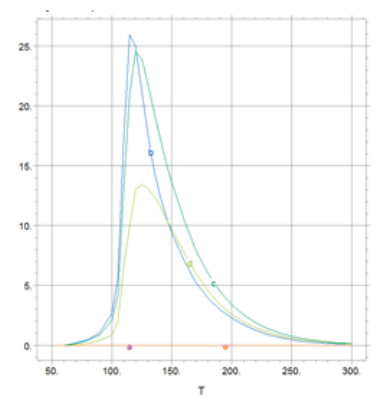
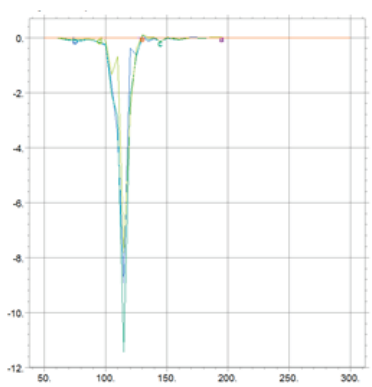


Fig. 3. Point at which the plots were generated

The heat transfer process in both phases of the medium is plotted in Table 3. As can be seen, the quantity of heat being transferred in the gas phase is negligible compared to the heat being transferred in the liquid phase. Overall, the medium consisting of the liquid and the skeleton transfers more heat than the skeleton-gas system.

Table 2. Results of computation of the time course of temperature in the skeleton and the pore fluid

Air	Carbon dioxide	Water
		

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The process of fluid flow through the two-phase medium can be best illustrated by the variation of the vertical component of the filtration velocity, presented in Table 4. Due to differences in viscosity of the fluids that are under consideration, significant differences in the filtration velocity are observed. Highest velocities occur in the case of air filtration, the lowest in the case of water filtration. The shape of the time course of the

Table 3. Time course of the quantity of heat flowing through the skeleton and the pore fluid

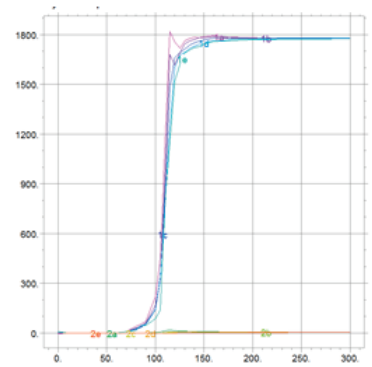
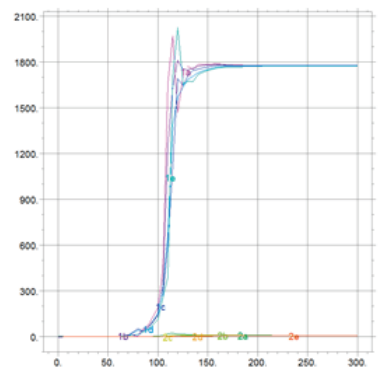
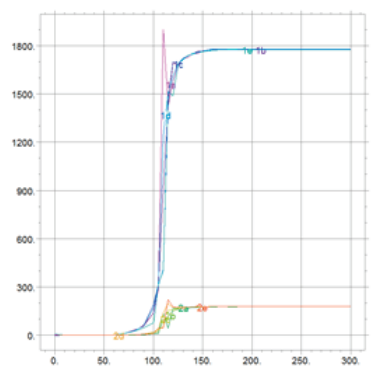
Air	Carbon dioxide	Water
		

