

## On Interpretation of the Plasma Beam Holographic Interferograms

A method of finding an analytic function for a proper description of a set experimental data obtained by measuring the phase shift occurring in a phase object with an interferometer is presented. The proposed method of hologram interpretation is next applied to determine the temperature distribution in the plasma jet.

The holographic interferometry is becoming more and more frequently used in interferometric examination of plasma. The introduction of lasers allowed to increase considerably the accuracy of interferometric metrology. Moreover, this enabled a development of some new examination methods, e.g. holographic interferometry in real time and by double exposure. The holographic interferometry in real time is particularly useful for investigation of phase objects, as it allows to observe the interference image in a continuous way. The optical scheme of the systems registering the interferograms in real time is shown in Fig. 1.

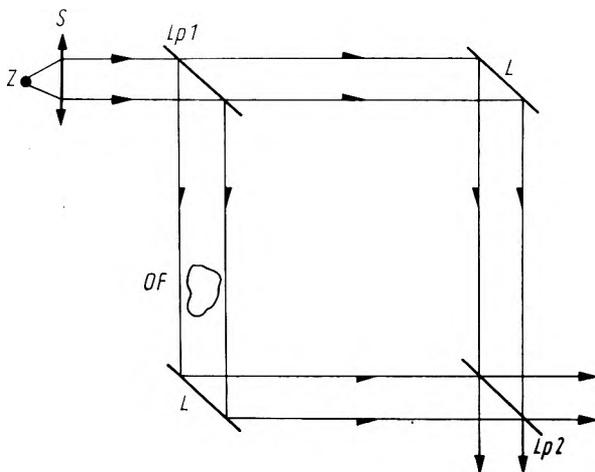


Fig. 1. A scheme of Mach-Zehnder interferometer  $L$  – totally reflecting mirrors,  $L_{p1}$ ,  $L_{p2}$  – half-transparent mirrors,  $S$  – lens,  $Z_f$  – source of light,  $OF$  – phase object

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The interference image produced in such an interferometer is based on a principle similar to that of Mach-Zehnder interferometer presented in Fig. 2.

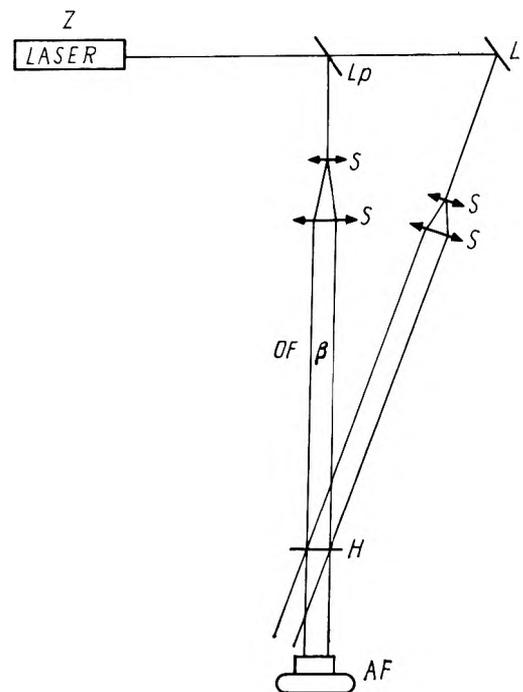


Fig. 2. Optical system of the holographic interferometer  $L$  – totally reflecting mirror,  $L_p$  – half-transparent mirror,  $S$  – lens,  $OF$  – phase object,  $H$  – hologram,  $AF$  – photographic camera

It may be noticed that there exists a correspondence between the Mach-Zehnder and the holographic interferometers. The wave traveling along the arm of the Mach-Zehnder interferometer, without the phase object, corresponds to the wave reconstructed from the hologram

in a holographic interferometer, while the wave travelling along the arm with the phase object in the Mach-Zehnder interferometer, corresponds to the wave passing through the object under test and the hologram in the holographic interferometer. An interferometric image of the plasma jet and that of the stabilized glow, both obtained by the holographic interferometry in the real time, are presented in Figs. 3 and 4. The plasma jet was produced in a plasmatron for the 250 A current and 250 l/h argon flow. At room temperature the



