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Influence of external magnetic field on the population trapping phenomenon in a resonant system

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A theory of population trapping in a resonant system with degenerate levels in the polarized laser radiation is developed. We are interested in the resonant transition $J_1 = 1 \rightarrow J_2 = 0$ because in this system the population trapping arises in the field of elliptically polarized wave due to Zeeman coherence between the magnetic sublevels $m_1 = -1$ and $m_1 = +1$. The theory is developed in the irreducible tensorial formalism, that allows us to take into consideration the relaxations related both to the non-uniform population and the coherence between magnetic sublevels. The exact formulae for the lser induced multipole momenta (population, orientation and alignment) are obtained. Modification of population trapping at the propagation of a polarized light in a constant magnetic field is investigated.

The population trapping phenomenon has been investigated in a number of theoretical [1]–[8] and experimental [9]–[11] papers. Interest towards this problem is not only due to fundamental character of the phenomenon, but is also caused by its applications in high-resolution spectroscopy [3], [12], in the systems of optical bistability [13], [14], for atom cooling [15]–[17], *etc.*

The essence of the effect lies in the formation of coherent superpositional state of two levels under two-photon resonance condition, in which the whole system population is practically being trapped. Investigation of the coherent population trapping effect is of great interest in resonant systems with degenerate levels in the field of polarized radiation. Correct account of the relaxation processes in the systems with degenerate levels is possible in the irreducible tensorial formalism (χ, q-representation), [18], [19]. At isotropic collisions the relaxation matrix is diagonalized with respect to χ and does not depend on q. In such systems in the field of polarized radiation the pultipole momenta of higher ranks (orientation, alignment, *etc.*) are being induced. Studies of these momenta are of great interest under conditions of the coherent population trapping. In the systems of degenerate levels the account of saturation effects becomes essential, as due to optical pumping the absorption saturation may take place at anomalously small light intensities. Such a convenient resonance system with degenerate levels is the transition $J_1 = 1 \rightarrow J_2 = 0$. The Zeeman coherence between magnetic sublevels of a ground state of this system occurs in the field of elliptically polarized wave and becomes the cause of the coherent population trapping [20].

The constant external longitudinal magnetic field shifts the magnetic sublevels and can influence the population trapping.

The aim of this paper is to develop a theory of the coherent population trapping in the resonant system $J_1 = 1 \to J_2 = 0$ in the external magnetic field taking into account the saturation effects.

In the irreducible tensorial formalism the density matrix of the system is the vector column with ${}_{ik}\rho_q^{\chi}$ elements, where i,k enumerate energetic levels of the atom (1 is the lower level, 2 is the upper level). In the resonant system $J_1 = 1 \to J_2 = 0$ in general case in the field of polarized radiation the following ${}_{ik}\rho_q^{\chi}$ density matrix elements exist: ${}_{11}\rho_0^0$ is the population of lower level 1, ${}_{11}\rho_q^1$ is the orientation of level 1 ($q = 0, \pm 1$); ${}_{11}\rho_q^2$ is the alignment of level 1 ($q = 0, \pm 1, \pm 2$); ${}_{22}\rho_0^0$ is the population of upper level 2; ${}_{21}\rho_q^1$ is the transition dipole current $1 \to 2$ ($q = 0, \pm 1$). In papers [20], [21] the calculation technique was developed, which made it possible to solve exactly the system of equations for density matrix of transition $1 \to 2$ in the field of arbitrary elliptically polarized radiation (accounting for saturation effects).

Furthermore, let us assume that the electromagnetic wave propagates along the axis z, elliptically polarized ($E_0 = 0$, $E_{\pm} = E_x \pm E_y \neq 0$ are circular components). The external constant magnetic field H also has z direction.

We have the following expressions for non-zero density matrix components in the stationary case:

— *population of level* 1

$$
{}_{11}\rho_0^0 = \left[\frac{N_1}{\sqrt{3}} - \frac{N\gamma'}{D_2 \Gamma_{11}^{(0)}}\left(1 - \frac{A_{21}}{\Gamma_{22}^{(0)}}\right)\left(2G + \frac{q}{2} G_+ G_-\right)\right], \tag{1a}
$$

