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SOME REMARKS ON TIME IN THE ANALYSIS OF ECONOMIC EVENTS

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Abstract: The paper presents the approaches to time in the theory of economics, and some problems resulting from the manner in which a time variable is used when describing and predicting economic events.

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1. Introduction

The author attended a seminar at the Association of Interdisciplinary Studies located near the Faculty of Theology in Wroclaw, which was devoted to the nature of time. The seminars were conducted by scientists from varied disciplines: history, physics, chemistry, genetics, theology and others; there also had to be a representative of economics. This article gives an adumbration of the seminar mentioned above, which aimed at acquainting auditors with the ways of perceiving the time economists. This author's appearance was linked to the book "Time in the Economy" by Czaja [2011]. Defining time and describing its properties has been a longtime problem tackled by great minds from Aristotle to Einstein that still arouses the emotions. One of the fundamental problems connected with time is the question regarding the emergence of the world – in time or with time? If we assume the definition that time results from the existence of things, the answer seems theoretically obvious, however it still evokes many emotions. Without doubt, time is a measure of the changeability of people, nature and things. This changeability in time is a fact experienced by all of us and although we do not have doubts that time runs in one direction: from the past that is known, to the future that is unknown, science has introduced the symmetry of time. Physics developed the concept of time's arrow, which is

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not quite unambiguous either. Almost all laws of physics are insensitive to the change in the direction of time, except for the second law of thermodynamics. If we represent a physical law (law of nature) by a mathematical equation with time typically denoted by *t*, then changing this time variable to $-t$, i.e. changing the direction of time, will still describe the same law. If we make use of the laws of cosmology, it turns out that a clock indicating the direction of time is the entire universe. If the universe expands according to Hubble's law, then the density of matter incessantly decreases.

Let us return to the above mentioned second law of thermodynamics worded in the language of entropy. Entropy is a measure of the dispersion of thermal energy, i.e. if energy in a given system is more dispersed, then its entropy is greater. The second law of thermodynamics states that no process occurring in an isolated system can lead to a decrease of entropy. Hence examining the levels of entropy of a system, we determine earlier states (with smaller entropy) and later states (with bigger entropy); this law allows to determine time's arrow, i.e. to distinguish earlier states from those later. It is worth noticing that the physics of elementary particles does not employ the direction of time, "classical" laws of physics, or specific "objects" as a consequence of Heisenberg's uncertainty principle resulting from the essence of quantum mechanics.

One can argue that economics is "similar" to the physics of elementary particles, especially if we confront provocative statements of two prominent economists. In his book *Economic Dilemmas*, Z. Czerwiński wrote: "The laws of nature are universal in time and in space, whereas the laws of economics are historical and local, because economic entities are much stronger integrated systems and less stable than natural (physical) systems". The other statement by J. Barrow is even more controversial: "Chaos rules economic systems, because almost identical initial states often lead to completely different later states". According to Poincaré's definition, Barrow's economic system is a random system. In the language of mathematics, the randomness of economic systems signifies a discontinuity of economic processes.

2. How to interpret time in the theory of economics?

We can specify the three approaches to the time variable in economics:

1. Local time is considered as a linear order and identified with the set of real numbers R; quantitative sciences deal with time series analysis. Economic models apply discrete or continuous construct of time.

2. Logical (operational) time is considered as a logical consequence of events occurring in an economic process. It is applied in theoretical modelling.

3. Circular time implies long-term processes, i.e. time passes by business cycles, and as a result, in space-time it follows spirals. Circular time is a consequence of vortices; e.g. the Earth's vortex defines days while the Moon's vortex defines months. On the other hand, depending on its direction, a vortex implies that a system either tends to the equilibrium or expands to infinity, i.e. to destabilization.

All real processes and economic events are embedded in real time, therefore time in these processes is considered in the calendar form such as charts of prices of commodities or share prices in stock exchanges. Whether a specific economic category is represented as a discrete or a continuous variable, typically depends on the frequency of the statistical measurement of these categories. In the real sphere of the economy we usually deal with measurement in time intervals, hence real economic processes are considered as functions of time, which is a discrete variable. In the financial sphere, the frequency of measurement is extremely high (data flowing from stock exchanges and from financial markets), therefore processes in this sphere are treated as functions of continuous time.

