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MULTI-PERIOD PORTFOLIO OPTIMIZATION OF POWER GENERATION ASSETS

The liberalization and deregulation of the energy industry in the past decades have been significantly affected by changes in the strategies of energy firms. The traditionally used approach of cost minimization was no longer sufficient, risk and market behavior could no longer be ignored and the need for more appropriate optimization methods for uncertain environments was increased. Mean-variance portfolio (MVP) theory is one of the more advanced financial methods that has been successfully applied to the energy sector. Unfortunately, this static approach is inadequate for studying multi-stage investment decision problems. The methodology proposed in this paper considering power generation assets is based on the model introduced by Mulvey, who suggests a reallocation approach using the analysis of various scenarios. The adoption of this methodology to power generation assets allows us to capture the impact of variations in the economic and technical parameters considered. The results of our study show that the application of a model for selection of multi-period portfolio can indeed improve the decision making process. Especially for the case of adding new investments to the portfolio mix, this rebalancing model captures new entries very well.

Keywords: *multi-period rebalancing model, portfolios of power generation assets*

1. Introduction

The liberalization and deregulation of the energy industry in the past decades have been significantly affected by changes in the goals of energy firms. Traditional optimization based on cost minimization is no longer satisfactory. Risk and market behav-

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ior cannot be ignored any longer and the need for more appropriate optimization methods for uncertain environments is increasing. In recent years, methods from finance such as mean variance portfolio (MVP) theory have been successfully adopted in the energy sector by producers, as well as by retailers. From an energy firm's point of view, MVP theory allows us to create low risk and high return portfolios of power generation technologies (power plants) with respect to economic, technical and social aspects, as well as resource availability.

A short review of the relevant literature shows that standard MVP analysis has been widely and intensively applied to the valuation of power generation assets (for more extensive reviews see, e.g. [3, 28]). The first application to the energy market (in the US) dates back to 1976 [2]. One of the earliest contributions considering the liberalized power markets of the European Union was by Awerbuch and Berger [1]. Other studies, e.g. [30, 31, 44, 49, 50], present a portfolio analysis of power generation plants undertaken in various countries or regions from a utility point of view using net present value (NPV) as a criterion for portfolio selection. Regarding portfolio risk, the concept of conditional value-at-risk (CVaR) [4] and semi-mean absolute deviation [16] to quantify risk have also been proposed. Moreover, fuzzy set theory has been applied as an alternative method of portfolio selection for power generation assets [16]. Other studies investigated the impact of new investments on the existing power generation mix [29], as well as the implications of increasing wind penetration [48].

The above mentioned studies concern the classical Markowitz mean-variance approach to portfolio selection, which is a static methodology where only a single investment period is considered and rebalancing of the portfolio is not envisaged. In contrast, a multi-period approach allows the modeling of portfolio rebalancing at different points in time. One of the first attempts to transform the Markowitz mean-variance methodology into a multi-period model for financial assets was presented by Mossin [33]. He proposes a recursive procedure as a method for solving multi-period portfolio selection problems. Recursive algorithms for portfolio selection were later also proposed by Östermark [39, 40] and Steinbach [47]. Elton and Gruber [6, 7] show that investors can obtain the solution of a multi-period model by solving a series of single-period portfolio problems. Brodt [5] proposes a multi-period linear programming model with uncertainty as an approach to solving dynamic problems of portfolio selection. Furthermore, economic developments in different periods can be described by a set of alternative scenarios (using a so-called tree structure or scenario tree), which are related to general economic conditions, as well as the state of the market. Using a scenario tree, the underlying random processes describing the value of parameters can be replaced by a discrete stochastic process. Korhonen [27] presents a model which is a combination of multi-stage and multi-objective programming techniques using a scenario tree. Further applications of scenario analysis can also be found in papers by Gülpinar and Rustem [17], Steinbach [46], Mulvey et al. [38] and Frauendorfer and Siede [13]. The authors of the last two papers combine scenario analyses with a rebalancing strategy.

Regarding the multi-period character of decision processes and uncertainty in the environment, it can be noticed that the application of portfolio theory to constructing a multistage and stochastic model is relatively new for the energy sector. However, when considering the energy sector one needs to distinguish between energy producers and energy retailers. The studies of Pereira and Pinto [41, 42] consider applications of a dynamic approach by energy producers. Both papers present an algorithm which can be used in the allocation of hydrothermal systems with the objective of minimizing the expected operation costs. This algorithm is based on stochastic programming and Bender's decomposition. Another application of multi-period portfolio optimization is presented in Kleindorfer and Li [25], who describe a multi-period value-at-risk constrained portfolio problem and apply this model to real and contractual assets in the energy sector. With regard to a retailer's point of view, multi-period stochastic models have been proposed, e.g. by Fleten et al. [12] (one of the earliest studies which suggests the integration of production planning and financial risk management), Hatami et al. [19], Hochreiter et al. [20], Kettunen et al. [24], Sen et al. [45], as well as Rocha and Khun [43]. These papers consider portfolio optimization models which show various ways to procure electrical energy while satisfying consumers' demand. They consider various financial instruments such as bilateral or spot contracts, power derivatives, as well as a firm's own production. Stochasticity enters into those models via uncertain energy prices and can be represented by a scenario tree. Furthermore, these studies show that a portfolio of derivative contracts can hedge very well against the financial risk which retailers meet on the energy market. Regarding risk measurements, average VaR and CVaR are proposed as an alternative to the classic variance used in the Markowitz approach (see, e.g. [19, 20 or 24]).