Fig. 3. Plasma jet interferogram

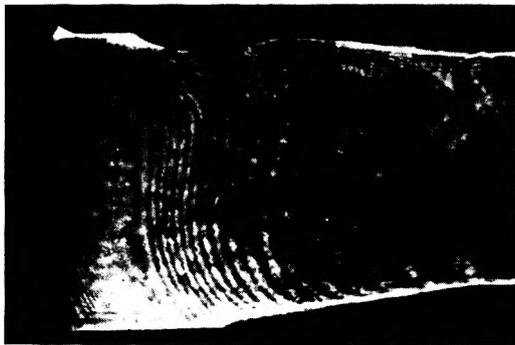


Fig. 4. Stabilized glow interferogram

refractive indices of argon, nitrogen and oxygen are close to each other, provided that their pressures are the same [5]. Consequently, a replacement of oxygen-nitrogen mixture by argon does not change practically the optical path of the light passing through such a region. The interference image appears first after introducing the argon plasma into this region. This means that it is the argon plasma that influences strongly the optical path difference of the inter-

fering waves as the results of the respective changes which occur in the refractive index of the medium, due to both temperature and ionization-induced variation in gas density. Hence, it follows that the interferograms of the jet contain information on its temperature. If the plasma exhibits a cylindrical symmetry around the  $z$ -axis of the coordinate system, then the plasma column cross-section in the  $x, y$ -plane has the property that the points equidistant from the origin of the coordinate system are of the same temperature  $T(r)$ , density  $\rho(r)$  and refraction  $(n(r) - 1)$  (see Fig. 5). The laser light passing parallelly to the  $y$ -axis through the medium of the refractive index  $n(r)$ , determined with respect to the surroundings, is subjected to optical path change. The respective phase shift  $f(x)$  is given by the formula

$$f(x) = 2 \int_0^{\sqrt{R^2 - x^2}} (n_0 - n(r)) dy. \quad (1)$$

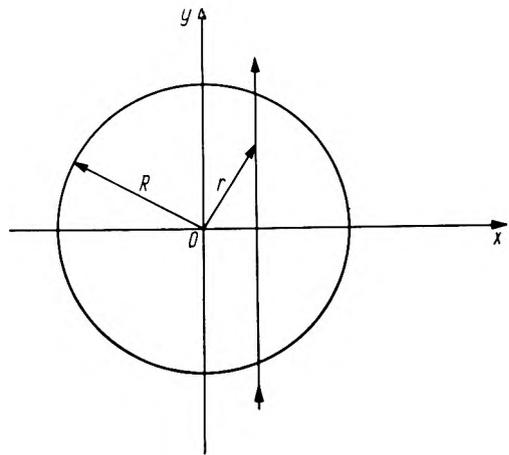


Fig. 5.

Its value may be read out of the interferogram. Much more important, however, is the distribution  $n(r)$  of the refractive index. In order to determine the latter it is necessary to solve the equation (1). This may be done by using the reverse Abel transformation [3]. Then the solution is of the form

$$n_0 - n(r) = \frac{1}{\pi} \int_r^R \frac{df(x)}{\sqrt{x^2 - r^2}} dx \quad (2)$$

and is, as easily seen, determined by the derivative  $\frac{df(x)}{dx}$ . Consequently, it becomes necessary to find the analytic form of  $f(x)$ , to the measurement data  $f(x_i)$  given by the experiment. The

values of  $f(x_i)$  read from the interferogram suffer from measurement errors. Consequently, the sought matching function  $f(x)$  should satisfy two opposite requirements. It should well approximate the experimental values at the measuring points, but at the same time it need not reproduce them too precisely, as the latter are charged with errors. Thus, the matching function should smoothen the experimental data. This condition is particularly important if in further calculation not the function but its derivative is used. If the matched function was led through the experimental points (charged with errors) then considerable oscillations in the value of the tangent of the slope angle may occur, resulting in unexpected maxima and minima of the derivative. Thus, the derivative may differ considerably at some places from that of the true experimental curve, of which it is known, for instance, that it should be regular.

