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INTRODUCING PROBABILISTIC MODELS FOR REDUNDANT SYSTEM RELIABILITY

Probabilistic models have been developed to evaluate the relationship between reliability measures and the performance of a repairable network with built in redundancy. Networks with built in redundancy have been considered and explicit expressions have been derived for three characteristics related to such systems including steady-state availability, period of repair, and a profit function. Various graphs have been plotted to discover the impact of availability and mean time to system failure on net profit, as well as the impact of the failure and service rate on the steady-state availability, net profit and mean time to system failure. The system was analysed using first order linear differential equations.

Keywords: *reliability, availability, mean time to system failure (MTSF), busy period, network, redundancy*

1. Introduction

The reliability of connections within networks can usually be increased by adding a number of redundant paths/units. Examples of such systems include water distribution, oil and gas supply networks, power generation and transmission networks, road and rail transport networks and telecommunication networks. Communication networks operating under normal conditions may experience random failures and abruptly cease functioning. The reliability and availability of such communication systems can be enhanced by using highly redundant structures in the design of units or subsystems. High system reliability and availability play a vital role in increasing production volume and thus contribute towards the profitability of industry.

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Since network flow systems are prevalent in power plants, as well as manufacturing and industrial systems, many researchers have studied their reliability characteristics and have introduced a great number of models to describe their behaviour and performance. The reliability of network flows with stochastic capacity and cost constraints was studied by Fathabadi et al. [3]. Markov models for analyzing the reliability of faults in wireless sensor networks was proposed by Vasar et al. [9]. Hassan [5] performed an evaluation of the reliability of a network with respect to the simplest system satisfying the capacity constraints. Ali [1] investigated the reliability of wireless body area networks which are used for monitoring the movement and health of individuals. A study of the system reliability of a multi-commodity limited-flow network was presented by Lin [6]. Rocco and Zio [7] presented cellular automata and the Monte Carlo sampling method adapted to solving problems involving the reliability of advanced networks. An approach based on cellular automata for the assessment of network reliability was studied by Rocco et al. [7, 8].

This paper presents a study of the reliability of a repairable network with built in redundancy. The study is used to develop a mathematical model. The objectives of this analysis are twofold: First, to capture the effect of the failure of individual units and repair rates on the mean time to system failure (MTSF), steady-state availability and profit. Secondly, to capture the impact of steady-state availability on the mean time to system failure and profit given the numerical values assigned to the system parameters.

The organization of the paper is as follows. Section 2 contains a description of the network flow system under study. Section 3 presents formulations of the models. The results of our numerical simulations are presented in section 4. Finally, we make some concluding remarks in Section 5.

2. Description of the network

The system is composed of three subsystems A, B and C, in series connected by two paths P_1 and P_2 (Fig. 1). Subsystem B consists of two identical units, B_1 and B_2, in cold standby. When the primary path P_1 fails, which occurs with the failure rate β_1, it is sent for repair with the service rate equal to α_1 and the standby path P_2 then carries out the function of the failed P_1. The system works whenever the subsystems A, C and either of unit B_1 or B_2 are working. It is assumed that switching from standby to operation is perfect and instantaneous. Signals from the subsystem A are received by units B_1 or B_2 through the path P_1 or P_2 and conveyed to the subsystem C. When one of the primary units, B_1 or B_2, in subsystem B fails, which occurs with the failure rate β_2 it is sent for repair with the service rate equal to α_2 and the standby unit is switched on to assume the role of the failed primary unit. System failure results from the failure of any of the subsystems A, B or C, or either of the paths P_1 and P_2. The subsystems A and C fail with failure rates β_3 and β_4 and are returned from repair at service rates α_3 and α_4, respectively (Fig. 3).

