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Modified Smith's Method for an Automatic Correction of Optical Systems

A modification of the non-linear Smith's method used for optimization procedures of optical systems has been developed. This modification deals with optimization directions. The choise of weighting factors of merit function has been selected so as to increase the effectivity of the minimizing procedure used.

1. Introduction

Several papers have been published in recent years on automatic lens design [1–3]. From the mathematical point of view the problem consists in solving the following system of equations:

$$\begin{aligned} f_1(x_1 \ldots\ldots\ldots\ldots x_n) &= f_{1s} \\ f_2(x_1 \ldots\ldots\ldots\ldots x_n) &= f_{1s} \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ f_m(x_1 \ldots\ldots\ldots\ldots x_n) &= f_{1s} \end{aligned} \tag{1}$$

with simultaneous satisfying the following constrains

$$\begin{aligned} f_{m+1}(x_1 \ldots\ldots\ldots\ldots x_n) &\geqslant 0 \\ f_{m+2}(x_1 \ldots\ldots\ldots\ldots x_n) &\geqslant 0 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ f_l(x_1 \ldots\ldots\ldots\ldots x_n) &\geqslant 0 \end{aligned} \tag{1a}$$

The following designations have been introduced in relations (1) and (1a).

$x_1 \ldots\ldots x_n$ — parameters of the system,

$f_1 \ldots\ldots f_n$ — aberration functions of the system.

As the analytic form of the functions f_i is, in general, unknown it is impossible to give any solution of system (1) in an analytic form. Obviously, a parameter group $x_1, \ldots, x_n$ is adopted as a solution of system (1) for which the merit function

$$\varphi = \sum_{i=1}^{m} w_i (f_i - f_{is})^2 \tag{2}$$

has a minimum, where w_i — weighting factors.

The oldest mathematical method of finding a minimum of a function of many variables is the steepest descent method given by Cauchy [4]. According to this method it is possible to find the minimum of the function of many parameters in the direction oposite to the gradient. However, convergence of this method is in general very slow especially for illconditioned functions.

In order to improve the convergance of Cauchy's method the following modifications have been introduced: change of metric in the parameters space proposed by H. B. Curry [5] and W. C. Davidon [6] or application of the conjugate gradient method [7].

Linearization methods from another group of methods, which may be exploited for solving the equation system (1). These methods work under the assumption that functions f_i are linear with respect to independent parameters x_i. However, the most frequently used methods are the damped least squared methods given by Levenberg [8] as well as various modifications of the Newton–Raphson method proposed (among others by D. P. Feder [1] and A. P. Grammatin [9]. Unfortunately, these methods require a time-consuming procedure of A matrix estimation $\left(A_{ij} = \dfrac{\partial f_i}{\partial x_j}\right)$ and a suitable choice of the damping factors.

For several reasons it seems to be profitable to use certain non-linearization methods. The most important reason is that the non-linearization method does not require either calculation of the matrix A or any choice of damping factors. In recent years scientists have discovered some non-linearization methods [10, 11, 12] of finding a minimum value of function of many parameters. The best known methods are those supplied by Davies, Svann, Campay, Powell and Smith. The differences among these methods are in choosing the directions p_l, along which attempts

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are made to determine the minimum of the function. The Powell methods were tested [13] on some functions of given analytic form and its convergence was compared with certain linearization methods [13].

However, the methods referred to above were not used so far in automatic lens designs. The first results of applying these methods to this purpose were presented during the SIMP Conference in Warsaw [14]. The present paper is a development of these ideas.

