Optica Applicata, Vol. XXIX, No. 3, 1999

Teaching optics

Transport of information in coherent optical systems in terms of diffraction. The amplitude object case

IRENEUSZ WILK

Institute of Physics, Wrocław University of Technology, Wybrzeże Wyspiańskiego 27, 50–370 Wrocław, Poland.

The aim of this paper is to show in the simplest possible way the diffraction mechanism of information transport as exemplified by a radically simplified model of an optical system being substituted by a single lens, thin but extended to infinity, while the illuminating system is reduced to a point source emitting the monochromatic light. The object is assumed to be of amplitude type. In contrast to these simplifications the imaging considered is of generalised (though coherent) type when the object and the observation plane are not necessarily interrelated by the lens law and thus an infinite number of transformations of optical information is possible. Under these conditions the law of information conservation is formulated.

1. Introductory remarks

The problem of information transport in optical instruments is of fundamental importance both for their designers and users. Therefore, it seems advisable to provide the relevant information in all the textbooks on instrumental optics. Unfortunately, it is not always the case probably because of relatively high complexity of an accurate description in many situations, in particular, if the problem is formulated in terms of diffraction. A general treatment of the problem is difficult due to enormous variety of optical devices being in use nowadays. For these reasons and in order to make the problem as simple as possible, we restrict our attention to one example which, however, provides a good introduction to the other more complicated optical systems. This case is that of information transport in a simplified coherent optical system if the object is of amplitude type.

As mentioned above, the purpose of the paper is to show the *diffraction mechanism* of the optical information transport as exemplified by an optical system reduced to a single lens which is thin but infinitely extended in its plane as illustrated in Fig. 1. (This lens can be considered as a substitute of a more complex optical system). Such an idealised optical system allows us to introduce a relatively simple description of the transport of optical information in terms of diffraction and may constitute something like "ideal reference level" when considering transport of information in real optical systems. Our analysis will be carried out for the case of

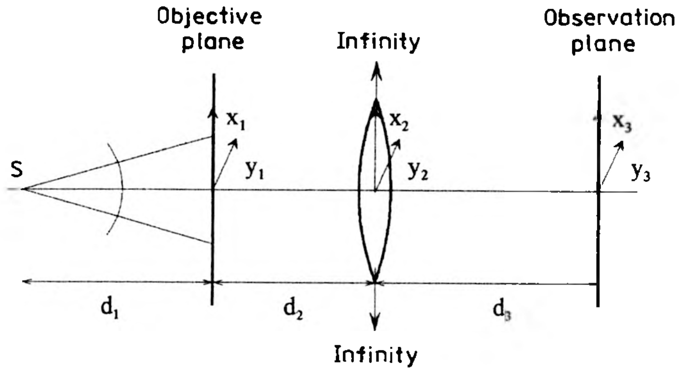

Fig. 1. Simplified optical system.

generalised imaging of an amplitude object when the object and image planes are not necessarily interrelated by the lens formula. Under these circumstances it is possible to formulate a law of information conservation. It is suggested that the derivation of two classic cases: of *identity imaging* when the image is perfectly similar to the object and *Fourier transforming* can constitute an interesting problem though simple enough to be solved by the students. The treatment will be ended with some critical remarks defining the relation of the offered idealised description to the real optical systems.

The above approach is novel and, therefore, it has been presented to the students of the third year of physics–optics and biooptics in order to verify its communicativeness. The results appeared to be encouraging.

2. Encoding of optical information in an amplitude object

As the first stage of analysis we will consider the transformation of a spherical wave at the amplitude object plane. For this purpose, three simple problems will be specified, *i.e.*, that of the *amplitude object structure*, that of *spherical wave structure* at the input to the object plane and finally that of a special case of a *plane wave incident on the object plane.*

— *Structure of the amplitude object*

Assume that the *optical amplitude object* located in the x_1, y_1 plane is expendable into a Fourier series and thus takes the form (compare [1], [2], for instance)

$$t(x_1, y_1) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} t_{mn} \exp -2\pi i (x_1 f_{xm} + y_1 f_{yn})$$

$$= \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} t_{mn} \exp \frac{-2i\pi}{\lambda d_1} [x_1 x_{1m} + y_1 y_{1n}] \tag{1}$$

where:

$$t_{mn} = t^*_{-m,-n}, \quad f_{xm} = m f_{x1}, \quad f_{yn} = n f_{y1} \quad \text{and} \quad x_{1m} = \lambda d_1 f_{xm}, \quad y_{1n} = \lambda d_1 f_{yn}, \tag{2}$$

for the case of even amplitude transmittance. Obviously, condition (2) assures that the amplitude transmittance of the object is real in spite of the complex notation used. The case of an amplitude object of odd amplitude transmittance can be described by the equation

$$t(x_1, y_1) = i \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} t_{mn} \exp -2\pi i(x_1 f_{xm} + y_1 f_{yn})$$
$$= \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} t_{mn} \exp \frac{i\pi}{\lambda d_1} [x_1(\lambda d_1 f_{xm}) + y_1 (\lambda d_1 f_{yn})]$$
$$= \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} t_{mn} \exp \frac{i\pi}{\lambda d_1} [x_1 x_{1m} + y_1 y_{1n}] \tag{3}$$

where $t_{mn} = -t_{-m-n}$ for the other conditions unchanged.