An important part of the time analysis in the theory of economics is the dynamics of economic events and the perception of economic processes in time. The most classic problems in this area concern modeling economic growth and modeling business cyclicity. "Dynamics" is a term denoting an analysis aimed either at the inspection and study of time paths for variables or at the examination of whether these variables tend to some values of an equilibrium. In these models, time is a fundamental independent variable. As mentioned above, time can be treated as a continuous variable or as a discrete variable. In the case of continuous time, integral calculus and differential equations are applied, whereas in the case of discrete time the methods of difference equations are used.

From the methodological viewpoint of economic modeling, models are dynamic if they take into account:

- General patterns of changes in levels of an economic phenomenon in the period under study;
- Change patterns of influence of factors in time;
- Lagged factors influence and lagged levels of event or process influence on its future shape.

Economic theories developed concepts of reversible and irreversible time; they are presented in Table 1. They reflect the two approaches to the relationship between mathematics and time. First, the Pythagorean approach "temporalized" mathematics, i.e. it became subordinated to time, in a sense. A number is a specific vehicle for quantity, measurement is a process occurring in time. Second, the Platonic approach treated mathematics as an archetype of timelessness. The archetypal entity for Plato was not a number, but geometry with its perfect immutability. A mathematical entity is always the same, the most perfect prototype of the Platonic realm of ideas.

The assumption of reversibility (symmetry) of time in economic models allows to develop attractive analytical themes, however it also can lead, inter alia, to: (1) separating economic theory from the history of economics, (2) necessary introduction of revolutionary investments and sources of innovations in the form of additional assumptions, (3) limitations with respect to the problems of resource depletion and environmental degradation; cf. [Czaja 2011].

Table 1. Interpretation of time in economics

Reversible time	Irreversible time
1. Statics – economic events occur in one interval of time or time is not a variable quantity	1. Irreversible time with risk – knowing past events determines probability distributions of future events
2. Relative statics – economic events occur in two separate, comparable periods	2. Irreversible time with uncertainty – future events are not related to past events
3. Reversible time – time is treated in the same way as spatial variables; direction of time is not defined, thus implying symmetry of time	3. Teleological order – denotes such ordering of activities and decisions that are a result of the objective (logical or operational time)

Source: [Czaja 2011].

Every human realizes that for us time is irreversible, and since economics is developed by people, therefore the assumption in economics that time is symmetrical contradicts the experience.

Business practice expects from science the delivery of tools to correctly predict economic events. Prediction can be seen as inferring unknown (future) events from known (past) events. It is worth noticing that forecasting is based on the assumed inertia (inactiveness) of a system. In economics, inertia results from the continuity of time and a large number of population and volume of capital. Typically, mathematical and statistical methods are distinguished where time series are employed – based on deterministic models, and methods based on econometric models. When analyzing a time series, one has to find three elements, which can influence levels of the event in time: a tendency of development called trend, seasonality and random fluctuations. In models based on time series, the only explanatory variable is often time.

Business cycles typically describe activities of national economies as fluctuations of aggregates representing the entire economy [Burns, Mitchell 1946]. The cyclical changes in time series are composed of waves with certain frequencies. Each business cycle consists of four phases:

1. Crisis (contraction): unemployment increases; output, employment, investment, demand, and prices decrease.
2. Recession (depression): decreases stop; all quantities remain at low levels.
3. Recovery: output, employment, investment, demand, and prices increase; unemployment is reduced.
4. Prosperity: increases stop; all quantities remain at high levels.

3. Typology of time in the theory of economics

Czaja [2011] presented an interesting, even if problematical typology of time in economic phenomena, considering time as a stimulant, nominal, and counter-stimulant. If processes develop in time in the desired manner, then time is said to stimulate the process and called a stimulant. The examples are the effects of compounding or discounting bank deposits. If the process under study behaves against the applied criteria, then it means that time behaves as a counter-stimulant. The examples are the depreciation and depletion of material resources or the exploitation of environmental resources. In cases when a given process takes a certain nominal value with deviations in any direction signifying the detrimental development of the phenomenon, then time is a nominal.

In my opinion such an approach, including terminology, is problematical because time cannot be treated as a stimulant. If that were the case, then time would have been considered as a cause of growth, therefore our knowledge of the economic process is null. As the saying goes, time is money, but this just means that well employed capital increases with time.

Despite my criticism, a typology proposed by Czaja [2011] will be now presented.

3.1. Time as a stimulant (compounding and discounting of value in time)

As regards the influence of time on the monetary value, two problems are of interest:

1. Why does time increase the volume of capital, i.e. what are the reasons for a positive interest rate?
2. What are the formal and quantitative relationships between time and the interest rate?