2. Modification of the Smith's Method

The starting point in our methods is Smith's method of optimization [11]. Pursuant to our suggestion each iteration is devided into optimization cycles. The individual cycles are realized in subspaces

$$\{x_1\}, \{x_1, x_2\}, \ldots, \{x_1, \ldots, x_n\}, \tag{3}$$

where $\{x_1, x_2, \ldots, x_k\}$ denotes subspace stretched on $x_1, x_2, \ldots, x_k$ axes. In the i-th cycle the starting point is distant from the minimum (obtained in the previous cycle) by trial step q_i in the direction of the x_i axis. The flow diagram of the optimization method is shown in schemes in Figs. 1 and 2 while the run of minimization procedure in two parameter space is depicted by a graph in Fig. 3. Where the following designations are introduced

P_0 — the initial point,

P_1 — the point obtained after the first cycle,

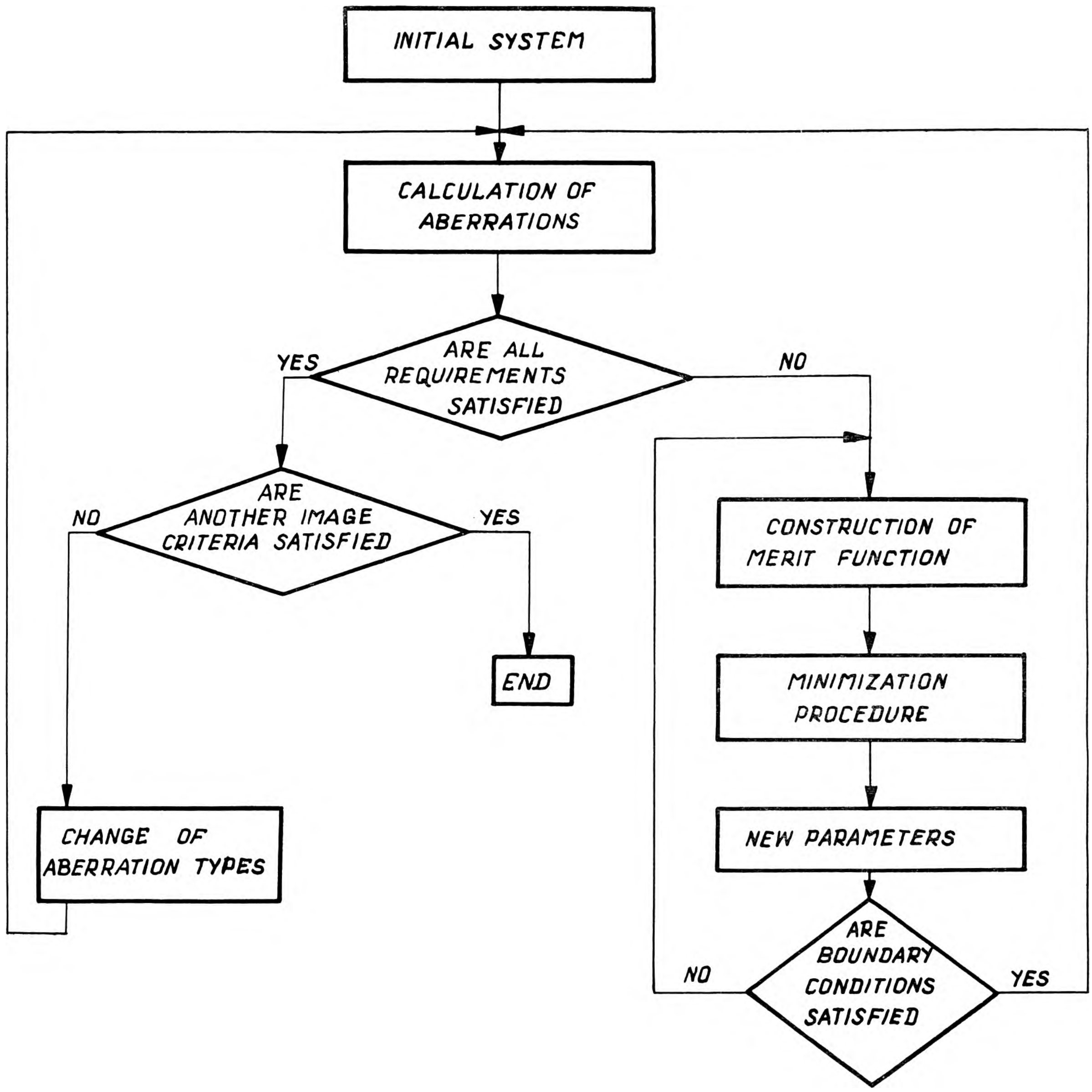

Fig. 1. A general flow diagram of the optimization method

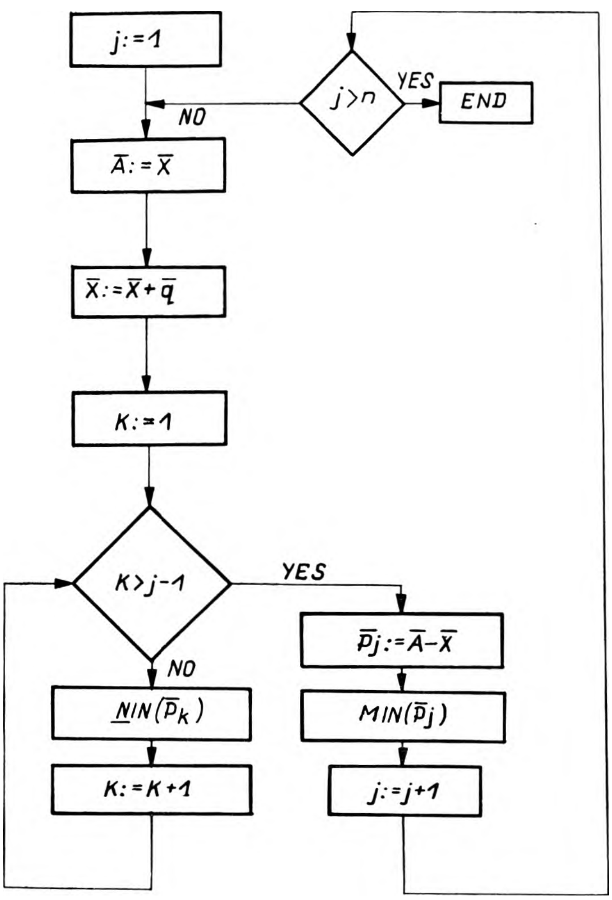

Fig. 2. Flow diagram of minimization procedure

P_2 — the starting point in the second cycle,
P_3 — the minimum obtained from starting point P_2 lying on direction x_1,
P_4 — the minimum in P_1 P_3 direction.

During one iteration the minimum value obtained after each cycle is not greater than that determined in the course of the previous cycle. Let the point $\boldsymbol{x}_{i-1}$ be a starting point and vector $\boldsymbol{p}_l$ represents a direction. If we try to find the minimum along this direction the next better approximation to the solution is a point $\boldsymbol{x}_i(x_{i1}, x_{i2}, \ldots, x_{in})$

