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Spectral degrees of cross-polarization of stochastic anisotropic electromagnetic beams in modified non-Kolmogorov atmospheric turbulence

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Based on the extended Huygens–Fresnel principle, variations in generalized Stokes parameters of stochastic anisotropic electromagnetic beams propagating through modified non-Kolmogorov atmospheric turbulence have been analyzed. The changes in generalized Stokes parameters with different turbulence parameters and source parameters were analyzed first. After that, the distributions of the spectral degrees of cross-polarization (SDCP) of isotropic beams and anisotropic beams were simulated. The results show that the profiles of distribution of SDCP of these two kinds of beams are very different in the near field, and will fluctuate through the propagation in atmospheric turbulence, but at last, when the propagation distance is long enough, the difference in the source makes a slight difference in the final profiles of SDCP in the output plane. They mainly depended on the turbulence perturbation, and in the weak turbulence, the profiles of final distribution show more flatter features.

Keywords: quasi-homogenous source, modified non-Kolmogorov turbulence, generalized Stokes parameters.

1. Introduction

The concept of cross-polarization of stochastic electromagnetic beam-like fields was first introduced by ELLIS and DOGARIU where the parameters are called the mutual degree of cross-polarization (MDCP) [1] and the quantity introduced in Volkov's paper is called the spectral degree of cross-polarization (SDCP) [2] which represent the correlation between the intensity fluctuations in terms of the degree of coherence and of the other two-point correlation properties of the beam. Recently, LI and PU have studied the degrees of cross-polarization of electromagentic beams propagation in turbulence media and uniaxial crystals [3, 4]. PU has found that the behavior of this quantity, not being similar to the behavior of the spectral degree of coherence or the ordinary degree of polarization, has been affected by all the parameters of the source. After the concept of a partially coherent Gaussian pulse was introduced [5], the spectrally partially coherent properties of such pulsed beams and the propagation of SDCP of stochastic electromagnetic pulsed beams in free space have been studied by DING and LAJUNEN [6–8]. Very recently, the statistic properties of stochastic anisotropic electromagnetic beams propagation in the atmosphere and oceanic water have been studied by DU and ZHOU [9, 10]. They have found that the changes in the degree of polarizaiton of anisotropic beams are different from those of isotropic beams, because they have no self-reconstructed properties. On the other hand, atmospheric turbulence is one of the most important turbulences which has a significantly degrading impact on the quality of imaging and laser communication systems. The classical Kolmogorov turbulence spectral model failed to predict the result of recent experiments in certain portions of the atmosphere. The propagation of light beams through the turbulence in portions of the troposphere and stratosphere may obey non-Kolmogorov statistics [11]. XUE *et al.* generalized the modified turbulence spectral model in non-Kolmogorov atmosphere turbulence which considers the finite turbulence in inner and outer scales and has a general spectral power law value in the range of 3 to 5, instead of the standard power low value of 11/3 [12]. CUI *et al.* obtained the long exposure turbulence modulation transfer function (MTF) based on this model [13].

To our best knowledge, the influence of turbulence perturbation on the evolution of SDCP of stochastic beams is not clearly pointed out in the articles we mentioned above, there are a few reports about the propagation properties of the electromagnetic beams traveling through modified non-Kolmogorov turbulence. In this paper, we use generalized Stokes parameters of anisotropic stochastic electromagnetic beams to calculate the SDCP of anisotropic beams propagation through modified non-Kolmogorov turbulence. The results are illustrated by numerical examples. Basing on these results, we have discussed the changes in SDCP distributions under different turbulence conditions and source parameters. The behaviors of SDCP of such beams propagating in turbulence media were summarized at last.

2. Anisotropic electromagnetic beams and modified non-Kolmogorov turbulence model

The elements of the cross-spectral density matrix (CSDM) for a random, statistically stationary electromagnetic beam generated by the Gaussian source are given by:

$$W_{ij}^{(0)}(\boldsymbol{\rho}'_1, \boldsymbol{\rho}'_2, \omega) = A_i A_j B_{ij} \exp\left\{-\frac{\boldsymbol{\rho}'^2_1}{4\sigma_i^2} - \frac{\boldsymbol{\rho}'^2_2}{4\sigma_j^2}\right\} \exp\left\{-\frac{|\boldsymbol{\rho}'_2 - \boldsymbol{\rho}'_1|^2}{2\delta_{ij}^2}\right\} \tag{1}$$

where the coefficients A_i, A_j, B_{ij} and variances σ_i, σ_j and δ_{ij} are both independent of the position but dependent on the frequency; $\boldsymbol{\rho}'_1$ and $\boldsymbol{\rho}'_2$ are two points on the source plane. The degree of polarization is the same at every point across the source plane if