Today's dynamic, uncertain and complex energy markets require portfolio based procedures that help energy providers to achieve their strategic objectives. In this paper, we aim to propose a model of portfolio selection for energy firms which considers more than one decision period (where the structure of the portfolio is evaluated) and the uncertain environment of energy markets. The proposed multi-period stochastic model of portfolio selection is based on the one introduced by Mulvey et al. [38] for an insurance company in the US[1]. Its adoption to power generation assets allows us to capture the impact of variations in the economic and technical parameters considered. The advantage of the proposed model for power firms lies in a different portfolio structure for different decision periods with respect to changes in the input data, instead of portfolio optimization for each single period.

The remainder is organized as follows: Section 2 presents the methodology with a specification of the model, definition of the portfolio selection criteria for power

[1]The approach of Mulvey was a basis for further developments in the theory of pension plans [35].

generation assets and the analytical procedure used in the study. Section 3 shows the results of a case study followed by Section 4 with some conclusions.

2. Methodology

The standard MVP optimization model describes a two-dimensional problem, which, on the one hand, maximizes a portfolio's expected return and, on the other hand, minimizes the portfolio's risk. Investors can either minimize risk at a certain level of return, maximize the return for a given (acceptable) risk level, or consider both return and risk in one combined objective function. This function can be interpreted as a so-called utility function, which maximizes the difference between the portfolio's return and its risk. These two portfolio characteristics are weighted in this function by a parameter which indicates the relative importance of variance (risk) compared to the expected return.

As many studies show, MVP analysis has been successfully applied to asset allocation, mutual fund construction, financial planning, and even power generation mixes, but nevertheless suffers from several limitations. One of them is its single-period character. In contrast to single-period models (which cannot capture investor's goals in long-term investment processes), multi-period models, properly formulated, can solve these limitations and moreover, take advantage of volatility by rebalancing the asset mix [15]. Investors from financial markets, as well as from the energy sector, use multi-period optimization models of portfolio selection to capture the uncertain changes in different market parameters over time. Such models are part of stochastic optimization problems [34]. The model proposed in this paper is based on the multi-period stochastic portfolio selection model introduced by Mulvey et al. [38] for financial assets. It allows the reallocation of portfolio assets during the investment period and represents the behavior of the market by a scenario tree. However, its application to real (power generation) assets requires some adaptations and simplifications to the original model, because of the specific character of portfolio selection problems for the power generation mix.

2.1. Description of the model

Regarding the description of the model provided in [38] and [32], the dynamic portfolio optimization problem adapted to power generation assets can be described as follows:

For each asset $i \in N$, time $t \in T$ and scenario $s \in S$ the following parameters and decision variables are defined.

- Parameters:

$r_{i,t}^{s}$ – uncertain return of technology i in period t, given scenario s,

α – parameter indicating the relative importance of variance compared to the expected value; $\alpha \in \langle 0, 1 \rangle$,

$x_{i,\max}$ – maximal possible share of technology i in the portfolio,

q^{s} – probability that scenario s (from S possible scenarios) occurs in the scenario tree, where $\sum_{s=1}^{S} q^{s} = 1$.

- Decision variables:

$x_{i,t}^{s}$ – percentage share of technology i at time t given scenario s.

Based on these definitions of parameters and variables, the description of the model is as follows: The objective function of this two-objective model rewards a higher expected return and penalizes greater risk weighted by α and $(1 - \alpha)$, respectively:

$$\alpha R_{p,T} - (1 - \alpha)\mathrm{Var}(R_{p,T}) \to \max \tag{1}$$

subject to:

$$\sum_{i=1}^{N} x_{i,t}^{s} = 1 \qquad \forall s \in S, \qquad t = 1, ..., T \tag{2}$$

$$0 \le x_{i,t}^{s} \le x_{i,\max} \qquad \forall s \in S, \qquad t = 0, ..., T, \qquad i = 1, ..., N \tag{3}$$

$$x_{i,t}^{s} = x_{i,t}^{s'} \qquad \forall s \in S, \qquad t = 0, ..., T, \qquad i = 1, ..., N \tag{4}$$

for all scenarios s and s' (s differs from s' and s, $s' \in S$ with an identical past up to time t.

In the objective function given by Eq. (1) the expected portfolio return $R_{p,T}$ is determined by summing across all the scenarios covering the entire planning horizon T as follows:

$$R_{p,T} = \sum_{s=1}^{S} q^{s} R_{p,T}^{s} \tag{5}$$

where $R_{p,T}^{s}$ defines the expected portfolio return for scenario s at the end of the planning horizon T.

This is given by the geometric mean return of the portfolio under the assumption that each possible outcome is equally likely:

$$R^s_{p,T} = \left(\prod_{t=0}^{T} \sum_{i=1}^{N} r^s_{i,t} x^s_{i,t} \right)^{1/T} \tag{6}$$

In the second part of the objective function (1), the portfolio variance $\mathrm{Var}(R_{p,T})$ denotes the variance across all the scenarios at the end of the planning horizon *T*, and is specified as follows:

$$\mathrm{Var}\left(R_{p,T}\right) = \sum_{s=1}^{S} q^s \left(R^s_{p,T} - R_{p,T}\right)^2 \tag{7}$$

Regarding the specific character of portfolio optimization for power generation assets, the presented model also includes the constraints on the upper bound necessary to obtain technically feasible solutions (see Eq. (3)).