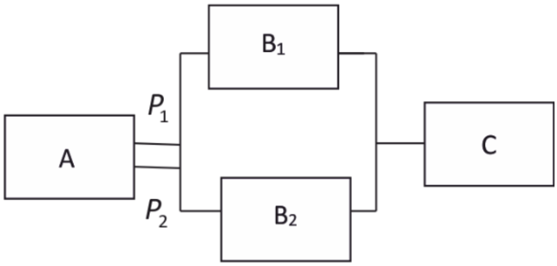

Fig. 1. Reliability block diagram of the network

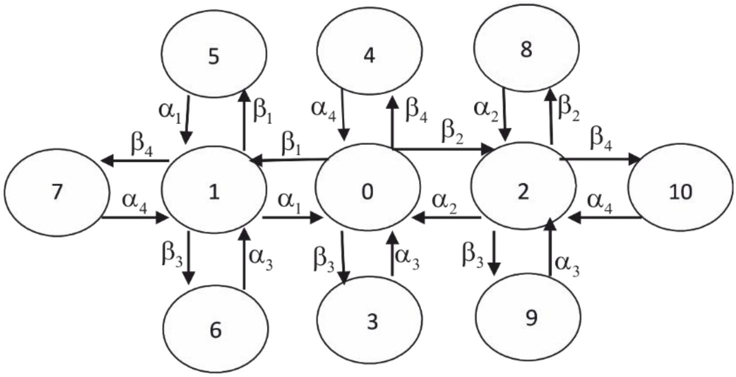

Fig. 2. Transition diagram for the network considered

S_0. Initial state. Path P_1, subsystems A and C, and unit B_1 in subsystem B are working; path P_2 and unit B_2 in subsystem B are on standby. The system is working.

S_1. Path P_1 has failed and is under repair; Path P_2, subsystems A and C, and unit B_1 in subsystem B are working; unit B_2 in subsystem B is on standby. The system is working.

S_2. Unit B_1 has failed and is under repair. Path P_1, subsystems A and C and unit B_2 in subsystem B are working; path P_2 is on standby. The system is working.

S_3. Path P_1, unit B_1 in subsystem B and subsystem C are idle, path P_2 and unit B_2 in subsystem B are on standby; subsystem A is down. The system is inoperative.

S_4. Path P_1, unit B_1 in subsystem B and subsystem A are idle; path P_2 and unit B_2 in subsystem B are on standby; subsystem C is down. The system is inoperative.

S_5. Path P_1 has failed and is waiting for repair; subsystems A and C, and unit B_1 in subsystem B are idle; unit B_2 in subsystem B is on standby; path P_2 is down and is under repair. The system is inoperative.

S_6. Path P_1 has failed and is under repair; path P_2, subsystem C and unit B_1 in subsystem B are idle; unit B_2 in subsystem B is on standby; subsystem A is down. The system is inoperative.

S_7. Path P_1 has failed and is under repair; path P_2, subsystem A and unit B_1 in subsystem B are idle; unit B_2 in subsystem B is on standby; subsystem C is down. The system is inoperative.

S_8. Unit B_1 has failed and is waiting for repair; path P_1, subsystem A and C are idle; path P_2 is on standby; unit B_2 in subsystem B is down. The system is inoperative.

S_9. Unit B_1 has failed and is waiting for repair; path P_1, unit B_2 in subsystem B and subsystems C are idle; path P_2 is on standby; subsystem A is down. The system is inoperative.

S_{10}. Unit B_1 has failed and is waiting for repair; path P_1, unit B_2 in subsystems B and A are idle; path P_2 is on standby, subsystem C is down. The system is inoperative.

3. Formulation of the model

3.1. Availability, busy period and profit of the network

In order to analyze the system availability of the network, we define $P_i(t)$ to be the probability that the system at $t \geq 0$ is in state S_i. Also let $P(t)$ be the row vector of these probabilities at time *t*.

The initial condition for this problem is:

$$P(0) = [P_0(0), P_1(0), P_2(0), \ldots, P_{10}(0)] = [1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]$$

We obtain the following differential equations:

$$P_0' = -(\beta_1 + \beta_2 + \beta_3 + \beta_4) P_0(t) + \alpha_1 P_1(t) + \alpha_2 P_2(t) + \alpha_3 P_3(t) + \alpha_4 P_4(t)$$