$\sigma_i = \sigma_j$, but if $\sigma_i \neq \sigma_j$, the source is not uniformly polarized. For convenience, Eq. (1) can be rewritten as:

$$W_{ij}^{(0)}(\boldsymbol{\rho}'_{12}, \omega) = A_i A_j B_{ij} \exp\left\{-\frac{ik}{2}\boldsymbol{\rho}'^{T}_{12}\, \mathbf{M}'^{-1}_{ij}\, \boldsymbol{\rho}'^{T}_{12}\right\} \tag{2}$$

where $\boldsymbol{\rho}'^{T}_{12} = (\boldsymbol{\rho}'^{T}_{1}, \boldsymbol{\rho}'^{T}_{2}) = (x'_1, y'_1, x'_2, y'_2)$, and

$$\mathbf{M}'^{-1}_{ij} \begin{bmatrix} A & 0 & C & 0 \\ 0 & A & 0 & C \\ C & 0 & B & 0 \\ 0 & C & 0 & B \end{bmatrix}$$

while $A = -\dfrac{i}{2k}\sigma_i^{-2} - \dfrac{i}{k}\delta_{ij}^{-2}$, $B = -\dfrac{i}{2k}\sigma_j^{-2} - \dfrac{i}{k}\delta_{ij}^{-2}$, $C = \dfrac{i}{k}\delta_{ij}^{-2}$.

The CSDM of stochastic beams propagation through turbulence can be derived by the extended Huygens–Fresnel principle:

$$W_{ij}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, z; \omega) = \frac{k^2}{4\pi^2 z^2}\iint \mathrm{d}^2\boldsymbol{\rho}'_2 \iint \mathrm{d}^2\boldsymbol{\rho}'_1\, W_{ij}^{(0)}(\boldsymbol{\rho}'_1, \boldsymbol{\rho}'_2, \omega) \times$$
$$\times \exp\left\{-ik\frac{(\boldsymbol{\rho}_1-\boldsymbol{\rho}'_1)^2 - (\boldsymbol{\rho}_2-\boldsymbol{\rho}'_2)^2}{2z}\right\}\left\langle \exp\{\psi^*(\boldsymbol{\rho}_1, \boldsymbol{\rho}'_1) + \psi(\boldsymbol{\rho}_2, \boldsymbol{\rho}'_2)\}\right\rangle_m \tag{3}$$

where $\boldsymbol{\rho}_1$, $\boldsymbol{\rho}_2$ are two observation points on the output plane, z is the propagation distance and $k = 2\pi/\lambda$ is the wave number of the beam. The last term describes the correlation function of the complex phase perturbed by random medium, the subscript m denotes the average over medium realization, and it takes the form:

$$\left\langle \exp\{\psi^*(\boldsymbol{\rho}_1, \boldsymbol{\rho}'_1) + \psi(\boldsymbol{\rho}_2, \boldsymbol{\rho}'_2)\}\right\rangle =$$
$$= \exp\left\{-4\pi^2k^2z\int_0^1\int_0^\infty \kappa\Phi_n(\kappa)\Big[1 - J_0\big|(1-\xi)(\boldsymbol{\rho}'_1-\boldsymbol{\rho}'_2) + \xi(\boldsymbol{\rho}_1-\boldsymbol{\rho}_2)\kappa\big|\mathrm{d}\kappa\mathrm{d}\xi\Big]\right\} \tag{4}$$

where J_0 stands for the 0th order Bessel function. Under the approximation which was used in [14], Eq. (4) is simplified to:

$$\left\langle \exp\{\psi^*(\boldsymbol{\rho}_1, \boldsymbol{\rho}'_1) + \psi(\boldsymbol{\rho}_2, \boldsymbol{\rho}'_2)\}\right\rangle =$$
$$= \exp\left\{-\frac{\pi^2k^2zT}{3}\Big[(\boldsymbol{\rho}'_1-\boldsymbol{\rho}'_2)^2 + (\boldsymbol{\rho}'_1-\boldsymbol{\rho}'_2)(\boldsymbol{\rho}_1-\boldsymbol{\rho}_2) + (\boldsymbol{\rho}_1-\boldsymbol{\rho}_2)^2\Big]\mathrm{d}\kappa\right\} \tag{5}$$

where $T = \int_0^\infty \kappa^3 \Phi_n(\kappa)\mathrm{d}\kappa$ denotes the intensity of the turbulence atmosphere which can only be determined by the spatial power spectrum of the refractive index fluctuations.

In the last decade, several researchers studied the propagation properties of beams traveling through non-Kolmogorov turbulence. Toselli has researched the long-term beam spread, scintillation index, probability of fait and mean bit error date as a variation of the spectrum exponent of non-Kolmogorov turbulence [15]. GERÇEKCIOLU and BAYKAL scrutinized the behavior of intensity fluctuations of a flatted-topped beam in non-Kolmogorov weak turbulence [16]. CUI *et al.* investigated the influence of power law on the irradiance scintillation of a Gaussian-beam wave [17]. KOROTOKOVA and SHCHEPAKINA revealed the dependence of spectral shifts and switches in optical stochastic beams propagating through non-Kolmogorov media on the slope of the power spectrum of fluctuations in the refractive index [18]. The temporal power spectrum of the irradiance and the AOA fluctuations of plane and spherical waves in non-Kolmogorov turbulence whose power law varies from 3 to 4 were derived by DU *et al.* [19].