2.2. Portfolio selection criteria for power generation assets

Regarding the energy sector, especially energy firms, portfolio optimization can be viewed from various perspectives, depending on the firm's position on the market (e.g. whether it is a producer or a retailer). As retailers, energy market participants can use portfolio theory to optimize the portfolio of contracts they buy, e.g. to maximize their profit. Producers can optimize their portfolio of power generation technologies (i.e. power plants). The application of portfolio theory to energy contracts which are quoted on the energy exchange is similar to the portfolio selection problem in financial markets. On the other hand, the application of portfolio optimization to real assets, such as power plants, results in the necessity to define a suitable measure of return as a selection criterion. Different power generation technologies are described by many different technical, as well as economic, parameters. Moreover, these parameters are related to the energy market and are characterized by volatility in the market. Some of the studies applying portfolio theory to power generation assets use only the costs of electricity production as the portfolio selection criterion (see e.g. [1, 2]). Other studies try to capture the real value of power plants and propose to consider also the revenue side. Their authors introduce, e.g. the NPV as a portfolio selection criterion for power generation assets (see, e.g. [31]). The NPV allows us to capture all the parameters associated with various power generation technologies and is based on annual cash flows (CF_t). In this paper, NPV is also proposed as a measure of return and given in € per unit of installed capacity (€/kW). During the operational phase of a plant, the annual cash flows consider all revenues r_p and costs (i.e. fuel costs, c_f, carbon dioxide mitigation costs, c_{CO_2}, operational and maintenance costs, $c_{O\&M}$, cost of capital, c_c, and capital depreciation, δ), and are calculated as follows:

$$CF_t = r_p - c_f - c_{CO_2} - c_{O\&M} - c_c - \delta \tag{8}$$

2.3. Analytical procedure for the multi-period problem for selecting the power generation mix

Considering the specification of the model (Section 2.1), Fig. 1 shows a possible representation of the multi-period model.

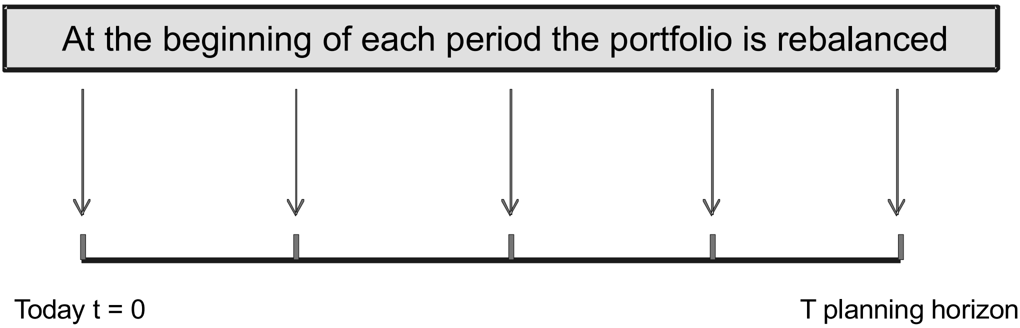

Fig. 1. Graphical representation of the multi-period model.
Source: author's illustration based on [32]

In our study, we solve the multi-period problem using multi-stage stochastic optimization. Uncertainties in the economic environment are represented by a number of distinct realizations of various parameters which provide a discrete set of scenarios. This model is solved by considering all the possible scenarios and periods. At the beginning of each period, the portfolio structure is rebalanced, which means that the weight of each asset (power technology) i is newly defined with respect to the input data for the period considered. It is important to note that power plants are never operated at full capacity. Therefore, the portfolio mix can easily be adjusted to meet the desired goals by increasing or decreasing the utilization rate of a particular power plant. Using this approach, energy providers can easily observe the structure of their portfolios, i.e. at what point in time a specific technology (power plant) can be used considering its life-time, and when new technology (a power plant) can start to be used with respect to its construction time. Such an approach gives decision makers more flexibility, can better represent future developments in the market and should perform better than single-period mean-variance models (resulting in a so-called "buy-and-hold policy"[2]).

The analytical procedure used for efficient computations based on the model of portfolio selection presented above is composed of the following steps:

- Step 1: NPV for each technology, with the discounted cash flow model (see Eq. (8)) being implemented.
- Step 2: historic time series of electricity, fuel and CO_2 prices were used to analyze their distributions, as well as to calculate the volatility and cross-correlations of these parameters[3]. In this step, the possible development of the market was defined (three possible directions of market development were considered: high, medium and low price trajectories).
- Step 3: Monte Carlo simulation (conducted with the Crystal Ball software) was run (100 000 runs) to compute the NPV for all technologies under various market scenarios (considering the calculations made in step 2 and technical information about the technologies analyzed) and their distributions.
- Step 4: the NPVs of the technologies analyzed (step 3) were used to determine their cross-correlation between technologies for each market scenario[4].
- Step 5: using the calculations from previous steps, the multi-period portfolio selection problem (from Section 2.1) was applied to generate efficient portfolios for the technologies analyzed. Because of the nonlinear and non-convex character of the model presented, the efficient frontier was obtained through the implementation of nonlinear programming algorithms in the dynamic object-oriented programming language Python 2.7.
- Step 6: for the purpose of comparison, the efficient frontier for the so-called "buy-and-hold policy" was calculated and compared with the efficient frontier obtained for the multi-period portfolio rebalancing model.

3. Results of the case study

The case study presented here regards various existing power generation technologies in use in Germany today (nuclear power plants, conventional fossil fuel power

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[2]The term "buy-and-hold policy" refers to the single-period portfolio selection problem, where the investor makes a decision at the beginning of the investment period and holds this portfolio during the whole period of time. For more information see [36, 37].

[3]For more information see Appendix, Table 1.

[4]Regarding the liberalized energy market, the correlation of the NPV of various technologies was additionally computed using the Monte Carlo simulation because it is not possible to infer it directly from empirical data [44].

plants, as well as renewables including onshore and offshore wind power) and owned by the E.ON company. The technical characteristics (such as the capacity factor or net thermal efficiency), as well as the assumed values of the economic parameters (such as investment costs, and operation and maintenance costs) were obtained from a literature search and are included in the Appendix in Table 2. The calculations were made for 2-, 3-, 4- and 5-year decision making periods[5] considering high, medium and low price trajectories, which leads to an increase in the number of scenarios from 3 to 283. Considering, for example, a 4-year decision making period, the portfolio for power generation was constructed from technologies available in the first year and then rebalanced (the percentage shares of each of these technologies in the portfolio were newly defined in each successive year) for the second, third and fourth year. Moreover, the multi-period rebalancing model considers information about the start year and necessary lead-times which allows to add new capacity to the feasible generation mix in a certain period. Also, information about shutting down power plants is taken into consideration, which involves removing some capacity from certain technologies.