$$\boldsymbol{x}_i = \boldsymbol{x}_{i-1} + h\boldsymbol{p}_l. \tag{4}$$

The point x_i is obtained from $\boldsymbol{x}_{i-1}$ point in optimization procedure of a function

$$\psi(h) = \varphi(\boldsymbol{x}_{i-1} + h\boldsymbol{p}_l) \tag{5}$$

along the direction $\boldsymbol{p}_l$ passing through the point $\boldsymbol{x}_{i-1}$. The corresponding scheme is shown in Fig. 2. In order to find a minimum value of the function ψ the parabolic approximation is used. Practically, the values of the function have to be computed at three points lying on a straight line $\boldsymbol{p}$. If the condition

$$\psi(0) < \psi(q) \tag{6}$$

is satisfied where q — trial step, the choice of points a, b, c, approximating the parabola is as follows

$$a = -q, b = 0, c = q, \tag{7}$$

because condition (6) suggests that the solution is to be sought along a vector, which has an opposite turn. The following transformation is, therefore, necessary

$$\boldsymbol{p} = :-\boldsymbol{p}. \tag{8}$$

In the case, when the condition (6) is not satisfied, which results in diminishing of the function value in the $\boldsymbol{p}$ direction, the points on a straight line are chosen in the following way

$$a = b, \; b = q, \; c = 2q \tag{9}$$

and the $\boldsymbol{p}$ vector remains unchanged in this case. A vertex point of the parabola given by the three points a, b, c lies at the point, whose coordinate is determined by the following value of the parameters h

$$h = \frac{1}{2}\frac{(b^2-c^2)\psi(a)+(c^2-a^2)\psi(b)+(a^2-b^2)\psi(c)}{(b-c)\psi(a)+(c-a)\psi(b)+(a-b)\psi(c)}. \tag{10}$$