— *orientation of level* 1

$$
{}_{11}\rho_0^1 = \frac{N\alpha_1}{\sqrt{2} D_2}(G_- - G_+), \tag{1b}
$$

— *alignment of level* 1

$$
{}_{11}\rho_0^2 = -\frac{N\alpha_2}{\sqrt{6} D_2}\left(2G + \frac{q}{2} G_+ G_-\right),
$$

$$
{}_{11}\rho_2^2 = \frac{N\alpha_2 (1 - i\Delta)\xi_+ \xi_-^*}{D_2 \Gamma_{12}^{(1)} \gamma'}\left(1 + \frac{p}{a} G\right),
$$

$$
{}_{11}\rho_{-2}^2 = {}_{11}\rho_2^{2*}, \tag{1c}
$$

— *population of level* 2

$$
{}_{22}\rho_0^0 = N_2 + \frac{N\gamma'}{D_2 \Gamma_{22}^{(0)}}\left(2G + \frac{q}{2} G_+ G_-\right), \tag{1d}
$$

— *dipole transition currents for different circular components of light polarization*

$$_{12}\rho_1^1 = -\frac{iN(1-i\Delta)}{D_2\Gamma_{12}^{(1)}}\left(1+\frac{p}{a}G_+\right)\xi_+^*, \tag{1e}$$

$$_{12}\rho_{-1}^1 = -\frac{iN(1+i\Delta)}{D_2\Gamma_{12}^{(1)}}\left(1+\frac{p^*}{a^*}G_-\right)\xi_-^* \tag{1f}$$

where:

$$p = \frac{\alpha_1}{1-i\Delta} - \frac{\alpha_2}{1+2i\lambda}, \quad a = 1+i\Delta+\frac{\alpha_2 G}{1+2i\lambda},$$

$$D_2 = 1+\Delta^2+G\left(\frac{2}{3}+\frac{1}{3}\alpha_2+\alpha_1\right)+\frac{1}{2}G_+G_-\left(\frac{2}{3}+\frac{1}{3}\alpha_2\right)q,$$

$$q = \frac{p(1-i\Delta)}{a}+cc, \quad \Delta = \frac{M_1H}{\Gamma_{12}^{(1)}}, \quad \lambda = \frac{M_1H}{\Gamma_{11}^{(2)}}, \quad \alpha_{1,2} = \frac{\gamma'}{\Gamma_{11}^{(1,2)}},$$

$$1/\gamma' = 1/\Gamma_{11}^{(0)}+3/\Gamma_{22}^{(0)}-A_{21}/\Gamma_{11}^{(0)}\Gamma_{22}^{(0)}, \quad N = \frac{1}{3}N_1-N_2,$$

$$G_\pm = \frac{|\xi_\pm|^2}{\gamma'\Gamma_{12}^{(1)}}, \quad \xi_\pm = \frac{d_{12}E_\pm}{\hbar}, \quad G = 1/2(G_+ + G_-).$$

The following designations are introduced: $\Gamma_{ik}^{(\chi)}$ are the relaxation constants of atomic multipole moment, A_{21} is the Einstein coefficient, $N_{1,2}$ are atom densities of energetic levels 1, 2 without field, $d_{21} = d/\sqrt{3}$, d is a reduced matrix element of the transition, $M_{1,2} = \mu_B g_{1,2}/h$, $g_{1,2}$ are Lande factors of levels 1, 2, μ_B is the Bohr magneton.

For simplicity, in formulae (1) we give only the case of exact resonance. It has been assumed that before the interaction with radiation field, we have a uniform population of incoherent magnetic sublevels. The formulae obtained show that in the field of wave the induced multipole momenta of higher ranks occur with both non-uniform population of the system magnetic sublevels and Zeeman coherency.

Along with the analyses of the expression for density matrix in χq-representation, of certain interest is the study of density matrix elements in common JM-representation. The $_{ik}\rho_q^\chi$ are connected with density matrix elements in JM-representation in the following way:

$$_{ik}\rho_q^\chi = \sum_{M_i, M_k} (-1)^{J_k-M_k} C(J_iJ_k\chi/M_i-M_kq)\rho_{J_iM_i;J_kM_k} \tag{2}$$

where $C(\ldots|\ldots)$ are the Clepsch–Gordan coefficients.

The general formulae (1) we obtained are essentially simplified on the following assumption: the upper level relaxation unpopulated before the interaction ($N_2 = 0$) is defined by a spontaneous decay ($A_{21} = \Gamma_{22}^{(0)}$), the lower level is a ground ($\Gamma_{11}^{(0)} = 0$), the relaxation widths of orientation $\Gamma_{11}^{(1)}$ and alignment $\Gamma_{11}^{(2)}$ are equal ($\alpha_1 = \alpha_2$).