Table 4. Time course of fluid velocity

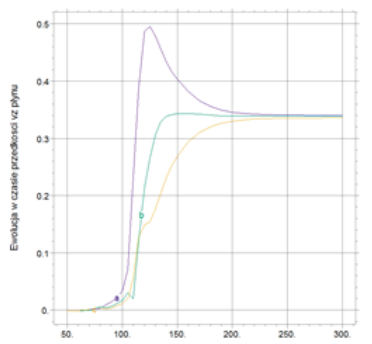
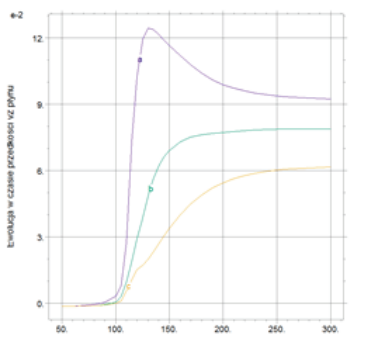
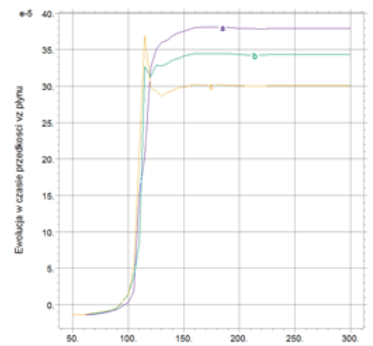
Air	Carbon dioxide	Water
		

Table 5. Time course of fluid viscosity

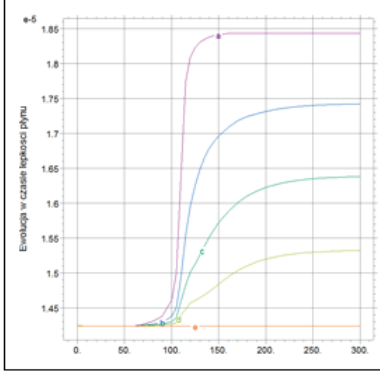
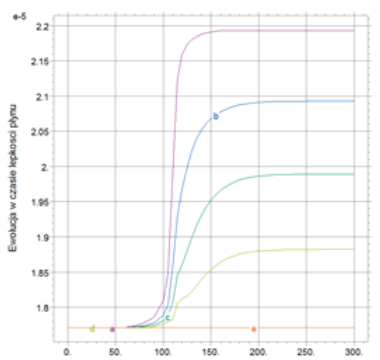
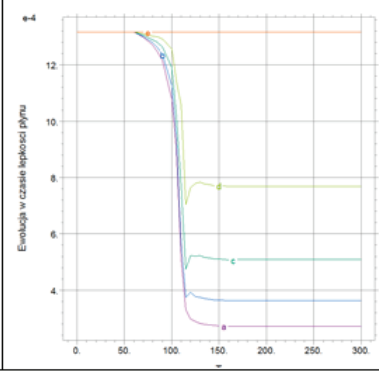
Air	Carbon dioxide	Water
		

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Table 6. Time course of the filtration coefficient

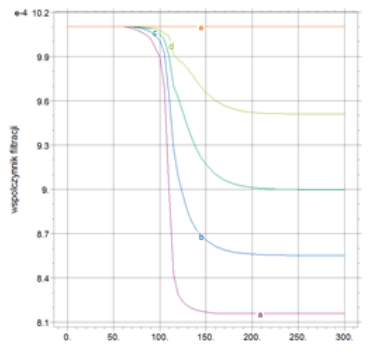
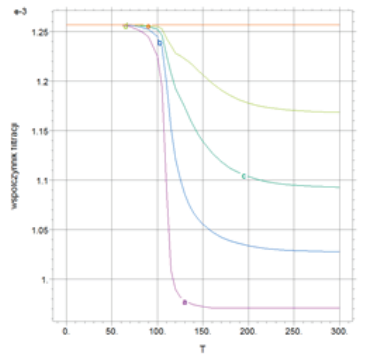
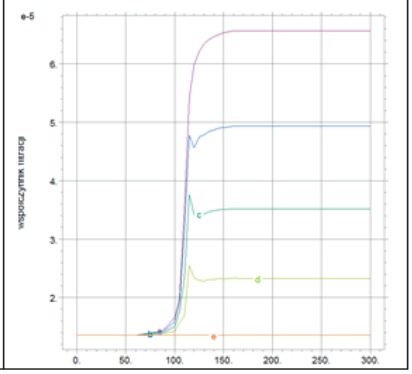
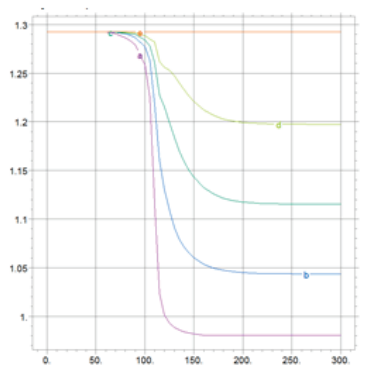
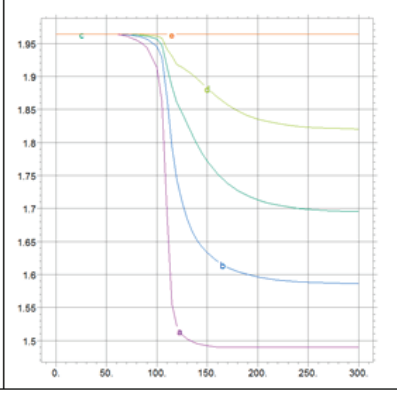
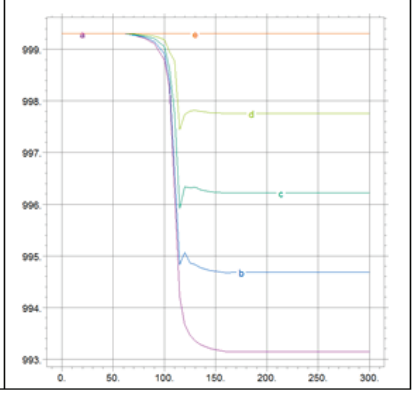
Air	Carbon dioxide	Water
		