Obviously, the parabola has a minimum at this vertex point, if the inequality

$$\frac{(b-c)\psi(a)+(c-a)\psi(b)+(b-c)\psi(c)}{(b-c)(c-a)(a-b)} < 0 \tag{11}$$

is satisfied. This choice of points according to (8) or (9) guarantees a positive value of the denominator in (11) so that the condition (11) may be replaced by the following inequality

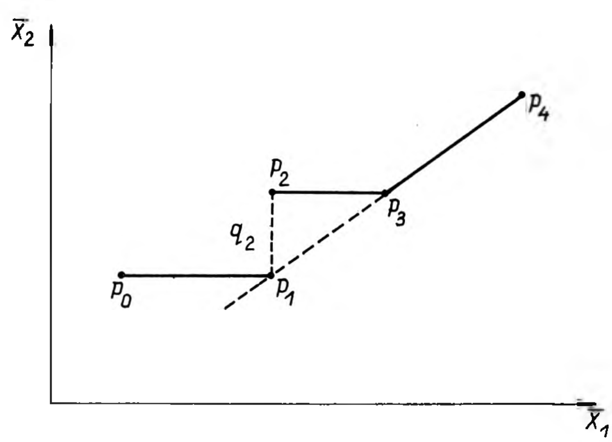

Fig. 3. A graphical presentation of the minimization procedure in two-dimensional space

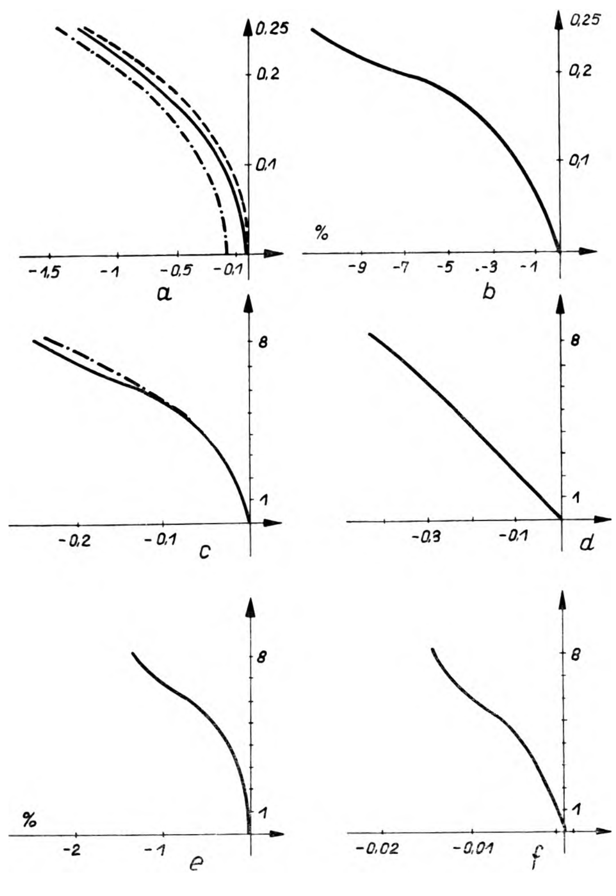

Fig. 4. Aberration curves of initial system
a) spherochromatic aberrations, b) sine condition, c) field curvature and astigmatism, d) meridian coma, e) distortion, f) chromatic difference of magnification

$$(b-c)\psi(a)+(c-a)\psi(b)+(a-b)\psi(c) < 0. \quad (12)$$

If the condition is not fulfilled the following transformation is to be carried out

$$a = b, \quad b = c, \quad c = c+q \quad (13)$$

and computations are continued.

