*Bożena Janowska, Jadwiga Szydłowska**

An Analysis of the Quartz Crystal Vibration by a Holographic Method

The application of holographic technique to object vibrations has been desicribed and a detailed description of the experiment with a quartz cristal presented. The holograms were performed in argon laser light with the wavelength $\lambda = 488$ nm.

Holographic technique may be employed to recording of vibrating objects [1, 2]. One of its methods consists in a sufficiently long holographic exposure of the vibrating object yielding. Then a time-averaged amplitude record of each of the object points is produced. Under such conditions the wave producing the virtual image during the reconstruction step will be modulated in a way characteristic of this vibration.

For an arbitrary object point P the wave function producing the virtual image is

$$F(P) = F_{st}(P)\frac{1}{t_n}\int\limits_0^{t_n} e^{i\Phi(t)}dt, \qquad (1)$$

where F_{st} is the wave function producing the stationary virtual image, t_n – is the exposure time, $\Phi(t)$ represents the phase change caused by vibration of point P.

The integral

$$\frac{1}{t_n}\int\limits_0^{t_n} e^{i\Phi(t)}dt$$

is the averaged change of the object wave in time-interval t_n. From the simple geometrical considerations it follows that

$$\Phi(t) = \frac{2\pi}{\lambda}(\cos\theta_1+\cos\theta_2)W(P,t), \qquad (2)$$

where θ_1 is the angle between the vibration illumination and the directions, while θ_2 denotes the angle between the direction of vibrations and that of observation, $W(P,t)$ is the deflection of object vibrations at the point P. For harmonic vibrations

$$W(P,t) = A(P)\cos[\omega t+\Phi(P)], \qquad (3)$$

where $A(P)$ is an amplitude at the point P, ω is the frequency of object vibrations, $\Phi(P)$ is the phase of point P at $t = 0$. Substitution of (2) and (3) for (1) results in

$$F(P) = F_{st}(P)\frac{1}{t_n}\int\limits_0^{t_n} e^{i\frac{2\pi}{\lambda}(\cos\theta_1+\cos\theta_2)A(P)\cos)[\omega t+\Phi(P)]}dt$$

$$= F_{st}(P)J_0\left[\frac{2\pi}{\lambda}A(P)(\cos\theta_1+\cos\theta_2)\right] \qquad (4)$$

on the assumption, that the exposure time is much greater than the vibration period. $J_0\left[\frac{2\pi}{\lambda}A(P)(\cos\theta_1+\cos\theta_2)\right]$ is the Bessel function of first kind and zero order. The observed intensity of the image point P reconstructed from the hologram is

$$I = |F(P)|^2 = I_{st}J_0(x), \qquad (5)$$

where $I_{st} = |F_{st}(P)|^2$ and $x = \frac{2\pi}{\lambda}A(P)(\cos\theta_1+\cos\theta_2)$.

A number of bright and dark fringes appear on the object surface. Their distribution depends on the values of squared Bessel function, which, in turn, are conditioned by the vibration amplitude $A(P)$. Thus, the value of $A(P)$ may be inferred from the distribution

*) Laboratory of Non-linear Optics and Chemical Physics IPPch, University of Warsaw, Warsaw, Poland.

of fringes. Immobile nodal points of the surface for which $x = 0$, will be the brightest ones, since $J_0^2(0) = 1$. Since the intensity of points, for which $A(P)$ is such that $J_0^2(x) = 0$, is equal to zero they can be digitinguished from the object points, and their vibration amplitude calculated from the formula

$$A(P) = \frac{\lambda}{2\pi}(\cos\theta_1 + \cos\theta_2)^{-1} j_{0,n}, \qquad (6)$$

where $j_{0,n}$ denotes the n-th zero point of the Bessel function of zero order.

For a given geometry of the experimental setup the value $\lambda/2\pi(\cos\theta_1 + \cos\theta_2)^{-1} = a$ is constant and may be written as

$$A(P) = a j_{0,n}. \qquad (7)$$

The accuracy in which the position of the n-th zero point is determined on the interferogram depends on the contrast of the fringes. Due to the specificity of the Bessel function the contrast of points compared with that of nodal points decreases with the amplitude value; provided that the vibration amplitude is sufficiently great; this effect restricts the measurements vibration amplitude values.

1. Description of experiment

For recording the vibration of a quartz crystal a holographic setup was used; its scheme is presented in Fig. 1 with constant $a = 566.5$ nm. The quartz crystal cut at $x + 5°$ was excited by an *RC* generator to flexural vibrations of resonance frequency 7460 Hz.