The general modified atmospheric spectrum is a turbulence spectral model which considers the influence of finite, inner and outer scales. For non-Kolmogorov turbulence, it has the form:

$$\Phi_n(\kappa, \alpha) = A(\alpha)\tilde{C}_n^2 \kappa^{-\alpha} f(\kappa, l_0, L_0, \alpha) \tag{6a}$$

$$f(\kappa, l_0, L_0, \alpha) = \left[1 - \exp\left(-\frac{\kappa^2}{\kappa_0^2}\right)\right]\left[1 + a_1\left(\frac{\kappa}{\kappa_l}\right) - b_1\left(\frac{\kappa}{\kappa_l}\right)^{7/6}\right]\exp\left(-\frac{\kappa^2}{\kappa_l^2}\right) \tag{6b}$$

where $\kappa_0 = C_0/L_0$, $\kappa_l = c(\alpha)/l_0$, $c(\alpha)$ is the scaling constant, L_0, l_0 are the out scale and the inner scale of turbulence. The intensity of modified non-Kolmogorov turbulence can be obtained by the numerical integral method of Matlab 2011a. Figure 1 shows a typical distribution of the intensity of the turbulence with power law. At first, T increases when the power law increases, and gets the maximum value at $\alpha = 3.11$, then it will decrease when the power law increases. For the homogeneous and isotropic turbulence, the power law α varies from 3 to 4 when Markov's approximation is used.

From Equations (1)–(6), the cross-spectral density matrix of the output plane of the beam propagating through turbulence can be obtained by a tedious integral, what has been done in the previous work [5, 6]

$$W_{ij}(\boldsymbol{\rho}_{12}, z, \omega) = A_i A_j B_{ij}\left[\mathrm{Det}(\bar{\mathbf{I}} + \bar{\mathbf{B}}\bar{\mathbf{P}} + \bar{\mathbf{B}}\mathbf{M}_{ij}'^{-1})\right]^{-1/2} \times$$
$$\times \exp\left\{\frac{ik}{2}\boldsymbol{\rho}_{12}^T\left[\left(\bar{\mathbf{B}}^{-1} + \bar{\mathbf{P}}\right) - \left(\bar{\mathbf{B}}^{-1} - \frac{1}{2}\bar{\mathbf{P}}\right)^T\left(\bar{\mathbf{B}}^{-1} + \bar{\mathbf{P}} + \mathbf{M}_{ij}'^{-1}\right)^{-1}\left(\bar{\mathbf{B}}^{-1} - \frac{1}{2}\bar{\mathbf{P}}\right)\right]\boldsymbol{\rho}_{12}\right\} \tag{7}$$

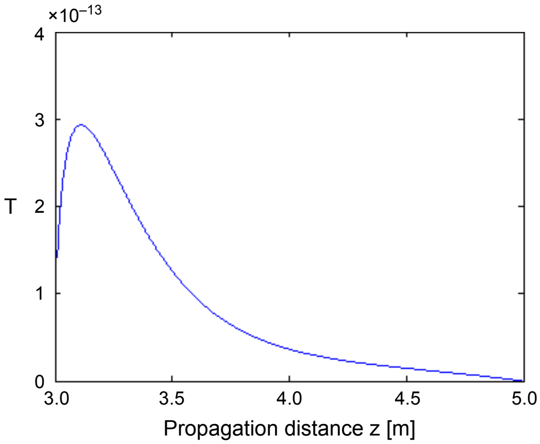

Fig. 1. The variation of T with power law α, other parameters are $L_0 = 10$ m, $l_0 = 1$ mm, $C_n^2 = 10^{-13}$ m$^{\alpha-3}$.

where

$$\overline{\mathbf{B}} = \begin{bmatrix} z\mathbf{I} & 0 \\ 0 & -z\mathbf{I} \end{bmatrix}$$

$$\overline{\mathbf{P}} = \frac{2\pi^2 kzT}{3i} \begin{bmatrix} \mathbf{I} & -\mathbf{I} \\ -\mathbf{I} & \mathbf{I} \end{bmatrix}$$

and $\boldsymbol{\rho}_{12}^T = (\boldsymbol{\rho}_1^T, \boldsymbol{\rho}_2^T) = (x_1, y_1, x_2, y_2)$, $\overline{\mathbf{I}}$ is 4×4 unitary matrix, and $\mathbf{I}$ is 2×2 unitary matrix; Det is the determinant of a matrix.