$$
\begin{aligned}
P_1'(t) &= -(\alpha_1+\beta_1+\beta_3+\beta_4)P_1(t)+\beta_1P_0(t)+\alpha_1P_5(t)+\alpha_3P_6(t)+\alpha_4P_7(t)\\
P_2'(t) &= -(\alpha_2+\beta_2+\beta_3+\beta_4)P_2(t)+\beta_2P_0(t)+\alpha_2P_8(t)+\alpha_3P_9(t)+\alpha_4P_{10}(t)\\
P_3'(t) &= -\alpha_3P_3(t)+\beta_3P_0(t)\\
P_4'(t) &= -\alpha_4P_4(t)+\beta_4P_0(t)\\
P_5'(t) &= -\alpha_1P_5(t)+\beta_1P_1(t)\\
P_6'(t) &= -\alpha_3P_6(t)+\beta_3P_1(t)\\
P_7'(t) &= -\alpha_4P_7(t)+\beta_4P_1(t)\\
P_8'(t) &= -\alpha_2P_8(t)+\beta_2P_2(t)\\
P_9'(t) &= -\alpha_3P_9(t)+\beta_3P_2(t)\\
P_{10}'(t) &= -\alpha_4P_{10}(t)+\beta_4P_2(t)
\end{aligned}
\tag{1}
$$

This can be written in the matrix form as

$$\dot{P} = MP \tag{2}$$

where Eq. (2) is expressed explicitly in the form

$$
\begin{bmatrix} P_0 \\ P_1' \\ P_2' \\ P_3' \\ P_4' \\ P_5' \\ P_6' \\ P_7' \\ P_9' \\ P_9' \\ P_0' \end{bmatrix}
=
\begin{bmatrix}
A_1 & \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 & 0 & 0 & 0 & 0 & 0 & 0\\
\beta_1 & A_2 & 0 & 0 & 0 & \alpha_1 & \alpha_3 & \alpha_4 & 0 & 0 & 0\\
\beta_2 & 0 & A_3 & 0 & 0 & 0 & 0 & 0 & \alpha_2 & \alpha_3 & \alpha_4\\
\beta_3 & 0 & 0 & -\alpha_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
\beta_4 & 0 & 0 & 0 & -\alpha_4 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & \beta_1 & 0 & 0 & 0 & -\alpha_1 & 0 & 0 & 0 & 0 & 0\\
0 & \beta_3 & 0 & 0 & 0 & 0 & -\alpha_3 & 0 & 0 & 0 & 0\\
0 & \beta_4 & 0 & 0 & 0 & 0 & 0 & -\alpha_4 & 0 & 0 & 0\\
0 & 0 & \beta_2 & 0 & 0 & 0 & 0 & 0 & -\alpha_2 & 0 & 0\\
0 & 0 & \beta_3 & 0 & 0 & 0 & 0 & 0 & 0 & -\alpha_3 & 0\\
0 & 0 & \beta_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\alpha_4
\end{bmatrix}
\begin{bmatrix} P_0(\infty) \\ P_1(\infty) \\ P_2(\infty) \\ P_3(\infty) \\ P_4(\infty) \\ P_5(\infty) \\ P_6(\infty) \\ P_7(\infty) \\ P_8(\infty) \\ P_9(\infty) \\ P_{10}(\infty) \end{bmatrix}
$$

and

$$A_1 = -(\beta_1 + \beta_2 + \beta_3 + \beta_4)$$

$$A_2 = -(\alpha_1 + \beta_1 + \beta_3 + \beta_4)$$

$$A_3 = -(\alpha_2 + \beta_2 + \beta_3 + \beta_4)$$

The above substitutions(A_1–A_3) adopted for the editorial reasons apply also to the matrices in (5a) and (7).

The steady-state availability (the proportion of time the system is in a functioning condition or equivalently, the sum of the probabilities of operational states) and busy period (the sum of the probabilities of states involving repair) are given by

$$A_V(\infty) = P_0(\infty) + P_1(\infty) + P_2(\infty) \tag{3}$$

$$B_P(\infty) = P_1(\infty) + P_2(\infty) + P_3(\infty) + \ \dots \ + P_{10}(\infty) \tag{4}$$