The problem stated in this way is a typical example of approximation of the  $f(x)$  function by a combination (mostly linear) of the function  $g_k(x)$  belonging to a class  $\{g_k(x)\}$   $k = 1, 2, \dots, m$ . In this case the function  $f(x)$  can be put into the form

$$f(x) \simeq a_0 g_0(x) + a_1 g_1(x) + \dots + a_m g_m(x), \quad (3)$$

where  $a_0, \dots, a_m$  are constant coefficients. Some rational approximations of the form

$$f(x) = \frac{a_0 g_0(x) + a_1 g_1(x) + \dots + a_m g_m(x)}{b_0 g_0(x) + b_1 g_1(x) + \dots + b_m g_m(x)} \quad (4)$$

may also be applied. These, however, are much more difficult to realize than the linear ones.

The approximations may be divided into some categories; depending on the way the constants  $a_0, \dots, a_m$  are chosen:

i) The interpolation approximations, in which the coefficients are chosen so that the approximating and approximated functions have the same derivatives and the same values at certain points  $x_i$  ( $i = 1, \dots, N$ ) called nodes;

ii) the r.m.s. approximations in which the minimum of the sum of squared differences between the values of the approximated and approximating functions are sought for a certain set of (measurement) point;

iii) the uniform approximation, which allows to find the least value of the maximum discrepancy between the approximating and approximated functions.

The interpolation approximation is used when the nodal point values of the function

approximated are known exactly and the intermediate point values are required.

The uniform approximation is used when the error of approximate value of the function is requested to be not greater than the allowed one within the whole interval  $\langle a, b \rangle$ .

In the case of a discrete set of values of the function  $f(x_i)$ ,  $i = 1, \dots, N$ , which are known up to certain errors, the r.m.s. approximation appears to be the most convenient method.

## 1. The r.m.s. approximation

For a linear approximation a class of function  $\{g_j(x)\}$ , ( $j = 0, \dots, m$ ) should be chosen according to the formula (3). In order to solve the integral equation (2) it is necessary to know the derivative of the function. Therefore, the analytic form of the approximating function is desired. This allows to calculate the derivative as well as to perform simple integration. The class of functions satisfying the above conditions are polynomials. Their integration and differentiation being simple they assure also a satisfactory accuracy of approximation. This follows from the Weierstrass theorem which states:

*If the function  $f(x)$  is continuous within the finite interval  $\langle a, b \rangle$  then for each  $\varepsilon > 0$  there exist an integer  $n = n(\varepsilon)$  and a polynomial  $P_n$  of the order  $n$  such that  $|f(x) - P_n(x)| < \varepsilon$  for all  $x \in \langle a, b \rangle$ .*

Let  $f(x)$  denote the function, which is to be approximated and let  $\{x_i\}$ , ( $i = 1, \dots, N$ ) be a set of those points at which the values of the function  $\bar{f}(x_i) \equiv \bar{f}_i$  have been measured with some error. The measurement error is  $E_i = f(x_i) - f_i$ , where the true and measured values of the function at the measurement point  $x_i$ , are denoted by  $f(x_i)$  and  $f_i$ , respectively.

Let  $\{g_j(x)\}$ , ( $j = 0, \dots, m$ ) be a finite sequence of functions determined for each  $x_i$ . The approximation to the function with the values  $\bar{f}_i$ , performed by the linear combination of functions belonging to the class  $g_j(x)$  may be written in the form

$$\bar{f} \simeq y_m(x_i) = \sum_{j=0}^m a_j^{(m)} g_j(x_i). \quad (5)$$

The index  $(m)$  at the coefficients  $a_j$  underlines their dependence on  $m$ .