A condition sufficient to finish the minimization procedure pursuant to a given direction $\boldsymbol{p}$ requires the parameter changes not to be smaller than the manufacturing tolerances.

3. Construction of the Merit Function

The merit function must be defined in such a way that the smaller value of the function the better quality of the optical system. As regards this procedure it is essential to properly choose both the aberration functions f_i and the weighting factors, which influence the computing procedure.

The proper choice of functions f_i is not difficult for an experienced lens designer but the optimum adjustment of weighting factors remains unsolved. An intuitive adjustment is difficult, time-consuming and in most cases not adequate at all.

In a further part of this paper we introduce an automatized method of choosing the weighting factors w_i. We propose to classify all the defects of optical systems like aberrations, deviations of focal lenght and magnification, technological tolerances etc. into three groups:

1. Defects, which have to be diminished to satisfy the following conditions

$$\text{abs}(f_i - f_{is}) < \Delta f_i, \quad (14)$$

where abs — absolute value, Δf_i — admissible tolerance on i-th defect.

This group is represented by the defects, which have to be led to and then kept within required tolerances. The group will be represented by such defects as, for instance, focus defect, magnification deviation and these aberrations, which have to be compensated if the system to be optimized is predicted to cooperate with the other optical systems of definite aberrations, e.g. the chromatic difference of magnification in the microscopic objectives have to be compensated by the eye-piece.

2. Aberrations, which should satisfy the conditions

$$\text{abs} f_{rs} > \text{abs} f_r. \quad (15)$$

It is clear from (15) that the majority of geometrical aberrations belong to this group.

3. Aberrations which always satisfy conditions (15) including the beginning stage of calculations of aberrations consist of all boundary conditions and other aberrations of the optical system, the values of which are not greater than those allowed. Weighting factors w_i should be chosen in such a way that a proportional contribution of each individual component to the merit function is ensured.

The proper weighting was realized by the following choice of the weighting factors (the treatment hereafter is concerned with aberrations of the first and the second group only).

The initial weighting factors w_{i0}

$$w_i = : w_{i0} = : 1/(f_{i0} - f_{is})^2, \quad (16)$$

where f_{i0} — the initial value of i-th aberration of initial system are selected in this way.

By assuming the weighting factors in the form determined by (16) each component of function φ is normalized to unity at the beginning of the procedure. But the different degree of non-linearity of functions