In the steady state, the derivatives of the state probabilities become zero and therefore Eq. (2) become

$$MP = 0 \tag{5}$$

which is in a matrix form

$$\begin{bmatrix} A_1 & \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 & 0 & 0 & 0 & 0 & 0 & 0 \\ \beta_1 & A_2 & 0 & 0 & 0 & \alpha_1 & \alpha_3 & \alpha_4 & 0 & 0 & 0 \\ \beta_2 & 0 & A_3 & 0 & 0 & 0 & 0 & 0 & \alpha_2 & \alpha_3 & \alpha_4 \\ \beta_3 & 0 & 0 & -\alpha_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \beta_4 & 0 & 0 & 0 & -\alpha_4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \beta_1 & 0 & 0 & 0 & -\alpha_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \beta_3 & 0 & 0 & 0 & 0 & -\alpha_3 & 0 & 0 & 0 & 0 \\ 0 & \beta_4 & 0 & 0 & 0 & 0 & 0 & -\alpha_4 & 0 & 0 & 0 \\ 0 & 0 & \beta_2 & 0 & 0 & 0 & 0 & 0 & -\alpha_2 & 0 & 0 \\ 0 & 0 & \beta_3 & 0 & 0 & 0 & 0 & 0 & 0 & -\alpha_3 & 0 \\ 0 & 0 & \beta_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\alpha_4 \end{bmatrix} \begin{bmatrix} P_0(\infty) \\ P_1(\infty) \\ P_2(\infty) \\ P_3(\infty) \\ P_4(\infty) \\ P_5(\infty) \\ P_6(\infty) \\ P_7(\infty) \\ P_8(\infty) \\ P_9(\infty) \\ P_{10}(\infty) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{5a}$$

Subject to following normalizing conditions:

$$P_0(\infty) + P_1(\infty) + P_2(\infty) + \dots + P_{10}(\infty) = 1 \tag{6}$$

Following [10] and [11], we substitute (6) in the last row of (5) to compute the steady-state probabilities.

$$\begin{bmatrix} A_1 & \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 & 0 & 0 & 0 & 0 & 0 & 0 \\ \beta_1 & A_2 & 0 & 0 & 0 & \alpha_1 & \alpha_3 & \alpha_4 & 0 & 0 & 0 \\ \beta_2 & 0 & A_3 & 0 & 0 & 0 & 0 & 0 & \alpha_2 & \alpha_3 & \alpha_4 \\ \beta_3 & 0 & 0 & -\alpha_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \beta_4 & 0 & 0 & 0 & -\alpha_4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \beta_1 & 0 & 0 & 0 & -\alpha_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \beta_3 & 0 & 0 & 0 & 0 & -\alpha_3 & 0 & 0 & 0 & 0 \\ 0 & \beta_4 & 0 & 0 & 0 & 0 & 0 & -\alpha_4 & 0 & 0 & 0 \\ 0 & 0 & \beta_2 & 0 & 0 & 0 & 0 & 0 & -\alpha_2 & 0 & 0 \\ 0 & 0 & \beta_3 & 0 & 0 & 0 & 0 & 0 & 0 & -\alpha_3 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} P_0(\infty) \\ P_1(\infty) \\ P_2(\infty) \\ P_3(\infty) \\ P_4(\infty) \\ P_5(\infty) \\ P_6(\infty) \\ P_7(\infty) \\ P_8(\infty) \\ P_9(\infty) \\ P_{10}(\infty) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \quad (7)$$

Solving (7),we obtain the steady-state probabilities

$$P\ P_0(\infty),\ P_1(\infty), P_2(\infty), P_3(\infty),\ \ldots, P_{10}(\infty)$$

The expressions for the steady-state availability and busy period given in (3) and (4) above are

$$A_V(\infty) = \frac{\alpha_1\alpha_2\alpha_3\alpha_4(\alpha_1+\beta_1+\beta_2)}{D_0} \quad (8)$$

$$B_P(\infty) = \frac{N_0}{D_0} \quad (9)$$

where

$$N_0 = \alpha_1\alpha_2^2\alpha_3\alpha_4\beta_1 + \alpha_3\alpha_2^2\alpha_4\beta_1^2 + \alpha_1^2\alpha_2\alpha_3\alpha_4\beta_2 + \alpha_1^2\alpha_3\alpha_4\beta_2^2 + \alpha_1^2\alpha_2^2\alpha_4\beta_3$$
$$+ \alpha_1\alpha_2^2\alpha_4\beta_1\beta_3 + \alpha_1^2\alpha_2\alpha_4\beta_2\beta_3 + \alpha_1^2\alpha_2\alpha_3\beta_4^2 + \alpha_1\alpha_2\alpha_3^2\beta_1\beta_4 + \alpha_1^2\alpha_2\alpha_3\beta_2\beta_4$$