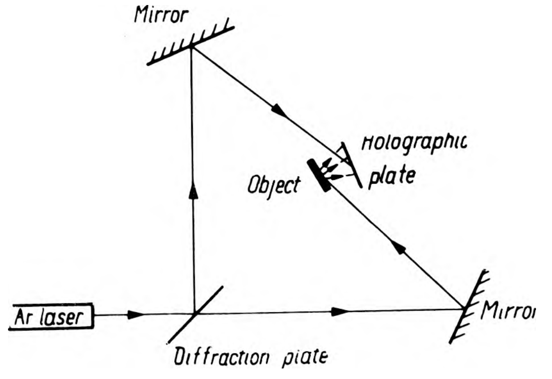

Fig. 1. Scheme of the holographic setup

This generator rendered possible the changes in the exciting signal amplitude within the range 0–18 V. The quartz crystal was fastened by locating its two supporting points at the distances amounting to 1/4 and about 3/4 of its length. The remaining points were allowed to vibrate with the natural frequency. The crystal was supported with the help of elastic wires fixed to a rigid steel bar structure. Its lateral surfaces were covered with silver electrodes diffusing the light.

The holograms were performed in the argon laser light, $\lambda = 488$ nm, by using the Agfa-Gevaert 8E56 holographic plates. The time of holographic plate exposure was $t_n = 4$s.

2. Results of the experiment

The holographic interferograms of the vibrating quartz crystal have been obtained for different exciting signal amplitudes ranging within 0–18 V. For each interferogram nodal points and the amplitude distribution along the quartz crystal may be determined. In the present paper the distribution was calculated for the interferogram of quartz excited with a 9 V signal. Figs. 2 and 3 present the interferogram and the amplitude distribution, respectively.

The dependence of the maximum amplitude of the quartz crystal vibrations on the exciting signal is presented in Fig. 4. For the range of

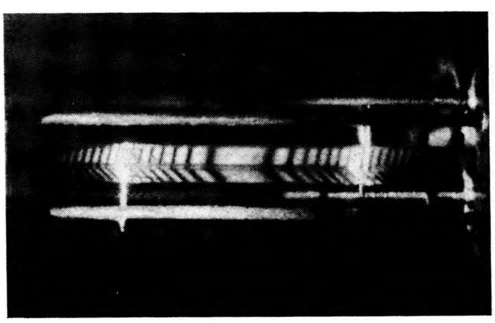

Fig. 2. Interferogram of the vibrating quartz crystal. Vibration frequency 7460 Hz, exciting signal amplitude 9 V

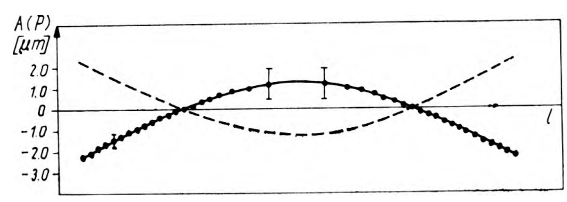

Fig. 3. The distribution of the vibrations along the quartz crystal, determined by the number and distribution of fringes obtained on the interferogram

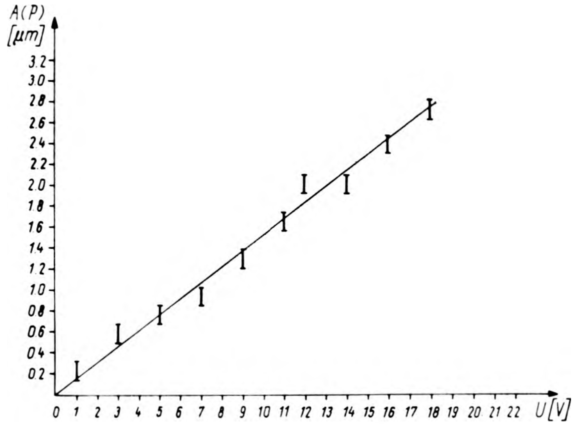

Fig. 4. Maximum vibration amplitude vs. exciting signal for quartz crystal

examined voltages this relation is linear within the experimental error.

The interferometric images of the vibrations have been recorded on each of the quartz crystal walls. For the parallel walls these images appeared to be identical.

No fringes have been observed on the interferograms of the vibrating objects if the vector of vibrations is perpendicular to the plane determined by the observation and illumination directions (Fig. 5). A slight change in the direction of observation (Fig. 6) results in the presence of interference fringes which appear on the same interferogram, since the vibration vector is no more perpendicular to the plane determined by the two directions mentioned above.

