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An Analysis of the Possibility of Applying Classical Formulae to the Position of Astigmatic Foci to Decentred Systems Assuming a Small Error of Centricity

A settlement of the tolerances for decentricity of an optical system assemblage requires a detailed analysis of the influence of small errors of centricity on the imaging quality. Methods of calculating the majority of geometrical aberrations (like coma, distorsion, chromatic differences of magnification) do not essentially differ from those employed for testing perfectly centric systems. In the case of field curvature and astigmatism the formulae (1) for the positions of astigmatic foci (S'_m and S'_s) of an infinitesimally thin beam of rays have been derived assuming that the chief ray of the beam lies in the plane of symmetry of the system (cf., for instance, [1], [2]). Examinations carried out previously concerned the influence of decentricity on the wave aberration and coma [3] without paying any detailed attention to the field curvature and astigmatism. Calculations were performed for the systems exhibiting a plane of symmetry [4] or were restricted to the region of third order aberrations [5], [6]. The problem was recently considered by Ch. Hofmann and J. Klebe [7], [8] who derived some general relations allowing to trace a thin astigmatic beam through arbitrarily decentred surfaces of arbitrary shape. Some formulae for the change of astigmatic foci caused by small decentricities in the optical systems consisting of spherical surfaces were also given in these papers. It was concluded from examples of calculations performed for the objectives of a special type that small decentricities in the sagittal plane have practically no influence on the change of the astigmatic foci positions in the system. In our opinion this conclusion is premature. It has not been proven that the conclusion is in accordance with calculations carried out with the help of other method for the microscopic objectives.

Relations obtained in paper [9] by S. C. Parker — concerning the caustic surfaces of ray beams which have passed the optical system — allow to determine the position of astigmatic foci. The given formulae have, however, a very general form and do not permit to formulate any conclusions as to the behaviour of the astigmatic beam in slightly decentred systems.

A possibility of applying the classical formulae to the position of the sagittal foci $|S'_m, S'_s|$:

$$\frac{n'_k \cos^2 i'_k}{S'_{mk}} - \frac{n_k \cos^2 i_k}{S_{mk}} = \frac{n'_k \cos i'_k - n_k \cos i_k}{r_k},$$

$$\frac{n'_k}{S'_{sk}} - \frac{n_k}{S_{sk}} = \frac{n'_k \cos i'_k - n_k \cos i_k}{r_k}, \tag{1}$$

$$S_{mk} = S'_{m\,k-1} - \bar{d}_k \qquad S_{sk} = S'_{s\,k-1} - \bar{d}_k,$$

where:

r_k — curvature radius of the surface,

n_k, n'_k — refractive indices in front of and behind the surface, respectively,

i_k, i'_k — angle of incidence and angle of refraction of the field ray,

$\bar{d}_k$ — distance between the surfaces along the field ray,

to the image curvature change examination caused by small decentricities would considerably simplify the work of the designers, when calculating the tolerances of both the centricity and assemblage of the optical systems.

This paper indicates — on the basis of performed analysis — that such a possibility exists and the decentricity range has been determined, for which formulae (1) gives correct results for field curvature and astigmatism calculation. The conclusions were for-

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mulated on the basis of a comparison of the results of position calculations carried out according to formulae (1) for the astigmatic foci of a infinitesimally thin beam of rays with those obtained with the help of the comparative method starting from the definition.

The comparative method consisted in finding the intersecting points of the two pairs of rays which are close to the field ray after their passage through the optical system. In the object space the pairs of rays are chosen in such a way that one lies in the meridional plane and the other in the sagittal plane of the system. The distances x'_m and x'_s of the estimated intersection points from the perfect image plane were assumed to be a measure of the meridional and sagittal curvature for the accepted field angle.