$$D_0 = \alpha_1^2\alpha_3\alpha_4\beta_2^2 + \alpha_1^2\alpha_2\alpha_3\alpha_4\beta_2 + \alpha_1^2\alpha_2\alpha_3\beta_2\beta_4 + \alpha_1^2\alpha_2^2\alpha_3\alpha_4$$
$$+ \alpha_1^2\alpha_2^2\alpha_3\beta_4 + \alpha_1^2\alpha_2\alpha_4\beta_2\beta_3 + \alpha_1^2\alpha_2^2\alpha_4\beta_3 + \alpha_1\alpha_2^2\alpha_3\alpha_4\beta_1$$
$$+ \alpha_1\alpha_2^2\alpha_3\beta_1\beta_4 + \alpha_1\alpha_2^2\alpha_4\beta_1\beta_3 + \alpha_3\alpha_2^2\alpha_4\beta_1^2$$

Let C_0 and C_1 be the revenue generated when the system is in a working state, equivalently loss of income when in an inoperative state and the cost of each repair respectively. The expected total profit per unit time generated by the system in the steady-state is

$$\text{Profit} = \text{Total revenue generated} - \frac{\text{Total maintenance}}{\text{Repair cost}}$$

$$PF = C_0 A_V(\infty) - C_1 B_P(\infty) \tag{10}$$

where PF is the net profit generated by the system.

3.2. Mean time to system failure of the network

Since it is difficult to evaluate the transient solutions, therefore the concept from [2], [4], [10] and [11] have been used to derive an explicit solution for evaluating the mean time to system failure (MTSF). These procedures require the deletion of rows and columns corresponding to the absorbing states (an absorbing state is a state from which there is a zero probability of exiting) of matrix M and take the transpose to produce a new matrix, say Q. The expected time to reach an absorbing state is obtained from the relation:

$$\text{MTSF} = P(0)\left(-Q^{-1}\right)\begin{bmatrix}1\\1\\1\end{bmatrix} = \frac{N_1}{D_1} \tag{11}$$

where

$$N_1 = (\alpha_1 + \beta_1 + \beta_3 + \beta_4)(\alpha_2 + \beta_2 + \beta_3 + \beta_4)$$
$$+ \beta_1(\alpha_2 + \beta_2 + \beta_3 + \beta_4) + \beta_2(\alpha_1 + \beta_1 + \beta_3 + \beta_4)$$

$$D_1 = \beta_4\left(2\beta_1\beta_4 + 2\beta_2\beta_4 + \beta_1^2 + \beta_2^2 + 3\beta_3^2 + 3\beta_3\beta_4 + \alpha_1\beta_4 + \alpha_2\beta_4 + \beta_4^2\right)$$
$$+ \beta_1\left(\alpha_2\beta_1 + \beta_1\beta_2 + \beta_1\beta_3 + 2\beta_3^2 + \beta_2^2\right)$$
$$\beta_3\left(2\alpha_2\beta_1 + 3\beta_1\beta_2 + 4\beta_1\beta_4 + 2\alpha_1\beta_2 + 4\beta_2\beta_4\right.$$
$$\left. + \alpha_1\alpha_2 + 2\alpha_1\beta_4 + 2\alpha_2\beta_4 + \beta_2^2 + 2\beta_2\beta_3 + \beta_3^2 + \alpha_1\beta_3 + \alpha_2\beta_3\right)$$
$$+ \beta_1\left(2\alpha_2\beta_4 + 3\beta_2\beta_4\right) + \alpha_1\left(2\beta_2\beta_4 + \beta_2^2 + \alpha_2\beta_4\right)$$

$$Q = \begin{bmatrix} -(\beta_1+\beta_2+\beta_3+\beta_4) & \beta_1 & \beta_2 \\ \alpha_1 & -(\alpha_1+\beta_1+\beta_3+\beta_4) & 0 \\ \alpha_2 & 0 & -(\alpha_2+\beta_1+\beta_3+\beta_4) \end{bmatrix}$$

4. Graphical analysis of the network

Numerical examples are presented to demonstrate the impact of service and failure rates on steady-state availability, net profit and mean time to system failure and the overall performance of the system based on given values of the parameters. For the purpose of numerical example, the following set of parameter values are used: $\alpha_1 = 0.3$, $\alpha_2 = 0.4$, $\alpha_3 = 0.6$, $\alpha_4 = 0.5$, $\beta_1 = 0.3$, $\beta_2 = 0.2$, $\beta_3 = 0.3$, $\beta_4 = 0.01$, $C_0 = 200\,000$, $C_1 = 80\,000$. The MATLAB package was used to program the simulations in this study.