The r.m.s. approximation consists in such a choice of the coefficients  $a_j^{(m)}$  that the value of the expression

$$H(a_0^{(m)}, a_1^{(m)}, \dots, a_m^{(m)}) = \sum_{i=1}^N w(x_i) \left( \bar{f}_i - \sum_{j=0}^m a_j^{(m)} g_j(x_i) \right)^2 \quad (6)$$

be the smallest;  $w(x)$  denotes the weighting function. This function determines the contribution of the separate experimental values to the expression (6), and may depend on the error made while measuring  $\bar{f}_i$ .

If, e.g.  $g_j(x) = x^j$ , ( $j = 0, \dots, m$ ), then the approximation takes the form

$$f(x) = y_m(x) = \sum_{j=0}^m a_j^{(m)} x^j. \quad (7)$$

The right-hand side of this equation is a polynomial of  $m$ -th order. Having  $N$  experimental data and applying the polynomial of a suitably high order ( $m+1 \geq N$ ) it is possible to carry the function  $y_m(x)$  through all the experimental points. Such an approximation, however, reproduces exactly the experimental values of the function, which are burdened with errors. This approximation does not "smoothen" the experimental data. When, moreover, the polynomials of higher order are employed, the values between separate measurement points, taken by the approximating function, may sometimes be considerably different from the expected ones. Hence, to approximate the experimental data it is recommended to use the polynomials of the order  $m+1 < N$ . This means a worse fitting of the function to the experimental data, but the course of approximating function is closer to the real one.

The value  $H(a_0^{(m)}, \dots, a_m^{(m)})$  determined by the formula (6) depends upon the coefficients  $a_j^{(m)}$ . Let  $a_j^{(m)}$  be variables of the function  $H$ . Then at the minimum of this function its partial derivatives with respect to the  $a_j^{(m)}$  will be equal to zero

$$\frac{\partial H}{\partial a_k^{(m)}} = -2 \sum_{i=1}^N w(x_i) \times \left( \bar{f}_i - \sum_{j=0}^m a_j^{(m)} g_j(x_i) \right) g_k(x_i) = 0. \quad (8)$$

This is a system of  $m+1$  equations called the normal system with  $m+1$  unknown coeffi-

cients  $a_k^{(m)}$ . If its determinant is different from zero then by solution of (8) the coefficients  $a_k^{(m)}$  may be determined.

If  $g_k(x) = x^k$  and  $w(x_i) = 1$ , then the system (8) will take the form

$$\sum_{i=1}^N \left( \bar{f}_i - \sum_{j=0}^m a_j^{(m)} x_i^j \right) x_i^k = 0, \quad (9)$$

$$k = 0, 1, \dots, m.$$

By a suitable rearrangement we get

$$\sum_{j=0}^m a_j^{(m)} \left( \sum_{i=1}^N x_i^{j+k} \right) = \sum_{i=1}^N \bar{f}_i x_i^k, \quad (10)$$

$$k = 0, 1, \dots, m.$$

Denoting

$$u_{jk} = \sum_{i=1}^N x_i^{j+k}, \quad z_k = \sum_{i=1}^N \bar{f}_i x_i^k \quad (11)$$

the normal system may be written in the form

$$\sum_{j=0}^m u_{jk} a_j^{(m)} = Z_k, \quad k = 0, 1, \dots, m. \quad (12)$$

Let, for instance,  $N$  experimental points be uniformly distributed within the interval  $\langle 0, 1 \rangle$ . Hence, it may be assumed that

$$u_{jk} = \sum_{i=1}^N x_i^{j+k} \simeq n \int_0^1 x^{j+k} dx = \frac{n}{j+k+1},$$

$$j, k = 0, 1, \dots, m. \quad (13)$$

The matrix  $U = [U_{jk}]$  of the normal system (12) takes the form

$$U = n \begin{vmatrix} 1 & \frac{1}{2} & \dots & \frac{1}{m+1} \\ \frac{1}{2} & \frac{1}{3} & \dots & \frac{1}{m+2} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1}{m+1} & \frac{1}{m+2} & \dots & \frac{1}{2m+1} \end{vmatrix}. \quad (14)$$

This is an example of an ill-conditioned matrix; i.e. the inverse matrix of the matrix normed so that its greatest element be of order of unity, contains very great elements. This yields considerable errors, due to the fact, that while solving the normal system of equations, the number in the computer were rounded. For  $m = 9$  the matrix inverse to (14) includes the elements of the order  $3 \cdot 10^{12}$ .