As, in reality, the ray beam calculated this way was not infinitesimally thin it is necessary to find out, for which angles of its divergence the most proper results of calculations of the astigmatic focus position can be obtained. For this purpose the field curvature and astigmatism was computed for several perfectly centred microscopic objectives by using formulae (1) and the coordinates of the intersection points of the corresponding pairs of rays representing the beams of different divergences and equal successively to 10^{-1}, 10^{-2}, 10^{-3}, 10^{-4} of the entire angle were estimated for the same systems. It has been stated that the most proper results (the error not exceeding 1%) are obtained for the beam divergences contained within the limits 10^{-2} — 10^{-3} of the complete aperture angle depending on the magnification, aperture and the optical system aberrations. For greater beam divergences the results are incorrect, which is in accordance with expectations, because the field curvature and astigmatism are disturbed by other aberrations. On the other hand the thinning of the beam causes considerable errors due to the increasing role of computer errors.

Examples of calculation results for beams of different divergence for the microscopic objectives of magnifications 10x (I) and 40x (II) are presented in Table 1. The first column of the Table shows results of calculations of field curvature and astigmatism performed on the basis of (1).

To check the accuracy of the calculation it has been verified whether the coordinates of the found intersection points of both ray pairs fulfil the equation of the straight line corresponding to the principle ray of the beam. A satisfactory accuracy has been achieved for the beam divergences, where the results of the field curvature and astigmatism calculations were the most exact.

Table 1

		1	2	3	4	5
			$10^{-1}A$	$10^{-2}A$	$10^{-3}A$	$10^{-4}A$
I	x'_m	−0.0451	−0.0629	−0.0396	−0.0439	—
	x'_s	0.0947	0.0790	0.0955	0.0856	—
	$x'_m - x'_s$	−0.1398	−0.1419	−0.1351	−0.1295	—
II	x'_m	−0.5057	2.955	2.862	−0.5073	−0.5
	x'_s	−0.3905	2.993	2.896	−0.3915	−0.4
	$x'_m - x'_s$	−0.1152	−0.038	−0.034	−0.1158	−0.1

In the case of a system containing surfaces decentred in arbitrary directions the pairs of rays in the image space are oblique with respect to each other and to the principle ray. The astigmatic foci will then have a sense of the positions of the greatest narrowings of the light beams. To find the position of these greatest narrowings for both the ray pairs traced the respective calculations of the coordinate of the point located on the first ray with respect to the perfect image plane have been made; the point fulfilling the condition of a minimum distance from the other ray lying in the same plane in the object space. With the help of the same method a point on the principal ray was found, which satisfies the condition of a minimum distance from each of the four calculated rays. Three different values for the meridional focus position as well as three different values for the sagittal focus position have been obtained.

By analyzing the dispersion of the received values for the case of different decentricities it has been stated that the deviations from the average value does not exceed 1%. Simultaneously, the position of the astigmatic foci have been estimated on the basis of the ordinary formulae for the coordinates (in this case — coordinates of the intersection points of the Gauss plane). The x-coordinate of the hit point for the rays given in such a way defines the position of the corresponding astigmatic focus. The results of these calculations did not differ from those obtained for the position of the greatest narrowing points by more than 1 %.

It has, therefore, been assumed that for a sufficiently small divergence angle the corresponding pairs of rays after passing the decentred system intersect each other (i.e. — that the smallest distance between the rays falls within the limits of the calculation tolerances). For such an assumption the position of the astigmatic foci in the decentred system has been calculated by applying for both the ray pairs ordinary formulae for the intersection coordinate for straight lines. Several microscopic objestives of different types and magnifications have been calculated in this manner. The optimal width of the given beam was fixed individually for each objective.