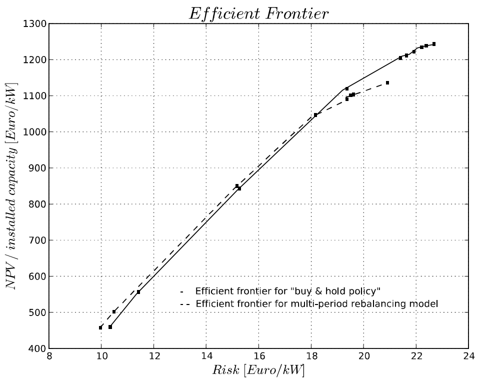
Figure 2 shows the efficient frontiers obtained for the multi-period rebalancing model in various situations analyzed in the case study and in comparison to the classical fixed mix strategy ("buy-and-hold policy"), which is outperformed.

We observe an increasing distance between the efficient frontiers for the "buy-and-hold policy" in comparison to the multi-period rebalancing model as the number of periods increases. This means that given a fixed risk level, by considering the rebalancing model, portfolios with higher returns can be obtained. Unfortunately, for high levels of risk the efficient frontier for the multi-period rebalancing model is located below the efficient frontier for the "buy-and-hold policy" which means these portfolios perform worse. In previous studies (see, e.g. [32]) the efficient frontier for rebalancing models was completely located above the frontier for the "buy-and-hold policy". This is not the case in our study. In both models, the portfolios with the highest return consist of the same technologies with similar shares (see Tables 4–7 in the Appendix). However, it should be noted that in the situation considered, as the number of periods increases, the maximum risk incurred using a rebalancing strategy decreases[6]. This also means that further research is necessary for more detailed explanations. When the number of analyzed periods increases, the part of the efficient frontier for the multi-period rebalancing model, which lies below the efficient frontier obtained for the "buy-and-hold policy", becomes smaller. On the other hand, an increase in the number of periods shifts the efficient frontier for the multi-period rebalancing model to the left and for almost all situations the portfolios obtained according to this model

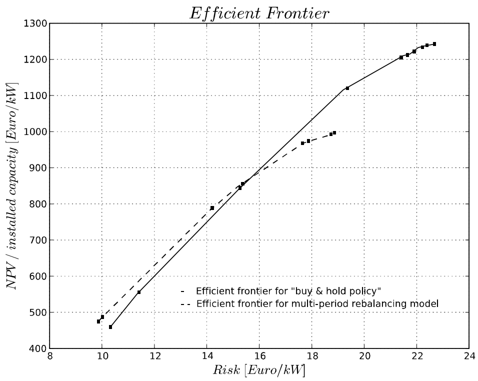
[5] 2, 3, 4, and 5-year decision making periods mean that decisions are made for 2, 3, 4 and 5 years, respectively, and each year the portfolio is rebalanced.

[6] The same observation can be found in [14, 15], where the multi-period portfolio rebalancing model was applied to only one scenario.

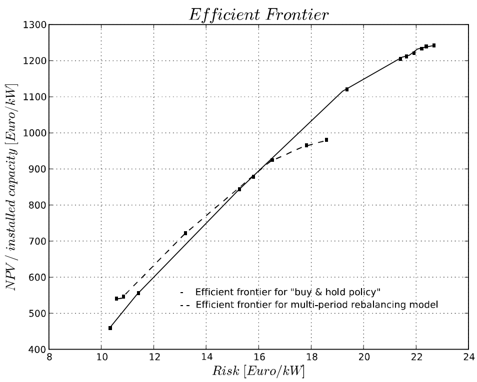
have a higher return for the same risk level (compared to the "buy-and-hold policy" model). Moreover, it can be expected that by further increasing the number of periods (by either lengthening the planning horizon or shortening the length of a period to, e.g. 6 months), the efficient frontier for the multi-period portfolio rebalancing model fully lies above the efficient frontier for the "buy-and-hold policy" model, which was initially expected.



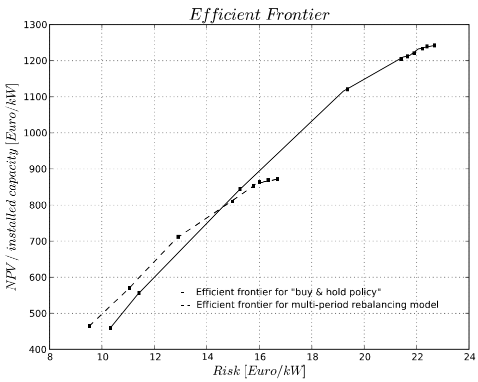
Situation 1 – 2-year decision-making period



Situation 2 – 3-year decision-making period



Situation 3 – 4-year decision-making period



Situation 4 – 5-year decision-making period

Fig. 2. Efficient frontiers obtained for the technologies analyzed (4 situations analyzed)

As the name reveals, the multi-period rebalancing model allows portfolio rebalancing at the beginning of each period. To obtain efficient portfolios, a special kind of "strategy" should be followed when applying this approach. This means that the investor changes the composition (shares of the technologies considered in the power generation mix) of his portfolio at the beginning of each period with the aim of achieving his goals. More precisely, the dots on the efficient frontier for the multiperiod rebalancing model are achieved using (as appropriate) two, three, four or five different portfolios selected for the corresponding decision making period (see Tables 4–7 in the Appendix). For energy providers, this means that the structure of their portfolio of technologies used for power production changes in each period without the necessity of renewed portfolio optimization.