There are many theories of positive interest rate r (beginning with Marx). One of them states that if a stream of incomes increases with time, then the law of diminishing marginal utility results in a positive interest rate. There are three basic methods to calculate interest earned by the principal amount K(0):

a) simple interest: $K(t) = K(0) + K(0) \times r \times t$,
b) compound interest: $K(t) = K(0) \times (1 + r)^t$,
c) continuous compounding: $K(t) = K(0) \times e^{rt}$.

It is worth adding that a reversed operation is also applied, i.e. discounting. This is a process of determining the present value K(0) when the future value $K(t)$, the interest rate r, and the length of the discounting period t are given. Compound discounting follows the formula $K(0) = K(t) / (1 + r)^t$; and continuous discounting: $K(0) = K(t) \times e^{-rt}$. Discounting is applied, among others, to evaluate the effectiveness of business decisions (Cost Benefit Analysis), and to calculate the value of life in a given country.

3.2. Time as a counter-stimulant

The management of natural resources is a good exemplification of negative correlation between a phenomenon and time. Environmental economics and ecological economics distinguish renewable and non-renewable resources. The renewable resources, after being processed into products, can become again useful resources, after time accepted by efficient management. The renewable resource is best exemplified by forests whose utility is determined both by resources providing timber and by non-economic values (recreation). According to the theory by Gordon, the equilibrium between renewable resources and the exploiting industry will be reached when the stream of incomes matches the cost level. In this case, a high market interest rate “shortens” the forest life, while a low interest rate makes it “longer”.

Non-renewable resources generate problems connected with the "passage" of time, especially as regards intergenerational equity, hence some questions arise:

- How can the time horizon of exploitation be estimated?
- How can non-renewable resources be allocated in time if they are negatively correlated with time?

3.3. Time as a nominal

Time occurs as a nominal in the process of creating and the destruction of human capital resources, more precisely, the age of employees is a nominal. "Human life is said to be capital, or – somehow reversed – that capital is embodied in a human, or that a capital resource is represented by people with their knowledge, abilities and health, or that knowledge, abilities and health are separate categories of capital incorporated in a given population" [Domański 1993]. At the microeconomic level, a human faces a time-related problem: to gain a higher income now given the current capabilities, or to invest more in enlarging their own capabilities so as to gain a higher income at a later time. There are three approaches to solve this problem:

1. Amount of investment in human capital and its time path determine the internal rate of return. Discounted cash outflows must match discounted cash inflows.
2. Maximizing discounted expected cash inflows results in optimal size and time distribution of accumulated human capital.
3. The optimization of investment in human capital is not based on effects in earnings, but on maximizing the utility function in a given time horizon (G. Becker).

Below some theoretical foundations of the Cost Benefit Analysis are presented as an example of time-related economic analysis.

Let Y_{ij} be the sum of earnings received during a professional career by a person *i* with education level *j*. Let X_i be a vector of observed capabilities, social and demographic attributes, and ε_i – a vector of unobserved factors influencing the earnings of the i^{th} employee. Then the total benefit from work can be defined as the function depending on these factors: $Y_{ij} = f\left(X_i, \varepsilon_i\right)$. If the cost of education to reach the level *j* for a person *j* equals C_{ij}, and if V_{ij} denotes the value of utility function of this person with respect to financial capital at hand, then the equality $V_{ij} = \max_j \left(Y_{ij} - C_{ij}\right)$

implies that the person will decide to invest in more education when the difference between the future earnings connected with this level of education and the total outlays needed to obtain it will be greater than future earnings connected with the elementary level of education.

The analytical formula is an extension of the model by Willis and Rosen [1979]. This model considers two states: an employee with medium education (*L*) and an employee with higher education (*H*). Let us assume that wages follow an increasing function of employment time, and that the rate of wage increase depends on the level of capabilities and knowledge acquired in the education process. This equals g_h for persons with higher education, and g_l for those with medium education. Employees willing to improve their levels of education must give up some time of professional activity. Let *S* denote the time needed to obtain a higher level of education. For persons selecting higher education, future earnings will be:

$$y_{hi}(t) = \begin{cases} 0 & \text{for } t \le S \\ \bar{y}_{h0} \cdot \exp\left(g_h (t - S)\right) & \text{for } t > S \end{cases},$$

where: $t - S$ is a measure of gaining professional experience (in terms of time), g_h denotes the rate of wage growth, and $\bar{y}_{h0}$ denotes the initial wage (at first employment).