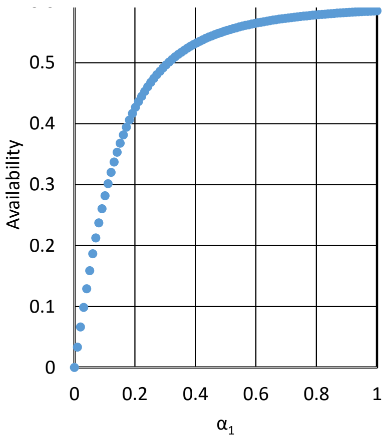

Fig. 3. Availability as function of α_1

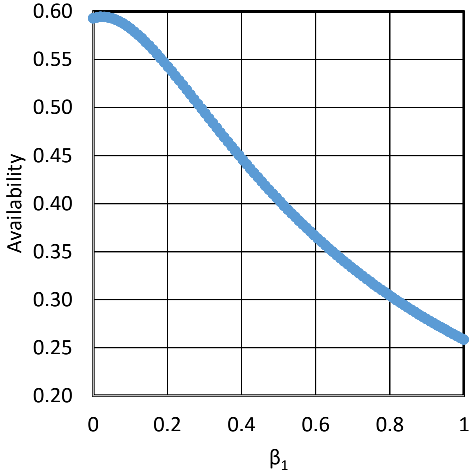

Fig. 4. Availability as function of β_1

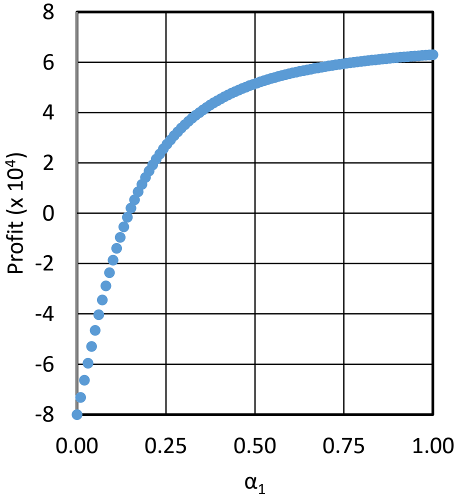

Fig. 5. Profit as function of α_1

Figures 3, 5 and 7 show the behavior of availability, net profit and mean time to system failure as function of service rate α_1. It is observed that the system availability, net profit and mean time to system failure increase in the service rate. This means that they can be improved by increasing the service rate, reducing the failure rate or by preventive maintenance action. Thus, this sensitivity analysis suggests one way of maximizing production output, system availability, mean time to system failure and net profit while minimizing cost.