In this approximation, writing a_{m_1} ($m_1 = 0, \pm 1$) and b_{m_2} ($m_2 = 0$) for the amplitudes of magnetic sublevels of ground and excited atomic levels, respectively, in JM-representation, we have the following expressions for magnetic sublevel populations:

$$
\begin{aligned}
|a_{+1}|^2 &= \frac{N_1}{3}\left[1+\frac{\alpha}{3D_2}\left(G_- - 2G_+ - \frac{\alpha G_+ G_-(\Delta+2\lambda)^2}{2[1-2\Delta\lambda+\alpha G)^2+(\Delta+2\lambda)^2]}\right)\right],\\
|a_{-1}|^2 &= \frac{N_1}{3}\left[1+\frac{\alpha}{3D_2}\left(G_+ - 2G_- - \frac{\alpha G_+ G_-(\Delta+2\lambda)^2}{2[1-2\Delta\lambda+\alpha G)^2+(\Delta+2\lambda)^2]}\right)\right],\\
|a_0|^2 &= \frac{N_1}{3}\left[1+\frac{\alpha}{3D_2}\left(G_+ + G_+ + \frac{\alpha G_+ G_-(\Delta+2\lambda)^2}{[1-2\Delta\lambda+\alpha G)^2+(\Delta+2\lambda)^2]}\right)\right],\\
|b_0|^2 &= \frac{N_1\gamma'}{3D_2\Gamma_{22}^{(0)}}\left(G_+ + G_- + \frac{\alpha G_+ G_-(\Delta+2\lambda)^2}{(1-2\Delta\lambda+\alpha G)^2+(\Delta+2\lambda)^2}\right),
\end{aligned}
\tag{3}
$$

and Zeeman coherency of sublevels $a_{\pm 1}$

$$
\begin{aligned}
a^*_{+1}a_{-1} &= \frac{N_1\alpha(1-i\Delta)\xi_+\xi^*_-}{3D_2\Gamma_{12}^{(1)}\gamma'}\left(1+\frac{i\alpha(2\lambda+\Delta)G}{(1+\Delta^2+\alpha G)-i\alpha\Delta G}\right),\\
a^*_{-1}a_1 &= \frac{N_1\alpha(1+i\Delta)\xi_-\xi^*_+}{3D_2\Gamma_{12}^{(1)}\gamma'}\left(1-\frac{i\alpha(2\lambda+\Delta)G}{(1+\Delta^2+\alpha G)-i\alpha\Delta G}\right).
\end{aligned}
\tag{4}
$$

From the formulae obtained it is obvious that induced non-uniform magnetic sublevel populations (3) as well as the Zeeman coherency (4) are caused by the degree of incident radiation ellipticity ($G_+ \neq G_- \neq 0$). The magnetic field shifting sublevels with $m_1 = \pm 1$ carries them out of the resonance, decreasing their population. Sublevel with $m_1 = 0$ is not shifted within the magnetic field and the possibility of its population increases.

Let us note that the formulae obtained are principally changed depending on the correlation between relaxation widths of orientation and alignment $\Gamma_{11}^{(1,2)}$ of the lower level and the widths of spontaneous decay of the upper level $\Gamma_{22}^{(0)}$. Indeed, the width $\Gamma_{11}^{(1,2)}$ is determined by the existence of disorientating atomic collision in the system, probability of which is extremely small compared with the spontaneous decay of the atoms. In reality, $\Gamma_{22}^{(0)} \approx 10^8\,\mathrm{s}^{-1}$ and $\Gamma_{11}^{(1,2)} \approx 10^{5\div 6}\,\mathrm{s}^{-1}$, *i.e.*, characteristic parameter $\alpha_{1,2} \approx \Gamma_{22}^{(0)}/3\Gamma_{11}^{(1,2)} \approx 10^{2\div 3}$. In the asymptotic case $\alpha_{1,2} \to \infty$, $|b_0|^2 \sim 1/\alpha_1$ tends to zero, that is, the effect of coherent population trapping takes place in the system.

When $H \to 0$ we have:

$$
\begin{aligned}
|a_{+1}|^2 &= N_1 G_-/2(G_+ + G_-) = N_1(1+\eta_2)/4,\\
|a_{-1}|^2 &= N_1 G_+/2(G_+ + G_-) = N_1(1-\eta_2)/4,\\
|a_0|^2 &= N_1/2, \quad |b_0|^2 = 0.
\end{aligned}
$$

Thus, sublevel population does not depend on wave intensity and is determined only by the wave polarization characteristics (Stock's parameters). Even in the field of the wave with very small intensity some essential redistribution of atomic population in magnetic sublevels occurs due to atomic optical pumping: for example, possibility of sublevel with $m_1 = 0$ population changes from $N_1/3$ to $N_1/2$. The Zeeman coherency completely compensates absorption, which leads to the medium "bleaching".

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