Because of the above limitations for the polynomial approximation of the form (7) numerical calculations may be applied for  $m$  not greater than 5 or 6.

## 2. Approximation by orthogonal polynomials

The troubles due to ill-conditioning of the normal system matrix may be avoided by using another class of functions  $\{g_j(x)\}$ ,  $j = 0, \dots, m$ . To this end the orthogonal polynomials may be applied. Let  $p_j(x)$  denote a polynomial of  $j$ -th order. The approximation, whose coefficients are to be determined, has the form similar to that of e.g. (7):

$$y_m(x) = \sum_{j=0}^m b_j^{(m)} p_j(x). \quad (15)$$

By following the rules of r.m.s. approximation (formulae (7) to (12)) the normal system may be presented in the form

$$\sum_{j=0}^m d_{jk} b_j^{(m)} = C_k, \quad k = 0, 1, \dots, m, \quad (16)$$

where

$$d_{jk} = \sum_{i=1}^N w(x_i) p_j(x_i) p_k(x_i),$$

$$C_k = \sum_{i=1}^N w(x_i) f_i p_k(x_i). \quad (17)$$

For arbitrary polynomials  $p_k(x)$  the difficulties with ill-conditioning of the matrix  $\{d_{jk}\}$  may be similar to those occurring in the case of ordinary polynomials discussed above. However, if the set of polynomials  $\{p_k(x)\}$  is defined and orthogonal in the set  $\{x_i\}$ , ( $i = 1, \dots, N$ ), i.e.

$$\sum_{i=1}^N w(x_i) p_j(x_i) p_k(x_i) = 0 \quad (18)$$

then  $d_{jk} = 0$  for  $j \neq k$ . Thus the system of equations (16) is reduced to

$$d_{kk} b_k^{(m)} = C_k, \quad (19)$$

$$k = 1, \dots, m$$

and has a simple solution

$$b_k^{(m)} = \frac{C_k}{d_{kk}}, \quad k = 0, 1, \dots, m.$$

Thus, the solution of the ill-conditioned normal system is avoided. It is also easy to find the coefficients  $b_k^{(m+1)}$  for a sequence of  $m+1$  orthogonal polynomials, namely

$$b_k^{(m+1)} = \frac{C_k}{d_{kk}}, \quad k = 0, 1, \dots, m.$$

Hence, it is clear that  $b_k^{(m)} = b_k^{(m+1)}$ , ( $k = 0, 1, \dots, m$ ), and this means that  $b_k$  does not depend on  $m$ . Consequently, to find  $b_k^{(m+1)}$ , it suffices to calculate  $c_{m+1}$  and  $d_{m+1, m+1}$ .

If the polynomials constructed are to be orthogonal in the set the recurrence formulae of the form [1] can be used:

$$p_{j+1}(x) = (x - \alpha_{j+1}) p_j(x) - \beta_j p_{j-1}(x), \quad (20)$$

where  $p_0(x) = 1$ , and  $p_{-1}(x) = 0$ ,  $\alpha_{j+1}$ ,  $\beta$  are constants, which may be determined from the following relations

$$\beta_k = \frac{\sum_{i=1}^N w(x_i) p_k^2(x_i)}{\sum_{i=1}^N w(x_i) p_{k-1}^2(x_i)},$$

$$\alpha_{k+1} = \frac{\sum_{i=1}^N w(x_i) x_i p_k^2(x_i)}{\sum_{i=1}^N w(x_i) p_k^2(x_i)}. \quad (21)$$