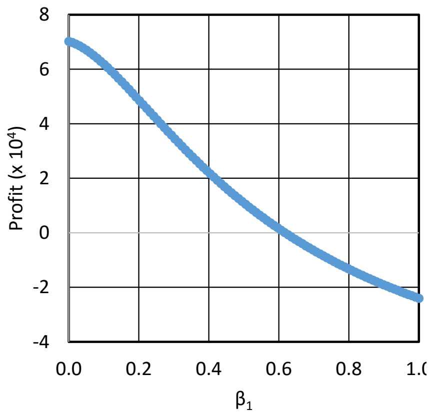

Fig. 6. Profit as function of β_1

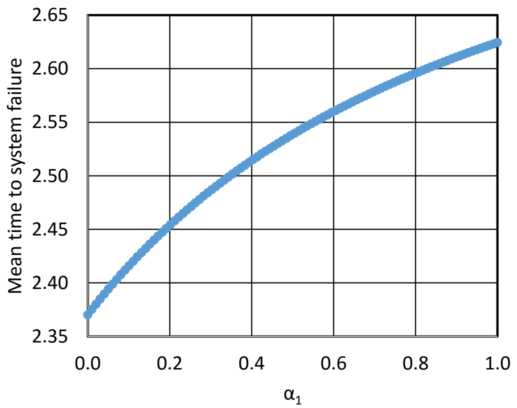

Fig. 7. MTSF as function of α_1

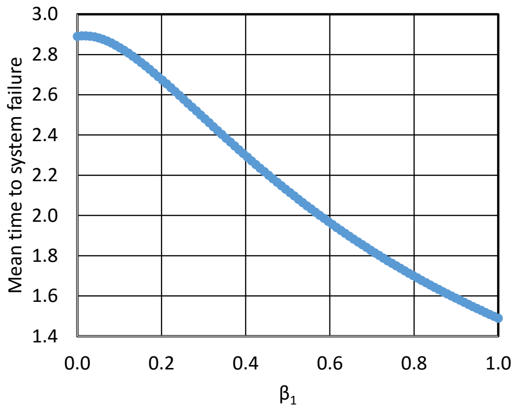

Fig. 8. MTSF as function of β_1

Figures 4, 6 and 8 present the impact of the failure rate β_1 on availability, net profit and mean time to system failure. It is evident from these figures that they decrease in the failure rate β_1. From these figures, it is clear that increasing the failure rate minimizes the production output, net profit and overall system performance.

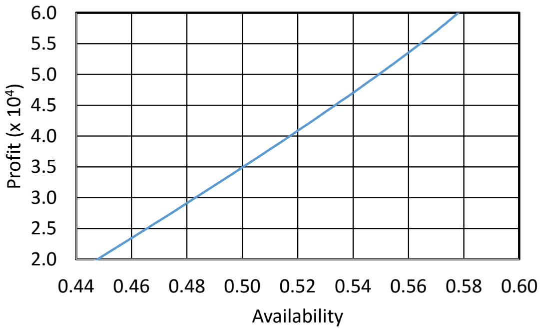

Fig. 9. Profit in function of the availability

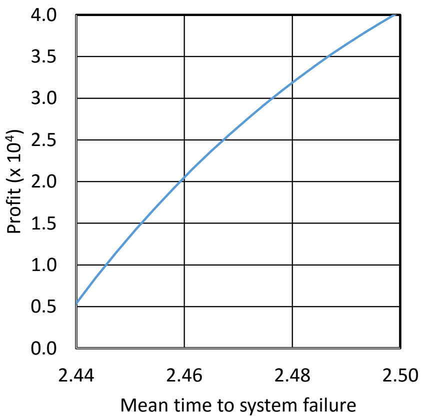

Fig. 10. Profit in function of the MTSF

The impact of availability and MTSF on net profit can be seen in Figs. 9 and 10. In these figures, profit is increasing in the steady-state availability and MTSF. These sensitivity analyses illustrate the behavior of profit as function of availability and MTSF.

It is of interest to know how sensitive profit is to availability and MTSF. Thus, net profit can be increase when availability and MTSF are enhanced through an increase in the service rate, reducing the failure rate or by preventive maintenance action.

5. Conclusion

Explicit expressions for the mean time to system failure, steady-state availability and profit of such system are derived. The numerical simulations presented in Figs. 3–8 provide a description of the effect of the failure rate β_1 and service rate α_1 on steady-state availability, profit and mean time to system failure (MTSF). From these simulations, the availability, profit and life span of the system is increased by improving the repair rate and is diminished by failures. It is evident from Figs. 9 and 10 that the associated net profit and mean time to system failure are increasing in availability. On the basis of the numerical and graphical results obtained for a particular case, it is suggested that the system availability, mean time to system failure and net profit of the system can be improved significantly by:

- Adding more paths and units in cold standby. Various redundant units/paths will lead to different results for the steady-state availability, profit and mean time to system failure. From the graphical study illustrated above, the steady-state availability, profit and mean time to system failure are sensitive to the failure and service rate. The steady-state availability, profit and mean time to system failure will be increasing in the number of redundant units/paths.
- Increasing the service rate.
- reducing the failure rate of the system by hot duplication.

Conflict of interests

The authors declare that there is no conflict of interests.

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