Another important advantage of the rebalancing model presented is the possibility of adding and removing technologies to or from the existing portfolio. In the case of the study analyzed here, new investments in offshore wind technology were planned (Amrum Bank West with an installed power of 288 MW) but the additional capacity so-obtained can only be included into the existing power generation mix from 2015 (start year) onwards.

4. Conclusions

As many practical studies have shown, optimization plays a very important role and can be successfully applied in financial planning, as well as risk management. The energy sector is trying to make good use of existing methods and contribute to their further development. The main advantage of optimization in decision making processes is its help in reducing the number of alternatives which decision makers need to consider.

In this paper, we have proposed a multi-period portfolio rebalancing model for power generation assets with the objective of capturing the impact of variations in economic and technical parameters and of achieving technically feasible solutions. The analysis was conducted using the NPV as a selection criterion (given in €/kW) and its variance as a risk measure. For a better illustration of the impact of the dynamization of the portfolio selection problem, the so-called “buy-and-hold policy” model was also introduced. The results obtained indicate that the consideration of portfolio rebalancing, and thereby of future market developments, has a positive impact on the decision making process and could improve the expected payoffs. Having different portfolio structures for different periods, energy providers can easily realize their long-term strategy. Despite the fact that the efficient portfolios with the highest return using a “buy-and-hold policy” cannot be achieved using the multi-period portfolio rebalancing model, the efficient portfolios obtained using this model have, in most cases, a smaller risk for the same return level. Moreover, efficient portfolios for the multi-period portfolio rebalancing model seem to be better suited to long-term investors, because more information about market developments can be included.

The analysis conducted shows that application of the multi-period rebalancing model allows changes in the portfolio structure during the period taken into consideration, and interactive addition of new technologies into the portfolio. This means that the rebalancing model can capture new investments and measure their impact on the portfolio mix very well. Furthermore, active rebalancing strategies can outperform single-period mean-variance strategies, especially in the case of long-term investors. This paper shows that multi-period strategies for portfolio selection can perform better than single-period mean-variance strategies. Nevertheless, many other multi-period models are possible and a comparison between different multi-period models seems to be necessary, and could be helpful in searching for better portfolio optimization methodologies. Moreover, other considerations that pertain to power generation assets (such as (im)possible combinations of technologies, grid constraints, preferential dispatch, or energy demand and import) also need to be taken into account in future research.

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Appendix

Table 1. Descriptive statistics and distribution of commodity prices used for calculations

Subject	Periodicity	Time period	Mean μ	Standard deviation σ	Distribution
Base power[1]	monthly	Jan. 2009–May 2012	47.80 €/MWh	7.61 €/MWh	beta
Peak power[1]			55.51 €/MWh	8.58 €/MWh	
Natural gas[1]			18.76 €/MWh	5.49 €/MWh	
Hard coal[2]			101.92 €/tonne	16.42 €/tone	
Oil[3]			476.29 €/tonne	126.06 €/tonne	
Lignite[4]		Jan. 2005–May 2008	93.17 €/t SKE	4.13 €/t SKE	
Uranium[5]		Jan. 2009–May 2012	562.84 €/kg	84.52 €/kg	logistic
CO_2[1]			12.74 €/tonne	2.79 €/tonne	min-extreme

Sources:
[1][10].
[2][11].
[3][23].
[4][22].
[5]Fuel costs including the uranium price (from <www.uraniumminer.net/market_price.htm> and <www.uxc.com>) and disposal cost.

Table 2. Assumed values of technical and economic parameters

Parameter		CCGT	CHP	Gas-fired GT	Oil-fired GT	Hard coal	Hydro	Lignite	Nuclear	Off-shore wind	On-shore wind
Total electricity capacity, MW_{el}[1]		1794.96	1297.00	446.00	946.00	5945.00	1993.80	874.00	5403.00	15.80	182.60
Net thermal efficiency, %[2]		60	35	30	30	40	90	40	37	100	100
Fixed O&M cost, €/kW_{el}		30.50[7]	21.90[7]	10.00[7]	9.00[5]	14.50[7]	17.50[3]	43.30[5]	56.80[5]	42.00[5]	33.80[5]
Variable O&M cost, €/MWh		–	–	0.33[7]	1.60[5]	2.16[7]	–	–	–	–	–
Specific CO_2 emissions, t CO_2/MWh[3]		0.351	0.351	0.351	0.737	0.917	–	0.929	–	–	–
Investment cost, €/kW_{el}		530.00[5, 6]	572.00[5]	400.00[5, 6]	200.00[4]	820.00[5, 7]	1750.00[3]	1100.00[5]	2400.00[6]	4000.00[5]	1000.00[5]
Depreciation period, a		40	40	40	40	45	100	45	50	20	20
Capacity factor, %[4]	Mean	49	10	0.3	17	56	52	76	90	35	20
	Standard deviation	4	4	1	2	10	2	3	4	2	3
	Distribution	normal									

Sources:
[1] [8].
[2] [18].
[3] [9].
[4] Calculation based on information from <http://www.eon-schafft-transparenz.de/kraftwerke>.
[5] [21].
[6] [26].
[7] [44].