These relations may be programmed, and the coefficients  $b_k$  as well as the form of the orthogonal polynomials may be easily calculated on an electronic computer. Finally, by determining the coefficients  $d_k$ ,  $\beta_k$  and  $b_k$  and grouping them according to the powers of  $x$ , the approximation (15) may be written in the form of an ordinary polynomial

$$y_m(x) = \sum_{j=1}^m a_j x^j. \quad (22)$$

## 3. Selection of the order of polynomials

The last problem concerns the selection of the power of the approximation polynomial. Let the approximation of the function  $f(x)$  by a linear combination of the orthogonal polynomials  $p_k(x)$  be exact for  $m = M$ ; i.e.

$$f(x) = \sum_{i=0}^M b_i p_i(x).$$

This means also that the fitting is not improved by an increase in the number of polynomials approximating and that for the approximation

$$f(x) = \sum_{j=0}^{M+1} b_j p_j(x),$$

$b_{M+1} = 0$ . Hence, it follows that for exact fitting ( $m = M$ ) the value of

$$D^2 = \sum_{i=1}^N w(x_i) \left( \bar{f}_i - \sum_{j=0}^m b_j p_j(x_i) \right)^2 \quad (23)$$

should not depend on  $m$  for  $m > M$ .

This intuitive remark may be proved.

In selection of the degree of the approximating polynomial the so-called zero hypothesis may be used [1]. According to this hypothesis

$$B_m^2 = \frac{D^2}{N - m - 1} \quad (24)$$

does not depend of  $m$ , starting with  $m = M$  for which the approximation is optimal. In practice, the value of  $B_m^2$  for the given  $M$  is not equal to zero, because of the errors in the experimental values  $\bar{f}_i$ . The zero hypothesis allows, however, to determine the  $m = M$  for which the fitting is good enough and "smooth" enough, simultaneously. This may be done in the following way: the value of the term  $B_m^2 \cdot m$  should be calculated for the particular  $m$ . This value of  $m$ , for which  $B_m^2$  decreases negligibly will be the sought optimal order of the polynomial  $M$ .

#### 4. Experimental results

The above method may be employed to determine the temperature distribution in the plasma jet. The argon plasma stream was produced in a plasmatron for 250 l/h argon flow rate and 250 A current. An interferogram of such a laminar plasma flow is shown in Figs. 3 and 4. The optical system, settled according to schema shown in Fig. 2, allows to observe the interference image in the real time and to record it at an arbitrary moment. The time of interferogram registration was 1/500 s. Photometric analysis of the photographic plate with interferogram enabled to determine the position of fringes with an accuracy not less than 0.2 mm. The experimental data of the phase shift were approximated by orthogonal polynomials by the method presented in this work. For the polynomials of the order ranging from 1 to 16 the value of  $B_m^2$  was calculated according to the formula (24). From the results presented in Table 1, it follows, that starting with  $m = 10$ , the value

of  $B_m^2$  changes considerably slower than it does for  $m < 10$ .

Table 1

$m$	$B_m^2 \cdot 10^{-4}$	$m$	$B_m^2 \cdot 10^{-4}$
1	40140.0	9	15.43
2	584.4	10	4.006
3	595.4	11	4.094
4	262.5	12	4.000
5	267.7	13	4.093
6	25.63	14	4.069
7	26.15	15	4.169
8	15.11	16	4.124

In accordance with former remarks the tenth power polynomial has been accepted as the optimum. Fig. 8 presents the graph of the accepted approximating polynomial with the marked experimental points. The photometric analysis allows to determine the position of fringes with a small error. Consequently, the values of the phase shift at certain positions are burdened with errors. In this case the difference between the phase shift determined experimentally and that calculated from the approximating function has been assumed as a measure of the approximation quality. These differences should be smaller than the error committed while determining the experimental data. This was the case in our experiment. The differences between experimental and calculated values determining the change in optical path were smaller than 0.02 (in the wavelength units). The value of the coefficient  $a_1$  at  $x$  (formula (22)) was of the order of  $10^{-9}$ . Hence, according to the expectations it may be assumed that the first derivative of the approximating function is equal to zero, for  $x = 0$ . The coefficient  $a_2$  at  $x^2$  was greater than zero. To choose optimally the order of the approximating polynomial it may be useful to check the analytic properties of the approximating function. From the viewpoint of physical conditions of experiment it is obvious that the true function describing the phase shift takes a minimum value for  $x = 0$ . This means that the first derivative should be equal to zero, while the second one should be positive. Consequently, if the approximating function is a polynomial then the coefficient  $a_1$  in (22) must be equal to zero, while  $a_2$  must be positive. Moreover, the function describing the true course of the phase change should not take the unexpected minima and maxima. By satis-