Similarly we can represent future earnings of persons who did not decide to gain a higher level of education (lower skilled employees):

$$y_{li} = \bar{y}_{l0} \cdot \exp\left(g_l \cdot t\right) \quad \text{for} \quad t \ge 0.$$

Earning cash flows depend on two parameters: the initial level of wages connected with each level of education, and *g* – the rate of wage growth. In order to compare both cases, future potential earnings are discounted. Assuming an infinite time horizon, constant rates of growth and constant discount rates r_i for person *j*, as well as $r_i > g_h, r_i > g_l$, the following yields:

$$PV_{hi} = \int_S^{\infty} y_{hi}(t) \cdot \exp(-r_i t) dt = \int_S^{\infty} \bar{y}_{h0} \cdot e^{g_h (t-S)} \cdot e^{-r_i t} dt = y_{h0} \cdot e^{-g_h S} \int_S^{\infty} e^{(g_{h0} - r_i)t} dt =$$

$$= e^{-g_h S} \cdot \bar{y}_{h0} \left[\frac{1}{r_i - g_{h0}} \cdot e^{(g_{h0} - r_i)S} \right] = \frac{\bar{y}_{h0}}{r_i - g_{h0}} \cdot e^{-r_i S} = \frac{\bar{y}_{h0}}{r_i - g_{h0}} \cdot \exp\left(-r_i S\right),$$

$$PV_{li} = \int_0^\infty y_{li}(t) \cdot \exp(-r_i t)dt = \int_0^\infty \bar{y}_{l0} \cdot \exp(g_l t) \cdot \exp(-r_i t)dt =$$

$$= \bar{y}_{l0} \int_0^\infty \exp(g_l - r_i)tdt = \frac{\bar{y}_{l0}}{r_i - g_l}.$$

If discounted costs of education equal $PV(C)$, then an employee will decide to increase the level of education when $PV_{hi} - PV(\mathrm{C}) > PV_{li}$, assuming the maximization of benefits.

4. Conclusion – time-related description of quality of life

Real time management, or more precisely, managing the process of life in time, since time can be neither accelerated nor decelerated, is the most important challenge of the theory of economics from the practical point of view. On one hand, it embraces researching relationships between work time, household chores and leisure, while on the other hand – the examination of free time itself, since it is treated today as a significant element of quality of life. The negative assessment of leisure time was a result of the traditional opinion that most significant for a human and for civilization is time spent for production (work). Free time was seen as lost. This condition changed as one began to treat free time in the wider context of well-being and quality of life. From the economic point of view, free time should be devoted to improving the humanity of every member of community, i.e. to enhance social capital. Results of research by the Central Statistical Office in Poland indicate that many social groups experience major changes in the amount of free time available, however, a disturbing result is that in free time roughly 90 per cent of the population watch TV (CBOS, 2010).

Average life expectancy at birth (QoL) or quality-adjusted life-year (QALY) are measures used to assess the quality of life of a community, to be precise its value of health (well-being). The example below illustrates the computation of QALY quantifying the state of health of a person (or a population). QALY is defined as the product of the years lived in a given state of health and the index value connected with that state, where this index takes values from the interval [0, 1], with 1 denoting one year lived in perfect health, and 0 in the case of death. If a person became sightless at the age of 30 years, then with the average life expectancy at birth equal to 70 years, the value of QALY at the moment of becoming blind equals $(70 - 30) \times 0.5 = 20$, assuming that the index value of being blind equals 0.5.

One of the methods to adjust the length of life using the quality of life is the Time-Trade-Off approach (TTO). The value of the state of health of a patient is determined based on the patient's answer to the following question: How many years of life would you give up to live for a shorter period in full health? Since QALY(sick) = QALY(healthy), let us assume that a sightless person is short of 30 years to the expected length of life and that the answer is 15 years.

Then

$$\text{years}(N) \times \text{index QoL}(N) = \text{years}(Z) \times \text{index QoL}(Z),$$

therefore

$$30 \times \text{QoL}(N) = 30 \times 1,$$

and finally,

$$\text{QoL}(N) = 15/30 = 0.5.$$

5. Ending

Professor Smoluk has depicted an interesting, philosophical time in the economics' interpretation, where he connected the periodicity of the processes (not only those economical) in the time with the Big Bang, claiming that the movement of the world in time is like an inner tube coiled in the one point, which is both the beginning and the end of "our" world, "because everything was born of God and seeks God".

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