Table 3. Efficient portfolios for "buy-and-hold policy" for each decision making period

No.	Return	Risk	Shares									
			CCGT	CHP	GT gas	GT oil	Hard coal	Hydro	Lignite	Nuclear	Offshore wind	Onshore wind
1	459.18	10.32	0.1417	0.0720	0.0280	0.0620	0.4880	0.0000	0.0000	0.1663	0.0040	0.0380
2	551.23	11.35	0.1077	0.0720	0.0280	0.0620	0.4880	0.1131	0.0000	0.0872	0.0040	0.0380
3	839.05	15.19	0.1630	0.0720	0.0279	0.0620	0.2604	0.1270	0.0000	0.2458	0.0040	0.0380
4	1115.26	19.20	0.1630	0.0720	0.0280	0.0482	0.0697	0.1270	0.0001	0.4500	0.0040	0.0380
5	1200.03	21.24	0.1630	0.0720	0.0280	0.0364	0.0277	0.1270	0.0540	0.4500	0.0040	0.0380
6	1209.78	21.47	0.1630	0.0719	0.0279	0.0381	0.0195	0.1270	0.0606	0.4500	0.0040	0.0380
7	1214.61	21.74	0.1630	0.0625	0.0024	0.0000	0.0976	0.1270	0.0556	0.4500	0.0040	0.0380
8	1232.02	22.05	0.1630	0.0719	0.0279	0.0014	0.0468	0.1270	0.0700	0.4500	0.0040	0.0380
9	1237.02	22.32	0.1630	0.0539	0.0000	0.0000	0.0944	0.1270	0.0696	0.4500	0.0040	0.0380
10	1241.16	22.61	0.1630	0.0000	0.0000	0.0000	0.1480	0.1270	0.0700	0.4500	0.0040	0.0380

Table 4. Efficient portfolios for the multi-period rebalancing model: 2-year decision making period

No.	Return	Risk	Shares									
			CCGT	CHP	GT gas	GT oil	Hard coal	Hydro	Lignite	Nuclear	Offshore wind	Onshore wind
1	456.47	9.95	0.1336	0.0720	0.0280	0.0620	0.4880	0.0027	0.0000	0.1717	0.0040	0.0380
			0.0271	0.0720	0.0280	0.0620	0.4807	0.0070	0.0000	0.2813	0.0040	0.0380
2	497.61	10.42	0.1217	0.0720	0.0280	0.0620	0.4879	0.0610	0.0000	0.1254	0.0040	0.0380
			0.0697	0.0720	0.0279	0.0620	0.4877	0.0831	0.0000	0.1556	0.0040	0.0380
3	845.69	15.11	0.1627	0.0719	0.0278	0.0618	0.2545	0.1269	0.0000	0.2524	0.0040	0.0380
			0.1613	0.0719	0.0279	0.0619	0.1755	0.1268	0.0000	0.3327	0.0040	0.0380
4	1,040.15	17.99	0.1629	0.0720	0.0280	0.0620	0.0628	0.1270	0.0000	0.4434	0.0040	0.0380
			0.1630	0.0720	0.0280	0.0479	0.0698	0.1270	0.0004	0.4500	0.0040	0.0380
5	1,085.94	19.30	0.1630	0.0720	0.0001	0.0000	0.1263	0.1270	0.0196	0.4500	0.0040	0.0380
			0.1327	0.0720	0.0280	0.0377	0.0685	0.1270	0.0421	0.4500	0.0040	0.0380
6	1,096.51	19.34	0.1630	0.0720	0.0132	0.0043	0.1190	0.1270	0.0094	0.4500	0.0040	0.0380
			0.1630	0.0719	0.0219	0.0274	0.0482	0.1270	0.0486	0.4500	0.0040	0.0380
7	1,098.68	19.45	0.1630	0.0718	0.0039	0.0000	0.1228	0.1270	0.0195	0.4500	0.0040	0.0380
			0.1574	0.0720	0.0280	0.0403	0.0378	0.1270	0.0455	0.4500	0.0040	0.0380
8	1,136.04	20.94	0.1630	0.0000	0.0000	0.0000	0.1480	0.1270	0.0700	0.4500	0.0040	0.0380
			0.1241	0.0586	0.0225	0.0295	0.1053	0.1262	0.0670	0.4494	0.0038	0.0137

Table 5. Efficient portfolios for the multi-period rebalancing model: 3-year decision making period

No.	Return	Risk	Shares									
			CCGT	CHP	GT gas	GT oil	Hard coal	Hydro	Lignite	Nuclear	Offshore wind	Onshore wind
1	472.96	9.85	0.0941	0.0720	0.0280	0.0620	0.4880	0.0127	0.0000	0.2013	0.0040	0.0380
			0.0919	0.0719	0.0279	0.0620	0.4846	0.0502	0.0000	0.1696	0.0040	0.0380
			0.0902	0.0718	0.0279	0.0552	0.4233	0.0636	0.0000	0.2260	0.0040	0.0380
2	481.69	9.95	0.1185	0.0720	0.0280	0.0620	0.4879	0.0445	0.0000	0.1451	0.0040	0.0380
			0.0732	0.0720	0.0280	0.0620	0.4878	0.0705	0.0000	0.1645	0.0040	0.0380
			0.0738	0.0719	0.0278	0.0617	0.4403	0.0859	0.0000	0.1968	0.0040	0.0379
3	783.88	14.14	0.1630	0.0438	0.0278	0.0555	0.3203	0.1270	0.0000	0.2245	0.0040	0.0342
			0.1501	0.0716	0.0274	0.0588	0.2632	0.1269	0.0000	0.2599	0.0040	0.0380
			0.1630	0.0720	0.0271	0.0328	0.1563	0.1270	0.0000	0.3799	0.0040	0.0380
4	848.38	15.19	0.1630	0.0719	0.0279	0.0617	0.1796	0.1270	0.0000	0.3268	0.0040	0.0380
			0.1525	0.0637	0.0215	0.0112	0.3274	0.1268	0.0141	0.2409	0.0040	0.0378
			0.1619	0.0713	0.0244	0.0541	0.0720	0.1238	0.0018	0.4498	0.0040	0.0369
5	962.89	17.57	0.1630	0.0708	0.0000	0.0325	0.1035	0.1270	0.0112	0.4500	0.0040	0.0380
			0.1424	0.0356	0.0098	0.0049	0.3286	0.1254	0.0376	0.2835	0.0040	0.0282
			0.1630	0.0720	0.0209	0.0179	0.0445	0.1270	0.0627	0.4500	0.0040	0.0380
6	968.96	17.70	0.1630	0.0720	0.0030	0.0116	0.1096	0.1270	0.0218	0.4500	0.0040	0.0380
			0.1479	0.0412	0.0076	0.0009	0.3172	0.1270	0.0304	0.2859	0.0040	0.0380
			0.1630	0.0717	0.0264	0.0085	0.0512	0.1270	0.0602	0.4500	0.0040	0.0380