Table 2

$r$ [cm]	$(n-1) \cdot 10^{-5}$	$T$ $10^3$ [K]
0.0	0.5086	14.785
0.04	0.5139	14.630
0.08	0.5297	14.200
0.12	0.5589	13.460
0.16	0.6088	12.350
0.2	0.7100	10.590
0.24	1.055	7.131
0.28	1.532	4.907
0.32	2.089	3.600
0.36	3.064	2.455
0.4	4.868	1.545
0.44	7.839	0.9593
0.48	12.05	0.6242
0.52	17.07	0.4404
0.56	21.87	0.3439
0.6	24.90	0.302
0.621	25.6	0.2966

fyng all these requirements we obtain an additional criterion for the choice of the order of the approximating polynomial.

The condition that the first derivative should be equal to zero for  $x = 0$  is, here, of a particular importance. Misfitting of this condition by the approximating function causes great error when calculating the integral (2). This, in turn, results in serious errors in the distribution of refraction determined for small  $x$  close to the axis, thus in the region of interest.

If  $a_1 = 0$  the integral (2) may be readily calculated analytically

$$n_0 - n(r) = \frac{1}{\pi} \int_R^0 \frac{\sum_{j=2}^{10} j a_j x^{j-1}}{\sqrt{x^2}} dx$$

$$= \frac{1}{\pi} \sum_{j=2}^{10} \int_R^0 j a_j x^{j-2} dx = \frac{1}{\pi} \sum_{j=2}^{10} \frac{j}{j-1} a_j x^{j-1}.$$

Thus, the refraction of the middle point in the plasma column may be determined.

For a chosen function, approximating the phase shift, the value of the integral (2) was calculated numerically by the method of Gauss-Tschebyshev [1]. The refraction estimated in this way made it possible to calculate the temperature, according to the relation

$$T(r) = \frac{n_0 - 1}{n(r) - 1} T_0,$$

where  $(n_0 - 1)$  — denotes the refraction of the plasma surrounding and  $(n(r) - 1)$  — is the refraction at a distance  $r$  from the middle point of the plasma column.

Numerical values of the refraction and temperature are presented in Table 2. The values of refraction and temperature have been estimated by taking account of the effect of free electrons on the total refraction of plasma, under assumption of local thermodynamic equilibrium. The refraction and temperature distribution shown in Figs. 6 and 7 have been obtained under certain conditions, which should be specified, while discussing numerical values given in Table 2. These values are affected by a number of factors, e.g. temperature of the surrounding, humidity, pressure, the accuracy of the determined position of interference fringes and the shape of wavefront used in the holographic interferometer etc.

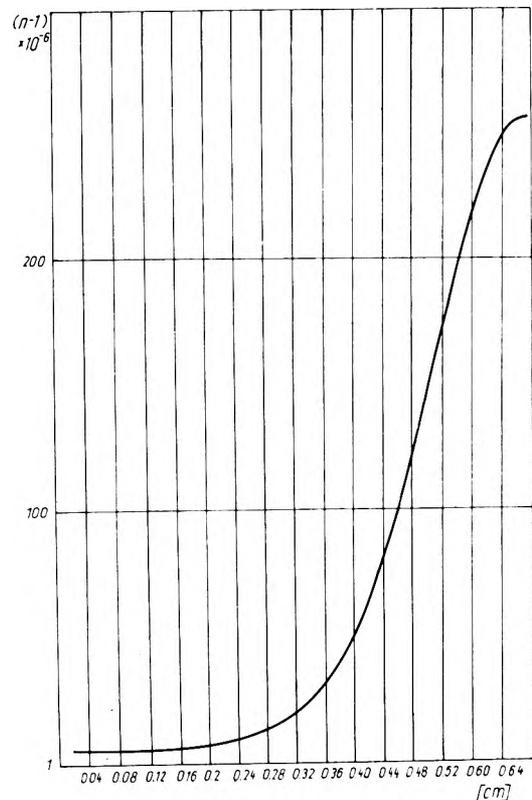


Fig. 6. A distribution of the plasma jet reflection calculated from the interferogram shown in Fig. 3

The influence of the particular factors on the results of holographic diagnosis of plasma, as well as the comparison of the latter with the spectroscopic diagnosis will be discussed in the next paper.