It can be deduced form Eq. (7) that the anisotropic electromagnetic beam in the output plane in turbulence media denpends on the parameters of the source, the perturbation of turbulence, and the propagation distance.

3. Generalized Stokes parameters and polarization properties

The polarization properties of an electromagnetic beam at a point in space can be determined by using Stokes parameters which have been generalized from one-point quantities to two-point counterparts recently [20]. They contained not only polarization properties but also coherence properties of beams [21] and can be measured by Young's interference experiment [22, 23]. The changes in generalized Stokes parameters in the optical system or random media are also an interesting problem which has attracted a lot of attention [24, 25]. The four parameters can be expressed as follows:

$$S_i(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) = \mathrm{Tr}\left[\mathbf{W}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, \boldsymbol{\omega}) \bullet \boldsymbol{\sigma}_i\right], \qquad i = 0, 1, 2, 3 \tag{8}$$

where **W** is the Pauli spin matrices, $\boldsymbol{\sigma}_0$ is the unit matrix:

$$\boldsymbol{\sigma}_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \sigma_1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \sigma_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \sigma_3 = \begin{pmatrix} 0 & -i \\ -i & 0 \end{pmatrix}$$

Substituting Eq. (7) into Eq. (8),we can get the changes in four generalized Stokes parameters of anisotropic beams propagation through turbulence media. Figure 2 illustrates the changes in normalized Stokes parameters of two pairs of points on the output plane of anisotropic beams traveling through turbulence with different power law α, other turbulence parameters are the same as those which we used in Fig. 1. The parameters of the source which we used in this paper are: $A_x = 3$, $A_y = 1$, $B_{xx} = B_{yy} = 1$, $B_{xy} = 0.2\exp(i\pi/6)$, $B_{yx} = 0.2\exp(-i\pi/6)$, $\delta_{xx} = \delta_{yy} = 1.5$ mm, $\delta_{xy} = \delta_{yx} = 2$ mm, and $\lambda = 632.8$ nm.

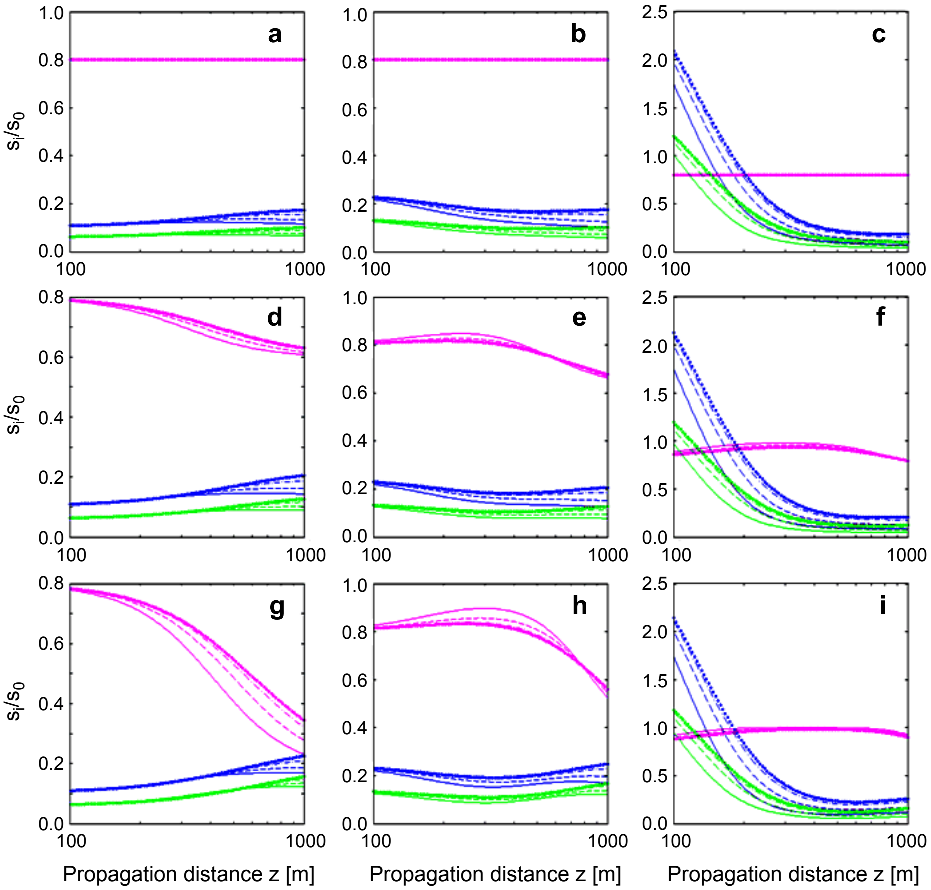

Fig. 2. Changes in normalized generalized Stokes parameters with the propagation distance of an anisotropic electromagnetic beam in turbulence. Two spatial points in the output plane are: $\boldsymbol{\rho}_1 = (0, 0)$, $\boldsymbol{\rho}_2 = (0, 0)$ (**a**, **d**, **g**); $\boldsymbol{\rho}_1 = (0.002, 0)$, $\boldsymbol{\rho}_2 = (0, 0)$ (**b**, **e**, **h**); $\boldsymbol{\rho}_1 = (0.003, 0)$, $\boldsymbol{\rho}_2 = (0, 0)$ (**c**, **f**, **i**).