No.	Return	Risk	Shares									
			CCGT	CHP	GT gas	GT oil	Hard coal	Hydro	Lignite	Nuclear	Offshore wind	Onshore wind
7	987.87	18.57	0.1630	0.0720	0.0081	0.0038	0.0649	0.1270	0.0693	0.4500	0.0040	0.0380
			0.1568	0.0391	0.0049	0.0010	0.3463	0.1266	0.0407	0.2510	0.0036	0.0300
			0.1630	0.0207	0.0000	0.0000	0.1273	0.1270	0.0700	0.4500	0.0040	0.0380
8	996.56	18.89	0.1629	0.0118	0.0070	0.0019	0.1291	0.1269	0.0697	0.4488	0.0039	0.0380
			0.1242	0.0256	0.0170	0.0050	0.3538	0.1232	0.0445	0.2990	0.0037	0.0041
			0.1630	0.0001	0.0001	0.0001	0.1478	0.1270	0.0700	0.4500	0.0040	0.0380

Table 6. Efficient portfolios for the multi-period rebalancing model: 4-year decision making period

No.	Return	Risk	Shares									
			CCGT	CHP	GT gas	GT oil	Hard coal	Hydro	Lignite	Nuclear	Offshore wind	Onshore wind
1	539.33	10.56	0.0730	0.0717	0.0278	0.0619	0.4154	0.0148	0.0000	0.2937	0.0040	0.0378
			0.0771	0.0720	0.0276	0.0619	0.4654	0.1152	0.0000	0.1390	0.0040	0.0379
			0.0723	0.0712	0.0279	0.0620	0.4358	0.1022	0.0001	0.1867	0.0039	0.0380
			0.1184	0.0512	0.0257	0.0618	0.2412	0.0525	0.0000	0.4073	0.0040	0.0379
2	541.71	10.77	0.0604	0.0720	0.0279	0.0619	0.3632	0.0513	0.0000	0.3214	0.0040	0.0379
			0.0590	0.0720	0.0259	0.0620	0.4614	0.0844	0.0000	0.1934	0.0040	0.0380
			0.0764	0.0720	0.0280	0.0620	0.4403	0.0582	0.0000	0.2211	0.0040	0.0380
			0.1085	0.0719	0.0279	0.0271	0.2899	0.0650	0.0000	0.3677	0.0040	0.0380
3	717.86	13.15	0.0944	0.0719	0.0208	0.0586	0.3253	0.0671	0.0000	0.3420	0.0040	0.0159
			0.1551	0.0698	0.0096	0.0619	0.2987	0.1091	0.0000	0.2537	0.0040	0.0380
			0.1171	0.0700	0.0152	0.0614	0.2283	0.0739	0.0000	0.3952	0.0009	0.0380
			0.1468	0.0710	0.0197	0.0450	0.1074	0.1259	0.0000	0.4423	0.0040	0.0380
4	871.99	15.64	0.1629	0.0470	0.0117	0.0511	0.1643	0.1269	0.0000	0.3942	0.0039	0.0379
			0.1588	0.0703	0.0261	0.0589	0.1206	0.1232	0.0010	0.3993	0.0040	0.0379
			0.1342	0.0324	0.0163	0.0340	0.2103	0.0886	0.0064	0.4394	0.0037	0.0347
			0.1629	0.0674	0.0258	0.0126	0.0606	0.1270	0.0521	0.4500	0.0040	0.0376
5	920.55	16.44	0.1627	0.0376	0.0227	0.0006	0.1982	0.1270	0.0048	0.4046	0.0040	0.0380
			0.1573	0.0717	0.0232	0.0130	0.1643	0.1270	0.0005	0.4010	0.0040	0.0380
			0.1626	0.0704	0.0260	0.0000	0.1006	0.1270	0.0214	0.4500	0.0040	0.0380
			0.1615	0.0720	0.0280	0.0344	0.0152	0.1270	0.0700	0.4500	0.0040	0.0380
6	960.58	17.66	0.1624	0.0401	0.0046	0.0007	0.1248	0.1270	0.0493	0.4494	0.0040	0.0379
			0.1623	0.0194	0.0129	0.0214	0.1991	0.1263	0.0106	0.4076	0.0025	0.0379
			0.1567	0.0402	0.0083	0.0160	0.1254	0.1166	0.0499	0.4476	0.0038	0.0354
			0.1625	0.0000	0.0000	0.0000	0.1485	0.1270	0.0700	0.4500	0.0040	0.0379
7	980.83	18.66	0.1627	0.0009	0.0011	0.0008	0.1551	0.1255	0.0698	0.4461	0.0038	0.0342
			0.1630	0.0002	0.0021	0.0000	0.1458	0.1270	0.0700	0.4500	0.0039	0.0380
			0.1476	0.0049	0.0015	0.0575	0.1323	0.1156	0.0695	0.4483	0.0038	0.0190
			0.1616	0.0216	0.0014	0.0263	0.2141	0.1269	0.0696	0.3761	0.0016	0.0009