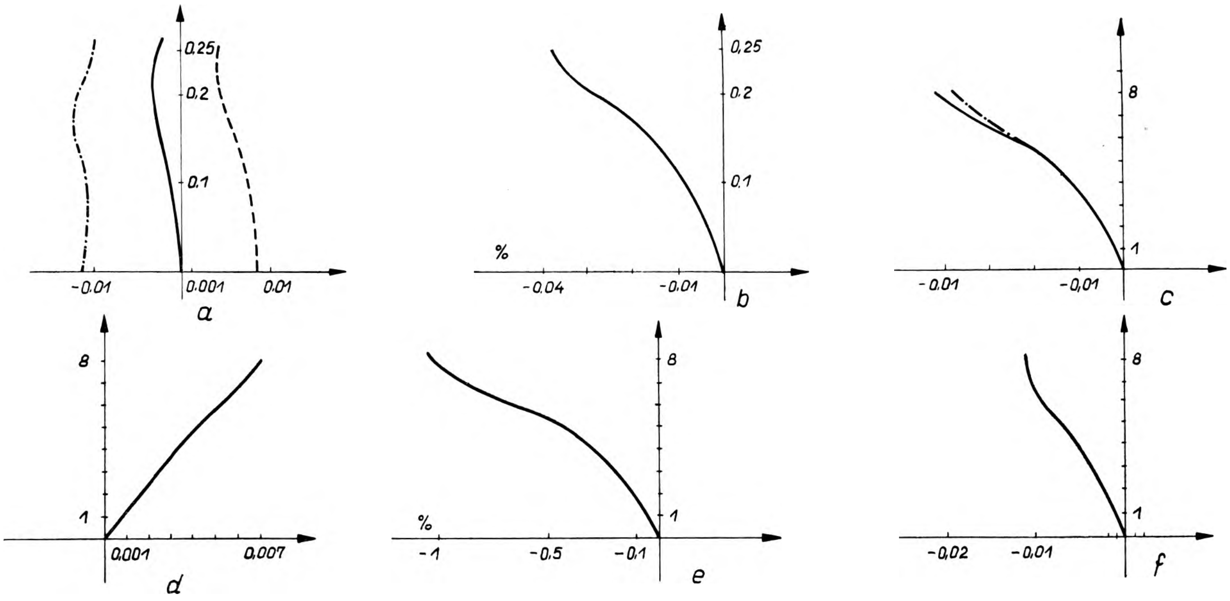

Fig. 5. Aberration curves as in Fig. 4 but for the system parameters from Table II

f_i and mutual relations between individual aberrations cause that the initial weighting factors w_{i0} may not assure a proper weighting of aberrations in further cycles of the correction process. In this instance, the weighting factors should be accordingly adjusted. The following correction has been proposed in this paper

$$w_i =: w_i\left(\frac{f_{is}-f_i}{f_{is}-f_{i0}}\right)^2, \tag{17}$$

where the earlier decomposition into aberration groups is still valid. For aberrations with indices $j > k$ the weighting factors were originally chosen as follows:

$$w_j =: w_{j0} =: 1/\mathrm{abs}(f_{j0}-f_{js}) \tag{19}$$

and then corrected by relations

$$w_j =: w_j \mathrm{abs}\left[(f_{js}-f_j)/(f_{js}-f_{j0})\right]. \tag{20}$$

Table I. Parameters of an initial system

Surface No.	1	2	3	4	5	6	7
Distances	2	1	1	1	1	1	0
Curvatures	.1428	0	0	0	0	0	0
Glasses	PSK 1	SF 8	AIR	LAK 9	AIR	LAF 3	AIR

which is valid for aberrations of the first and the second group not yet satisfying the corresponding conditions ((14), (15)).

However, these corrections are not supposed to be done oftener than after each cycle. It appears that the correction (17) does not guarantee a uniform diminution of all the aberrations even, if it is carried out after each cycle. In order to improve this disproportion and to have a better adjustment of the merit function to the non-linearization method used, the following form of the merit function is proposed

$$\varphi' = \sum_{i=1}^{k} w_i(f_i-f_{is})^2 + \sum_{j=k+1}^{m} w_i \cdot \mathrm{abs}(f_j-f_{js}), \tag{18}$$

The form of the merit function described by the relations (18) and proper choice of the weighting factors done on the basis of (16), (17), (19) and (20) ensured a more uniform convergence of aberrations to the desired values.

4. Computation Results

This method of optimization has been programmed for the "Odra-1204" computer (10^5 operations per second) and the results obtained are presented below. For the sake of illustration the method has been applied to design a flat field microscope objective (mag-