In Fig. 2, the changes in normalized Stokes parameters in modified non-Kolmogorov turbulence have been illustrated. The pink line expressed the changes in s_1. The blue line expressed the changes in s_2, the green line expressed the changes in s_3. Different line types show the change in the parameters under different power laws of turbulence: the solid line shows the changes in generalized Stokes parameters under the power law of $\alpha = 3.11$, the dashed line is under the power law of $\alpha = 3.4$, the dash-dotted line is under the power law of $\alpha = 3.7$, the dotted line shows the changes in Stokes parameters in free space ($T = 0$). In Figures 2**a**–2**c** σ_x and σ_y are assumed to be equal $\sigma_x = \sigma_y = 0.2$ mm, and we find that as the propagation distance increases, s_1 is stable for different pairs of spatial points on the output plane, and s_3 will fluctuate due to turbulence perturbation and these fluctuations will be different from each other under different turbulence power laws. These variations are also different from each pair of points. In Figures 2**d**–2**i** we show the changes in the parameters of the beams generated by anisotropic sources, *i.e.*, the sources with $\sigma_x = 0.2$ mm, $\sigma_y = 0.3$ mm for Figs. 2**d**–2**f**, and $\sigma_x = 0.2$ mm, $\sigma_y = 0.5$ mm for Figs. 2**g**–2**i**.

In these situations, s_1 shows more fluctuation features as the deviation of spot beam width increases ($\Delta = \sigma_x - \sigma_y$). The variations of other parameters are less influenced by the deviation Δ. However, the turbulence perturbation affects the changes in s_2 and s_3 much more than the changes in s_1 in Figs. 2**a**–2**f**. In Figures 2**g** and 2**h**, the intensity of turbulence influences the change in s_1 obviously, but in Fig. 2**i**, the changes in s_1 are less influenced by the turbulence. From numerical simulation results, we can summarize the characteristics of generalized Stokes parameters of stochastic beams: for isotropic beams, s_1 will hold a constant during the propagation whether in turbulence or free space, s_2 and s_3 will change in different turbulence conditions through the propagation. The trends of these changes depend on the positions of the two points in the output plane. For anisotropic beams, s_1 will fluctuate during the propagation; these fluctuations are influenced by the turbulence. We noticed that in Fig. 2**d**–2**f**, the changes in s_2 and s_3 are similar to the changes in the same parameters in Figs. 2**a**–2**c**, but in Figs. 2**g**–2**i**, the changes in these two parameters are different. It means that the changes in s_2 and s_3 will be similar to each other if the deviation Δ is small, but there is a significant difference between them when Δ increases to a certain extent.

From the discussion by Volkov *et al.* [2], the relationship of generalized Stokes parameters and SDCP is:

$$P(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) = \sqrt{s_1^2(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) + s_2^2(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) + s_3^2(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2)} \tag{9}$$

If $\boldsymbol{\rho}_1 = \boldsymbol{\rho}_2 = \boldsymbol{\rho}$, SDCP is equal to the degree of polarization (DOP) of the beam. In Fig. 2**a**, we can notice that s_2 and s_3 will reach the initial value if the propagation distance is long enough, s_1 will be a constant, so it can be deduced that DOP of the isotropic beam will return to its initial value when the beam propagates sufficiently long distance; the phenomenon is the so-called polarization self-reconstruction. But for an anisotropic beam, or for two-points SDCP cases like those presented in other