The purpose of this publication was to propose a method of digital processing of interfero-

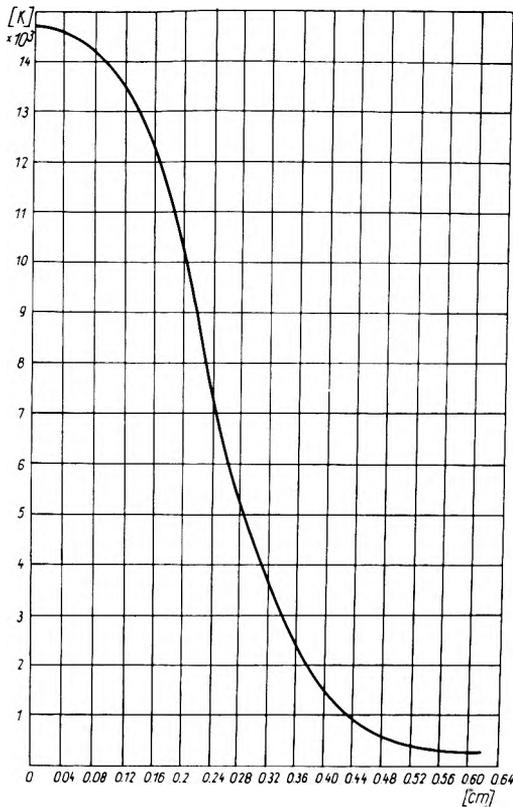


Fig. 7. The temperature distribution in the plasma jet corresponding to the refraction shown in Fig. 6

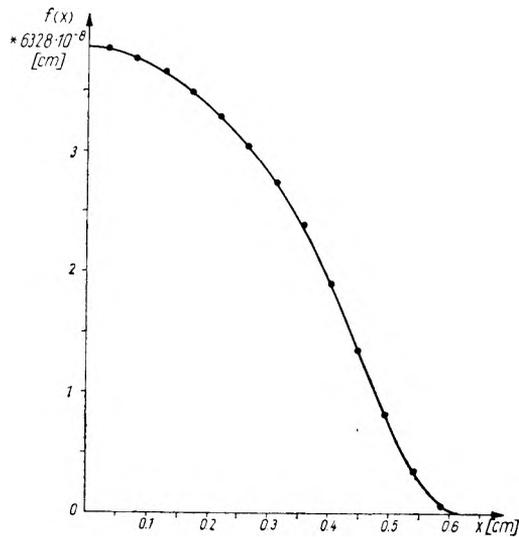


Fig. 8. The phase shift (change in optical path difference) determined from the hologram (Fig. 3). The continuous curve corresponds to the accepted approximating function (a tenth order polynomial in this case). The experimental data are marked with the dots

grams. Obviously, the method may be applied also to many other cases different from those described above.

### L'interprétation des hologrammes interférométriques du plasma

On a présenté la méthode de détermination de la fonction analytique qui décrit l'ensemble des données expérimentales obtenues par la mesure interférométrique du déphasage s'effectuant dans l'objet étudié. La méthode d'interprétation proposée a ensuite servi à déterminer la répartition des températures dans un flux de plasma.

### Об интерпретации интерферометрических голограмм

В работе изложен метод определения аналитической функции, описывающей множество экспериментальных данных, полученных путем интерферометрического измерения сдвига по фазе, совершающегося в фазовом объекте. Предложенный метод интерпретации применен затем для расчета распределения температуры в плазменном потоке.

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