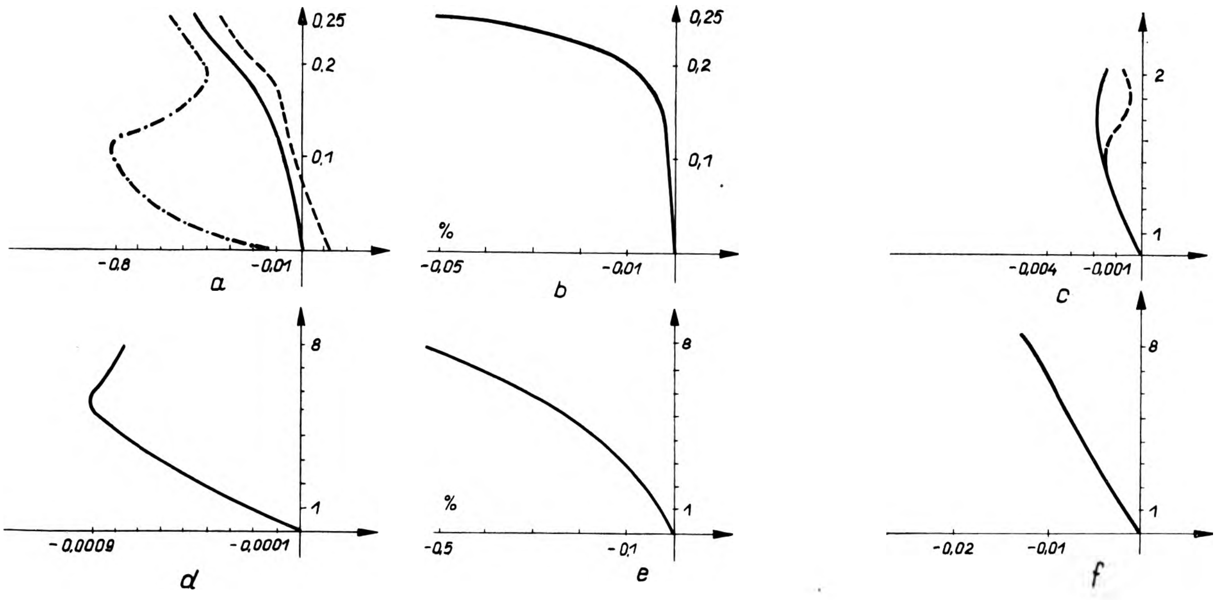

Fig. 6. Aberration curves as in Fig. 4 for a system with parameters from Table III

Table II. Parameters of a system obtained on the base of the merit function (2)

Surface No.	1	2	3	4	5	6	7
Distances	1.5	12.187	2.988	2.59	0.22	8.001	0
Curvatures	0.0722	−0.1001	−0.0044	0.0813	0.0145	0.1255	0.2181
Glasses	PSK 1	SF 8	AIR	LAK 9	AIR	LAF 3	AIR

Table III. Parameters of a system obtained after correction with the help of the merit function (18) for $k = 0$

Surface No.	1	2	3	4	5	6	7
Distances	3.1	1.812	15.041	1.626	.221	8.102	0
Curvatures	.0558	−.0584	−.0008	.0828	.0214	.125	.2188
Glasses	PSK 1	SF 8	AIR	LAK 9	AIR	LAF 3	AIR

gnification 5*). To show the rate of convergence of the employed method the system consisted of one lens with a finite focal length and three glass plates (of selected glasses) were chosen as an initial system. Parameters of the initial system are shown in Table I, and corresponding aberrations in Fig. 4:

a) longitual spherochromatic aberrations,
b) sine condition,
c) astigmatism and field curvature,
d) meridian coma,
e) distortion,
f) chromatic difference of magnification.

These aberrations have been corrected at the initial stage of the automatic correction process. Parameters of the system, which have been obtained after two iterations, are presented in Table II. In this case the optical system was corrected on the base of a merit function of the form (15) with the weighting factors determined by (16) and (17). The aberration curves in Fig. 5 characterize this optical system. Table III contains aberrations of a system, which has been obtained from the merit function (18) for $k = 0$ (all components of merit function were weighted with the help of relations (19) and (20). The aberrations of