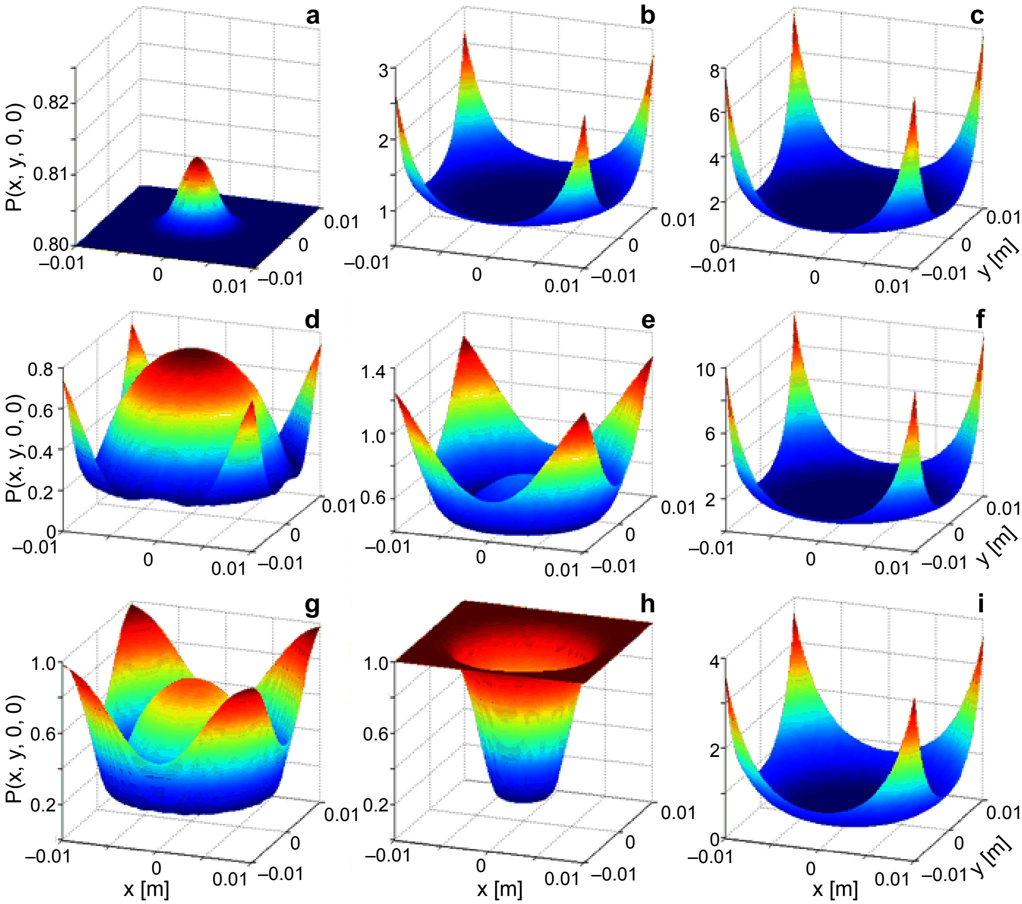

Fig. 3. Profiles of the degree of cross-polarization $P(x, y, 0, 0)$ of an anisotropic electromagnetic beam in modified turbulence $\alpha = 3.11$; $z = 10^2$ m (**a**, **d**, **g**), $z = 10^3$ m (**b**, **e**, **h**), and $z = 3\times10^3$ m (**c**, **f**, **i**).

figures in Fig. 2**a**, there is no such property. In order to study the evolution of transverse SDCP through turbulence, Fig. 3 is organized to show the two-dimensional distribution of SDCP of stochastic beams with the same source parameters setting as in Fig. 2. We can see that the central values of SDCP of the isotropic beam are much greater than circumference values at the propagation distance of $z = 0.1$ km, but for the anisotropic beam, the center and four corners (±0.01, ±0.01, 0, 0) of the values are greater than the remaining value of the area.

If the propagation distance is long enough, the profile of the distribution of transverse SDCP of the isotropic beam and the anisotropic beam will be the same. The SDCP values in the surrounding region will increase as the propagation distance increases, the final SDCP distribution profile is almost the same for the isotropic beam and the anisotropic beam, they only differ in quantity. The four corners of the SDPC will get to the maximum value at $z = 3$ km. The power law of turbulence is assumed as $\alpha = 3.11$, it means the perturbation is relatively strong in this situation.

In order to obtain how the turbulence perturbation affects the changes in SDCP, we show the variation of SDCP at different propagation distances with the difference power law of modified non-Kolmogorov turbulence in Fig. 4. These figures illustrate that the influence of atmospheric turbulence on the distribution of SDCP becomes obvious as the propagation distance increases. From Figs. 4**a**–4**c** to Figs. 4**g**–4**i**, we can see that the profile of transverse SDCP fluctuates much more in strong turbulence