Simultaneously, the calculation for the astigmatic foci positions were performed for each case of the decentred system by using the formulae (1). To determine the limit of applicability of these formulae the systems of surface decentricities Δc have been investigated within the range from 0.005 up to 0.5 (the measure of the decentricity Δc is a distance of the curvature centre of the de centred surface from the axis determined by the mechanical axis of the holder) and in different directions defined by the asimuth $\Theta(0° \leqslant \Theta \leqslant 360°)$. If the decentricities occur in the meridional plane of the system ($\Theta = 0$) the results of calculations carried out with the two methods are similar, they show a great accuracy to each other for the whole range of introduced decentricities. As regards the case of $\Theta \neq 0$ it has been proved that the formulae give correct results for the positions of the astigmatic foci (i.e. the results consistent with those obtained on the basis of calculation of the intersection points of the ray pairs) only up to some value of the decentricities introduced.

These limiting value oscillate between the decentricities equal to 0.03 mm in the well-corrected system and for sensitive surfaces up to 0.05 mm in the systems poorly corrected and for the surfaces, which are less sensitive. Usually decentricities appearing in the course of the optical system assembling do not exceed the values given above. Thus, it is possible to apply with sufficient accuracy the classical formulae for the positions of astigmatic foci to examine the influence of small decentricities on the field curvature and astigmatism of the optical systems. Because of the considerable simplicity of these formulae in comparison to the relations given by Ch. Hofmann and J. Klebe they may radically simplify the work of lens designers as far as the determining of the tolerances of single surfaces and the whole elements of the optical system are concerned.

In Tables 2 and 3 some exemplified results of positions of the astigmatic foci in the objectives of magnifications 10x and 40x are given for different decentricities of single surfaces as well as for the case of many surfaces decentred simultaneously in different directions.

Table 2

1	ΔC [mm]	Θ	4	5	6	7
1	0.01	0°	0.0369	0.0528	0.0366	0.0521
5	0.01	0°				
6	0.01	0°				
7	0.01	0°				
1	0.01	90°	−0.0681	0.0819	−0.689	0.0825
5	0.01	320°				
6	0.01	90°				
7	0.01	225°				
1	0.05	0°	0.3940	−0.0978	0.3940	−0.0981
5	0.05	0°				
6	0.05	0°				
7	0.05	0°				
1	0.05	90°	−0.1308	0.0477	−0.1422	0.05924
5	0.05	320°				
6	0.05	90°				
7	0.05	225°				

Column 1 — number of the surface
column 4,5 — position of the astigmatic foci x_m and x_s calculated from the classical formulae,
column 6,7 — position of the astigmatic foci x_m and x_s calculated from the ray intersection condition.

Table 3

1	ΔC [mm]	Θ	4	5	6	7
6	0.01	0°	−0.3405	−0.2045	−0.3408	−0.2050
8	0.01	0°				
10	0.01	0°				
11	0.01	0°				
6	0.01	45°	−0.2062	−0.1925	−0.2136	−0.1736
8	0.01	135°				
10	0.01	225°				
11	0.01	320°				
6	0.05	0°	0.2626	0.4895	0.2613	0.4877
8	0.05	0°				
10	0.05	0°				
11	0.05	0°				
6	0.05	45°	1.2359	0.6905	1.0327	−0.1236
8	0.05	135°				
10	0.05	225°				
11	0.05	320°				
4	0.02	90°				
8	0.02	90°	−0.3518	−0.4455	−0.5605	−0.2365

The meaning of these columns is the same as shown in Table 2.

References

[1] Born M., Wolf E., Principles of Optics, London 1964.
[2] Tudorovsky A. J., Teorija optičeskich priborov, Moskva 1952.
[3] Hopkins H. H., Tiziani H. J., Brit. J. Appl. Phys. **17** (1966), 33–55.
[4] Chojnacka A., Kryszczyński T., Biuletyn Inform. Optyka CLO No. 2, Warszawa 1968.
[5] Bartkowska J., Optica Applicata I (1971) No. 1.
[6] Slevogt H., Optik **20** (1963), 488–496.
[7] Hofmann Ch., Klebe J., Jenaer Jahrbuch 1964.
[8] Hofmann Ch., Klebe J., Optik **22** (1965), 95–122.
[9] Parker S. C., Properties and Application of Generalized Ray Find, University of Arisona, Technical Rapport, November 1971.