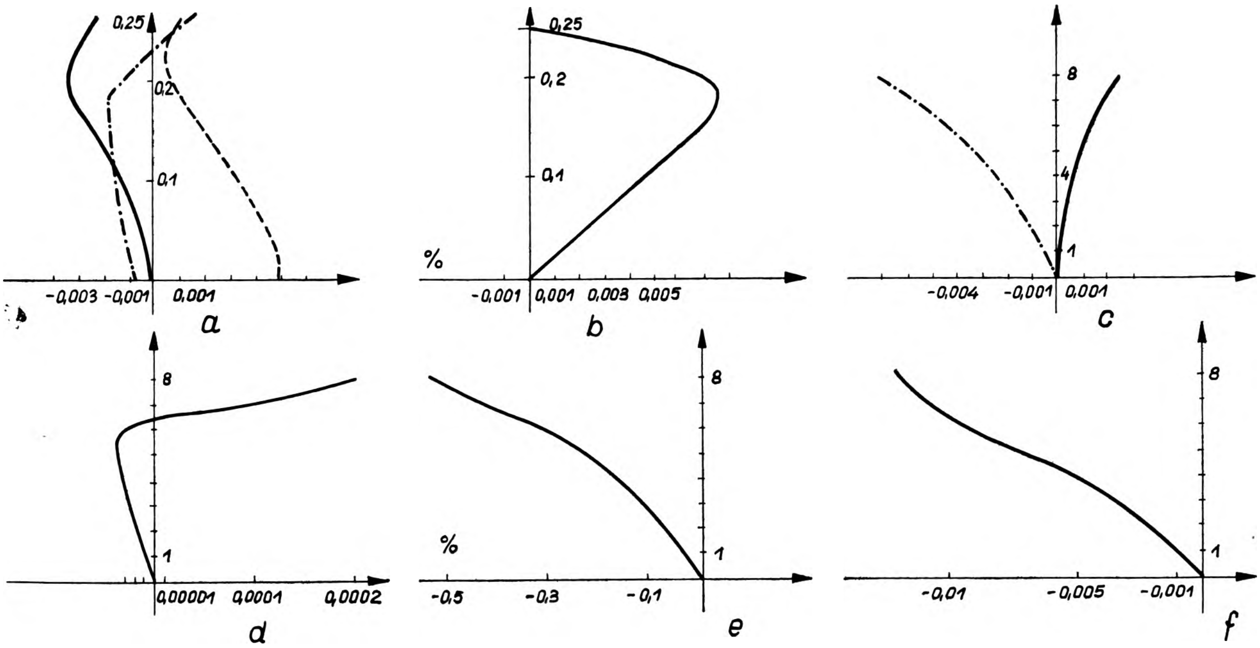

Fig. 7. Aberration curves as in Fig. 4 for a system with parameters from Table IV

Table IV. Parameters of a system obtained after correction on the base of the merit function (18) for $0 < k < m$

Surface No.	1	2	3	4	5	6	7
Distances	14.153	9.687	3.04	0.223	2.649	7.02	0
Curvatures	.0586	−.1036	0.0011	0.0809	0.0146	0.125	0.1542
Glasses	PSK 1	SF 8	AIR	LAK 9	AIR	LAF 3	AIR

the system are illustrated in Fig. 6. Parameters of the system, which has been obtained with the help of the function (18) (for $0 < k < m$) are shown in Table IV. In this case the function was composed of weighted squared deviations and weighted absolute deviations of aberrations (the corresponding aberration curves are in Fig. 7).

A "shifting" of any aberration from the "modulus" group to the "squared" group or vice versa has been executed only if the improvement of this aberration during the correction process was not satisfactory, though the change of the weighting factors connected with this aberration was great. Pursuant to presented aberration curves it is obvious that the best results are obtained, when the minimization method is based on the merit function (18). It is also obvious that the convergence of the optimization method referred to depends on the accepted permutation of the parameters x_i, $i = 1,2, \ldots, n$.

As regards a system consisting of a great number of surfaces it is profitable to initiate the design with a permutation, in which low indices are assigned to the parameters from the last part of the system. It enables to retain a great deal of current data in the computer memory, which may next be used repeatedly in further computations.

Thus, the necessity to trace rays over the whole optical system (from the object space to the image plane) is removed. Observing a design process one may eventually change permutation of parameters when results are not up to requirements.

This paper contains a part of investigations on this subject. It will be possible to do improvements upon these processes only after computer of greater capacity and higher speedes will be available.

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