Table 7. Efficient portfolios for the multi-period rebalancing model: 5-year decision making period

No.	Return	Risk	Shares									
			CCGT	CHP	GT gas	GT oil	Hard coal	Hydro	Lignite	Nuclear	Offshore wind	Onshore wind
			0.1186	0.0720	0.0280	0.0620	0.4879	0.0547	0.0000	0.1348	0.0040	0.0380
			0.0730	0.0720	0.0278	0.0620	0.4560	0.0770	0.0000	0.1903	0.0040	0.0379
1	464.02	9.51	0.0774	0.0719	0.0272	0.0591	0.2915	0.0532	0.0002	0.3780	0.0039	0.0376
			0.0910	0.0715	0.0279	0.0479	0.3891	0.0071	0.0001	0.3235	0.0040	0.0379
			0.0373	0.0717	0.0271	0.0588	0.3975	0.0724	0.0001	0.2932	0.0040	0.0380
			0.0874	0.0720	0.0280	0.0620	0.4666	0.0702	0.0000	0.1719	0.0040	0.0380
			0.1009	0.0720	0.0223	0.0279	0.4208	0.0971	0.0000	0.2173	0.0037	0.0380
2	565.43	10.98	0.1134	0.0719	0.0279	0.0245	0.2329	0.0540	0.0204	0.4230	0.0040	0.0280
			0.1209	0.0719	0.0279	0.0613	0.2843	0.0707	0.0000	0.3210	0.0040	0.0380
			0.0737	0.0706	0.0273	0.0549	0.3137	0.1261	0.0181	0.2827	0.0014	0.0314
			0.1061	0.0720	0.0280	0.0620	0.3339	0.1068	0.0000	0.2492	0.0040	0.0380
			0.1176	0.0720	0.0243	0.0384	0.3340	0.1066	0.0000	0.2651	0.0040	0.0380
3	707.76	12.84	0.1316	0.0718	0.0277	0.0230	0.1623	0.0999	0.0120	0.4298	0.0040	0.0379
			0.1630	0.0709	0.0203	0.0619	0.1112	0.1270	0.0000	0.4038	0.0040	0.0380
			0.1185	0.0719	0.0267	0.0043	0.1370	0.1270	0.0227	0.4500	0.0040	0.0379
			0.1152	0.0720	0.0280	0.0620	0.1032	0.1270	0.0006	0.4500	0.0040	0.0380
			0.1155	0.0565	0.0123	0.0237	0.3453	0.1017	0.0109	0.2998	0.0015	0.0329
4	803.47	14.80	0.1350	0.0629	0.0257	0.0220	0.1281	0.1242	0.0315	0.4433	0.0040	0.0233
			0.1630	0.0720	0.0003	0.0196	0.1083	0.1270	0.0178	0.4500	0.0040	0.0380
			0.1193	0.0313	0.0200	0.0003	0.1763	0.1270	0.0419	0.4500	0.0018	0.0320
			0.1629	0.0720	0.0280	0.0156	0.0920	0.1270	0.0105	0.4500	0.0040	0.0380
			0.1240	0.0502	0.0052	0.0264	0.3085	0.1090	0.0076	0.3295	0.0033	0.0365
5	848.79	15.70	0.1630	0.0718	0.0118	0.0026	0.0790	0.1270	0.0528	0.4500	0.0040	0.0380
			0.1630	0.0718	0.0167	0.0027	0.0625	0.1270	0.0643	0.4500	0.0040	0.0380
			0.1299	0.0342	0.0070	0.0061	0.1842	0.1270	0.0401	0.4298	0.0040	0.0377
			0.1520	0.0720	0.0280	0.0620	0.0350	0.1270	0.0321	0.4500	0.0040	0.0380
			0.1200	0.0155	0.0189	0.0305	0.3185	0.1192	0.0028	0.3329	0.0039	0.0378
6	857.76	15.83	0.1630	0.0720	0.0280	0.0155	0.0587	0.1270	0.0438	0.4500	0.0040	0.0380
			0.1630	0.0463	0.0000	0.0130	0.0887	0.1270	0.0700	0.4500	0.0040	0.0380
			0.1386	0.0578	0.0011	0.0161	0.1240	0.1270	0.0435	0.4500	0.0040	0.0380
			0.1393	0.0690	0.0212	0.0001	0.1107	0.1270	0.0407	0.4500	0.0040	0.0380
			0.1245	0.0513	0.0116	0.0225	0.3006	0.1042	0.0196	0.3333	0.0015	0.0310
7	864.16	16.18	0.1630	0.0498	0.0195	0.0200	0.0848	0.1270	0.0481	0.4500	0.0040	0.0339
			0.1630	0.0165	0.0000	0.0000	0.1315	0.1270	0.0700	0.4500	0.0040	0.0380
			0.1434	0.0264	0.0127	0.0101	0.1423	0.1270	0.0537	0.4499	0.0015	0.0331
			0.1592	0.0027	0.0070	0.0134	0.1345	0.1266	0.0698	0.4494	0.0012	0.0361
			0.0988	0.0587	0.0217	0.0403	0.2917	0.1154	0.0168	0.3403	0.0001	0.0163
8	871.95	16.73	0.1628	0.0038	0.0148	0.0004	0.1722	0.1247	0.0566	0.4497	0.0040	0.0108
			0.1630	0.0001	0.0001	0.0000	0.1479	0.1270	0.0700	0.4500	0.0040	0.0380
			0.1400	0.0034	0.0254	0.0115	0.1570	0.1217	0.0611	0.4484	0.0019	0.0296

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