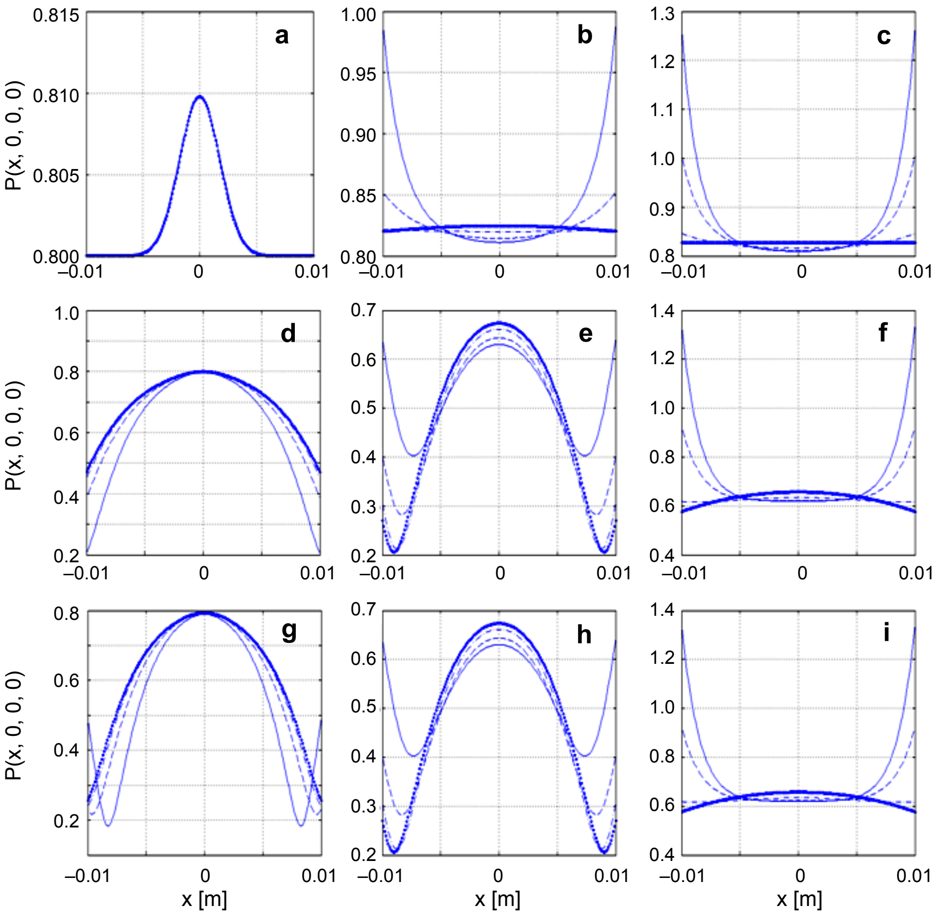

Fig. 4. Profiles of one-dimensional degree of cross-polarization $P(x, 0, 0, 0)$ of an anisotropic electromagnetic beam in modified turbulence with different propagation distance and power law of turbulence. $z = 10^2$ m (**a**, **d**, **g**), $z = 10^3$ m (**b**, **e**, **h**), and $z = 3\times10^3$ m (**c**, **f**, **i**). Solid line – $\alpha = 3.11$, dashed line – $\alpha = 3.4$, dash-dotted line – $\alpha = 3.7$, dotted line – free space.

than that in weak turbulence and free space during the propagation. The final profiles of SDCP of the isotropic and anisotropic beam in weaker turbulence also show more flat features than those in stronger turbulence.

4. Conclusions

In summary, the evolution of generalized Stokes parameters of stochastic beams propagation in modified non-Kolmogorov atmospheric turbulence was investigated, and the changes in SDCP of the beams were carried out directly on the basis of these parameters. The source parameters were chosen as an isotropic case and anisotropic case. Our results indicated that the profiles of distribution of SDCP of these two kinds of stochastic beams are different in the near filed, as the propagation distance increases. The influence of turbulence becomes obvious on the variations of the distribution, at last, the difference in the profiles of the distribution of SDCP between the isotropic beam and the anisotropic beam will disappear if the propagation distance is long enough. The difference in the sources only determine the quantity of final SDCP of the beam but the profiles of the distribution of SDCP mainly determine the turbulence perturbation. The research may be helpful for the application in optical communication, in imaging or remote sensing systems in the atmosphere.

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References

[1] ELLIS J., DOGARIU A., *Complex degree of mutual polarization*, Optics Letters **29**(6), 2004, pp. 536–538.
[2] VOLKOV S.N., JAMES D.F.V., SHIRAI T., WOLF E., *Intensity fluctuations and the degree of cross--polarization in stochastic electromagnetic beams*, Journal of Optics A: Pure and Applied Optics **10**(5), 2008, article 055001.
[3] JIA LI, YANRU CHEN, QI ZHAO, MUCHUN ZHOU, *Degrees of cross-polarization of a partially coherent electromagnetic beam in uniaxial crystals*, Optica Applicata **40**(1), 2010, pp. 93–100.
[4] JIXIONG PU, KOROTKOVA O., *Propagation of the degree of cross-polarization of a stochastic electromagnetic beam through the turbulent atmosphere*, Optics Communications **282**(9), 2009, pp. 1691–1698.
[5] PÄÄKKÖNEN P., TURUNEN J., VAHIMAA P., FRIBERG A.T., WYROWSKI F., *Partially coherent Gaussian pulses*, Optics Communications **204**(1–6), 2002, pp. 53–58.
[6] CHAOLIANG DING, ZHIGUO ZHAO, LIUZHAN PAN, XIAO YUAN, BAIDA LÜ, *Propagation of the degree of cross-polarization of stochastic electromagnetic pulsed beams in free space*, Optics and Laser Technology **43**(7) , 2011, pp. 1179–1183.
[7] LAJUNEN H., VAHIMAA P., TERVO J., *Theory of spatially and spectrally partially coherent pulses*, Journal of the Optical Society of America A **22**(8), 2005, pp. 1536–1545.
[8] LAJUNEN H., TURUNEN J., VAHIMAA P., TERVO J., WYROWSKI F., *Spectrally partially coherent pulse trains in dispersive media*, Optics Communications **255**(1–3), 2005, pp. 12–22.
[9] XINYUE DU, DAOMU ZHAO, KOROTKOVA O., *Changes in the statistical properties of stochastic anisotropic electromagnetic beams on propagation in the turbulent atmosphere*, Optics Express **15**(25), 2007, pp. 16909–16915.

