Letters to the Editor

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The Secondary Spectrum of Two-Lens Superachromats with an Air Space

The simplet optical system that meets the superachromatic correction is the two-lens system. Such a system may also serve as a starting point for calculations of more complex systems, but with a better correction and simultaneous increase in the aperture and field angle. For example, the simplet superachromatic system with a corrected Petzval curve is a two-lens system with an air space; but it has to be expanded in order to be of practical value. We shall deal with this problem in the paper "An analysis of the possibility of constructing a superachromatic objective with a flat field". We shall now analyse the secondary spectrum of two-lens systems with an air space. This problem was dealt with ČANŽEK who gave the following formula in papers [1, 2]:

$$s'_\lambda - s' = \frac{h(P_{\lambda 1} - P_{\lambda 2})}{h_{\nu 1} - \nu_2}, \tag{1}$$

where

$\nu_1, P_{\lambda 1}$ — the Abbe's number and the partial dispersion, respectively, of the first glass,

ν_2, $P_{\lambda 2}$ — the same for the second glass,

h — hight of incidence of the aperture ray on the second lens (the hight of incidence on the first lens is assumed to equal zero),

s'_λ — the last lens surface to focus distance for wave length λ,

s' — the last lens surface to focus distance for basic colour.

The following approximation was used to derive formula (1),

$$h^2 = s'_\lambda s' \tag{2}$$

and the term neglected

$$R = (1-h)[(1-h)a_F a_\lambda - h(n_1-1)(a_F + a_\lambda)], \tag{3}$$

where

$$a_F = n - n_F,$$

$$a_\lambda = n - n_\lambda.$$

It seems, however, that it is difficult to estimate the error we make when neglecting (3) without a detailed analysis. It follows that in some superachromatic systems the approximation resulting from neglecting the term (3) is not sufficient. We shall, therefore deal with this problem again. Henceforth we shall use the following definition of Abbe's number and partial dispersion

$$\nu = \frac{n_F - 1}{n_F - n_C},$$
$$P_\lambda = \frac{n_F - n_\lambda}{n_F - n_\lambda}. \tag{4}$$

Violet has been taken as the basic colour (denoted without index). We assume that the system is corrected for violet and red. For systems with the focal length normalized to a unity we may write

$$\varphi_1 + h\varphi_2 = \frac{h}{s'}, \tag{5}$$

where φ_1, φ_2 — focussing power of the first and second lens, respectively. The focussing power for any wavelength may be written as

$$\varphi_\lambda = \frac{n_\lambda - 1}{n-1}\varphi. \tag{6}$$

Formula (5) for any wavelength has the form:

$$\frac{n_{1\lambda}-1}{n_1-1}\varphi_1 + h_\lambda \frac{n_{2\lambda}-1}{n_2-1}\varphi_2 = \frac{h_\lambda}{s'_\lambda}. \tag{7}$$

Dividing eq. (5) by h and eq. (7) by h_λ, and subtratracting them we obtain

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$$\varphi_1\left(\frac{1}{h}-\frac{1}{h_\lambda}\frac{n_{1\lambda}-1}{n_1-1}\right)+\varphi_2\left(1-\frac{n_{2\lambda}-1}{n_2-1}\right)=\frac{s'_\lambda-s'}{s'_\lambda-s'}. \tag{8}$$

Multiplying eq. (8) by hh_λ we get

$$\varphi_1\left(h_\lambda-h\frac{n_{1\lambda}-1}{n_1-1}\right)+hh_\lambda\varphi_2\left(1-\frac{n_{2\lambda}-1}{n_2-1}\right)=(s'_\lambda-s')\frac{h_\lambda}{s'_\lambda}. \tag{9}$$

It may be shown that

$$\frac{h_\lambda}{s'_\lambda}=\frac{h+dh}{s'+ds'}=\left(\frac{h}{s'}+\frac{dh}{s'}\right)\left(1-\frac{ds'}{s'}\right)\approx 1. \tag{10}$$

From the conditions

$$h=1-d\varphi_1$$
$$h_\lambda=1-d\varphi_{\lambda 1} \tag{11}$$

we obtain

$$h_\lambda=h+d\varphi_1\left(1-\frac{n_{1\lambda}-1}{n_1-1}\right). \tag{12}$$

On inserting relations (10) (11) and (12) into eq. (9) we have

$$\varphi_1\left(1-\frac{n_{1\lambda}-1}{n_1-1}\right)+h^2\varphi_2\left(1-\frac{n_{2\lambda}-1}{n_2-1}\right)+$$
$$+hd\varphi_1\varphi_2\left(1-\frac{n_{2\lambda}-1}{n_2-1}\right)\left(1-\frac{n_{1\lambda}-1}{n_1-1}\right)=s'_\lambda-s'. \tag{13}$$

Making use of the identity

$$n_\lambda-1=(n-1)\left(1-\frac{P_\lambda}{\nu}\right). \tag{14}$$

Eq. (13) may be written as

$$\varphi_1\frac{P_{\lambda 1}}{\nu_1}+h_2\varphi_1\frac{P_{\lambda 2}}{\nu_2}+hd\varphi_1\varphi_2\frac{P_{\lambda 1}}{\nu_1}\frac{P_{\lambda 2}}{\nu_2}=s'_\lambda-s'. \tag{15}$$

Since the system was assumed to have the chromatic aberration corrected for violet and red colours,

$$\frac{\varphi_1}{\nu_1}+h^2\frac{\varphi_2}{\nu_2}=0. \tag{16}$$

From the normalization condition it follows that

$$\varphi_1+h\varphi_2=1. \tag{17}$$

Therefore φ_1, and φ_2 can be found

$$\varphi_1=\frac{\nu_1 h}{\nu_1 h-\nu_2}\qquad \varphi_2=-\frac{\nu_2}{h(\nu_1 h-\nu_2)}. \tag{18}$$

Calcullating d from the first of eq. (11) and inserting into (15) with simultaneous application of (18), we obtain

$$\frac{h(P_{\lambda 1}-P_{\lambda 2})}{\nu_1 h-\nu_2}-\frac{(1-h)P_{\lambda 1}P_{\lambda 2}}{\nu_1(\nu_1 h-\nu_2)}=s'_\lambda-s'. \tag{19}$$

The relation (19) becomes (1) if we neglect the second term. As seen from eq. (19) the approximate formula (1) may be used only when the product of the focussing power of the first lens and the distance between the lenses is relatively small. The magnitude of error we make when using formula (1) depends also on the dispersion of glass the first lens is made of, and also on the wavelength for which the secondary spectrum is calculated. As an example we estimate the secondary spectrum of an superachromatic system with the Petzval curve corrected. The first lens is made of fluorite, the second — of LaK 11. The focussing powers and distances between the lenses are, respectively;

$$\varphi_1=6.158,\quad \varphi_2=-7.140,\quad d=0.045. \tag{20}$$

This system is the strating system for calculating a superachromat with flat field. The magnitude of the secondary spectrum for the wavelength $\lambda=1.014$ μm obtained from formula (1) (the first term of formula (19)) is — 0.0012, whereas from formula (19) it is — 0.002. Therefore, the relation (19) can not be used in this case. The above considerations show that, in general the formula (1) can not used for estimating the magnitude of secondary spectrum. This formula can be used only when the second term in relation (19) is negligible.

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References

[1] Čanžek L., *Optik* 30, 1069, 17.
[2] Čanžek L., Private communication.

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