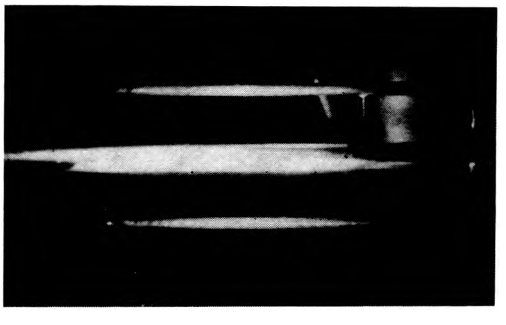

Fig. 5. Vibrations of quartz crystal on the plane parallel to the observed crystal wall; the amplitude of the exciting singal amounting to 9 V

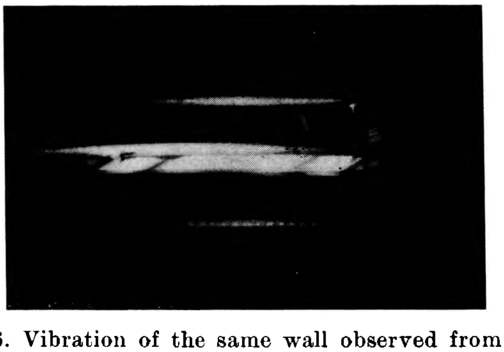

Fig. 6. Vibration of the same wall observed from the bottom

If, however, the direction of the object vibration vector coincides with that of observation, the maximum fringe density is recorded on the hologram (Fig. 7). In this case a slight change in observation direction results in a slight change in the fringe density (Fig. 8).

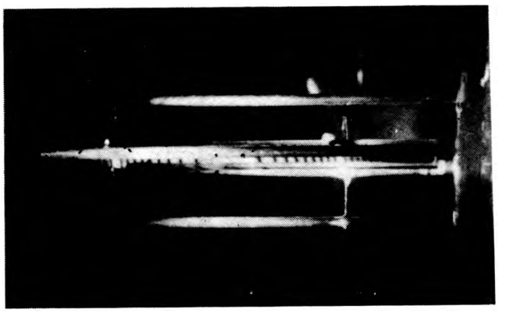

Fig. 7. Quartz vibration in the plane perpendicular to the observed wall. The exciting signal amplitude amounting to 9 V

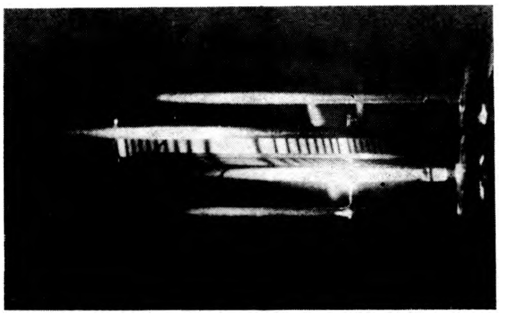

Fig. 8. Vibration of the same wall as observed from the bottom

Analyse des vibrations des cristaux par une méthode holographique

Les résultats d'une experimence ont permis d'obtenir des interferogrammes holographiques du quartz pour les amplitudes différentes d'un signal stimulant allant de 0 à 18 V. Les hologrammes ont été fait sous la lumière d'un laser à argon dont la longeur d'onde était s $\lambda = 488$ nm. On a constaté qu'il était possible de déterminer des endroits et la distribution des amplitudes de vibration au long du quartz pour chacun des hologrammes. On a calculé cette distribution pour l'interferogramme du quartz stimulé par un signal de 9 V. On a donné la dépendence d'une amplitude maximum des vibrations du cristal du signal stimultant. On a enregistré les image interferometriques des vibrations sur chaque coté du cristal de quartz. Pour les cotés parallels les images sont identiques.

Анализ колебаний кристалла голографическим методом

В результате произведенных опытов получены голографические интерферограммы колеблющегося кварца для разных амплитуд возбуждающего сигнала в пределах от 0 до 18 В. Голограммы выполнены в свете аргонового лазера длиной волны $\lambda = 488$ нм. Обнаружено, что для каждой из интерферограмм возможно определить узловые места и распределение амплитуд колебаний вдоль кварца. Это распределение рассчитано для интерферограммы кварца, возбужденного сигналом 9 В. Приведена зависимость максимальной амплитуды колебаний кристалла от возбуждающего сигнала. Записаны интерферометрические изображения колебаний на каждой из стенок кристалла кварца. Для параллельных друг другу стенок изображения один и те же.

References

[1] Powell R., Stetson K., J. Opt. Soc. Am. **55**, 1593 (1965).
[2] Stetson K., Powell R., J. Opt. Soc. Am. **55**, 1694 (1965).

Received, July 12, 1975.