[10] YING ZHOU, KAIKAI HUANG, DAOMU ZHAO, *Changes in the statistical properties of stochastic anisotropic electromagnetic beams propagating through the oceanic turbulence*, Applied Physics B **109**(2), 2012, pp. 289–294.

[11] ZILBERMAN A., GOLBRAIKH E., KOPEIKA N.S., *Propagation of electromagnetic waves in Kolmogorov and non-Kolmogorov atmospheric turbulence: three-layer altitude model*, Applied Optics **47**(34), 2008, pp. 6385–6391.

[12] BINDANG XUE, LINYAN CUI, WENFANG XUE, XIANGZHI BAI, FUGEN ZHOU, *Generalized modified atmospheric spectral model for optical wave propagating through non-Kolmogorov turbulence*, Journal of the Optical Society of America A **28**(5), 2011, pp. 912–916.

[13] CUI LIN-YAN, XUE BIN-DANG, CAO XIAO-GUANG, DONG JIAN-KANG, WANG JIE-NING, *Generalized atmospheric turbulence MTF for wave propagating through non-Kolmogorov turbulence*, Optics Express **18**(20), 2010, pp. 21269–21283.

[14] KOROTKOVA O., FARWELL N., *Effect of oceanic turbulence on polarization of stochastic beams*, Optics Communications **284**(7), 2011, pp. 1740–1746.

[15] TOSELLI I., ANDREWS L.C., PHILLIPS R.L., FERRERO V., *Free space optical system performance for laser beam propagation through non-Kolmogorov turbulence*, Optical Engineering **47**(2), 2008, article 026003.

[16] GERÇEKCIOLU H., BAYKAL Y., *Intensity fluctuations of flat-topped beam in non-Kolmogorov weak turbulence*, Journal of the Optical Society of America A **29**(2), 2012, pp. 169–173.

[17] LINYAN CUI, BINDANG XUE, LEI CAO, SHILING ZHENG, WENFANG XUE, XIANGZHI BAI, XIAOGUANG CAO, FUGEN ZHOU, *Irradiance scintillation for Gaussian-beam wave propagating through weak non-Kolmogorov turbulence*, Optics Express **19**(18), 2011, pp. 16872–16884.

[18] KOROTKOVA O., SHCHEPAKINA E., *Color changes in stochastic light fields propagating in non-Kolmogorov turbulence*, Optics Letters **35**(22) , 2010, pp. 3772–3774.

[19] WENHE DU, LIYING TAN, JING MA, YIJUN JIANG, *Temporal-frequency spectra for optical wave propagating through non-Kolmogorov turbulence*, Optics Express **18**(6), 2010, pp. 5763–5775.

[20] KOROTKOVA O., WOLF E., *Generalized Stokes parameters of random electromagnetic beams*, Optics Letters **30**(2), 2005, pp. 198–200.

[21] TERVO J., SETÄLÄ T., ROUEFF A., RÉFRÉGIER P., FRIBERG A.T., *Two-point Stokes parameters: interpretation and properties*, Optics Letters **34**(20), 2009, pp. 3074–3076.

[22] BHASKAR KANSERI, HEM CHANDRA KANDPAL, *Experimental determination of two-point Stokes parameters for a partially coherent broadband light beam*, Optics Communications **283**(23), 2010, pp. 4558–4562.

[23] BHASKAR KANSERI, SHYAMA RATH, HEM CHANDRA KANDPAL, *Direct determination of the generalized Stokes parameters from the usual Stokes parameters*, Optics Letters **34**(6), 2009, pp. 719–721.

[24] YINGBING ZHU, DAOMU ZHAO, *Generalized Stokes parameters of a stochastic electromagnetic beam propagating through a paraxial ABCD optical system*, Journal of the Optical Society of America A **25**(8), 2008, pp. 1944–1948.

[25] XINYUE DU, DAOMU ZHAO, *Changes in generalized Stokes parameters of stochastic electromagnetic beams on propagation through ABCD optical systems and in the turbulent atmosphere*, Optics Communications **281**(24), 2008, pp. 5968–5972.

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