Table 7. Time course of fluid density

Air	Carbon dioxide	Water
		

velocity function is certainly affected by the process of temperature equalization in the fluid and the skeleton. As can be seen from the plots, the shape of the function is different for gases and liquids. The difference in the fluid velocity resulting from the change of its density is due to the temperature change.

The discussed mathematical model of thermal consolidation is strongly non-linear, since effective parameters are temperature-dependent. The time variation of the effective parameters of fluids for the accepted scenario of thermal consolidation is presented in Tables 5 and 7. The variation of effective parameters for gases and water is described by formulas (8), (9) and (11). As can be seen from the time course of the viscosity in Table 5, the character of the plots is different for the gases and the water; this affects the filtration coefficients presented in Table 6. This table shows that the gas filtration coefficient decreases with the rise of temperature, and contrariwise in the case of water, it increases with the temperature rise. The time course of fluid density is presented in Table 7. As can be seen, for both gases and liquids, fluid density decreases due to a temperature rise. In the case of the issue studied, the fluid is getting denser with height due to a linear decrease in temperature. This explains the change in fluid velocity presented in Table 4.

4. SUMMARY

In the case of gases as pore fluids, the temperature plots are very similar; they differ mainly in terms of value. Heat is transferred mostly through the skeleton. In the case of water, the mechanism of heat transfer is different as the pore fluid plays a more significant role. Thus, for gas as the pore fluid, when the precise determination of an effective heat transfer coefficient is impossible and must be assessed – neglecting heat transfer in the pore fluid and accepting an average heat transfer coefficient calculated based only on the coefficient for the skeleton (depending on geometry,

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its upper limit is $\lambda^{\text{eff}} = \lambda^s (1 - n)$). This will not generate a huge error. However, it must be noted that for the precise averaging of the heat transfer coefficient, an accurate knowledge of the medium microscale geometry and usage of homogenization techniques is essential.

In the scale of the sample, strain and stress fields were very similar for both gases. For water the results were noticeably different. This was caused, unlike in the case of physical parameters, by using different constitutive equations for both cases.

In technical applications, assuming that the heat transfer is taking place in both phases simultaneously, this seems to be more practical. The heat transfer equation then takes the form

$$\begin{aligned} \lambda^{\text{eff}} \nabla^2 \vartheta &= T(3K\alpha^s + Q\alpha^l)\dot{\varepsilon} \\ &+ T(3Q\alpha^s + R\alpha^l)\dot{\theta} + \frac{T}{T_0}\rho c_v \dot{\vartheta} \end{aligned} \tag{12}$$

where λ^{eff} is the effective heat transfer coefficient of the medium, under the assumption that heat flows through the entire volumes of both the fluid and the skeleton, $\lambda^{\text{eff}} = \lambda^s (1 - n) + \lambda^l n$. Other formulas for averaging the heat transfer coefficient were introduced in examples by Coussy [3].

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