Since further arguments for the two (even and odd) cases would be carried out in a similar way, we restrict our discussion to the *even amplitude objects* defined by Eq. (1).

– *Illuminating system generating a spherical wave*

The illuminating system is here reduced to a single point source emitting a monochromatic light wave, and thus constitutes a coherent source. Under these circumstances the state of the optical field at the input of the object can be described approximately by the equation of the reduced spherical wave (compare [1], [2])

$$u_{\text{in}} = \frac{A'}{r} \exp -i\mathbf{kr} \approx A \exp \frac{i\pi}{\lambda d_1} (x_1^2 + y_1^2 - 2x_1 x_0 - 2y_1 y_0), \tag{4}$$

where $(x_0,\ y_0 - d_1)$ are the coordinates of the point source, which for $x_0 = y_0 = 0$ (position of the point source on the optical axis) takes a simpler form (see Fig. 1)

$$u_{\text{in}} = A \exp \frac{i\pi}{\lambda d_1} (x_1^2 + y_1^2). \tag{5}$$

– *Illuminating system generating a plane wave*

As is well known [1], [2], a plane wave propagating in the object space under an arbitrary angle to the axis of the optical system (the latter being identified with the axis z of the coordinate system) has the form

$$A \exp -2\pi i(x \cos\alpha/\lambda + y \cos\beta/\lambda + z \cos\gamma/\lambda) = A \exp -2\pi i(xf_x + yf_y + zf_z) \tag{6}$$

where: $f_x = \cos\alpha/\lambda$, $f_y = \cos\beta/\lambda$, $f_z = \cos\gamma/\lambda$ are the spatial frequencies and $\cos\alpha$, $\cos\beta$ and $\cos\gamma$ are the corresponding directional cosines of the propagating plane wave. However, if we assume that the object is located in the plane $z = 0$, the notation of the incident plane wave is reduced to the form

$$U_{\text{in}}(x_1, y_1) = A \exp -2\pi i(x_1 \cos\alpha/\lambda + y_1 \cos\beta/\lambda) = A \exp -2\pi i(x_1 f_x + y_1 f_y). \tag{7}$$

– *Transformation of the illuminating wave by an even amplitude object*

a) Illuminating the object by the spherical wave

For the sake of simplicity of the due calculations the equation of the spherical wave illuminating the amplitude object will be used in an approximate form (4) given above.

Transformation of the optical field on the even amplitude object (1) is defined by the equation

$$u_{\text{out}}(x_1,y_1) = u_{\text{out}}(x_1,y_1)\,t(x_1,y_1)$$
$$= A\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} t_{mn}\exp\frac{i\pi}{\lambda d_1}[x_1^2+y_1^2-2x_1(\lambda d_1 f_{xm})-2y_1(\lambda d_1 f_{yn})]$$
$$= A\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} t_{mn}\exp\frac{i\pi}{\lambda d_1}[x_1^2+y_1^2-2x_1x_m-2y_1y_n]. \tag{8}$$

This situation is illustrated in Fig. 2, in which the illuminating wave is marked by a thicker line.

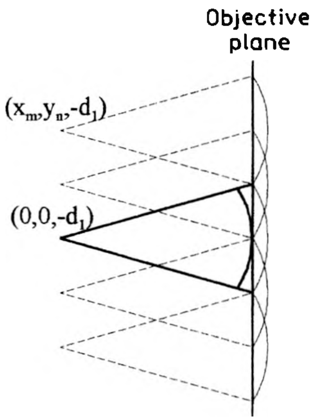

Fig. 2. Transformation of a spherical wave emerging from a point source on the axis and incident on the even amplitude object structure (objective plane).

Note that the prolongation of this wave on the other side of the object plane is marked by a thinner line symbolising the wave travelling in the same direction but weakened due to its transformation on the object. Note again that the other waves emerging in reality from the object as a result of transformation of the incident spherical wave on this object are represented by their wave fronts in the form of spherical waves as if they were emitted by the virtual point sources located at the points $(x_m, y_n, -d_1)$, *i.e.*, all positioned in the plane $z = -d_1$ perpendicular to the axis of the optical system and passing through the real illuminating point source $(x_0, y_0, -d_1)$. It is easy to notice that *the number of the emerging spherical waves equals the number of harmonics of spatial frequencies* ($f_{xm} = x_m/\lambda d_1$, $f_{yn} = y_n/\lambda d_1$) *constituting the structure of the amplitude object.* Thus, the transformation on the amplitude object consists in suitable multiplication of the incident illuminating spherical wave into a suitable number of partial spherical waves, each of them carrying information about the corresponding structural harmonics. *This is the way the structural information about the even amplitude object is encoded into a set of partial spherical waves emerging from this object.*

b) Illuminating the object by the plane wave

Let the object be illuminated by a plane wave travelling obliquely in relation to the axis of the optical system. Its equation has the form

$$u_{\text{in}}(x_1, y_1) = A\exp 2\pi i(f_{x0}\, x_1 + f_{y0}\, y_1) = A\exp \frac{2\pi i}{\lambda}(x_1 \cos\alpha_0 + y_1 \cos\beta_0) \tag{9}$$

where $\cos\alpha_0$ and $\cos\beta_0$ are the corresponding directional cosines defining the direction of propagation of the illuminating plane wave. Then the transformation of the even amplitude object (1) is described by the equation

$$\begin{aligned} u_{\text{out}}(x_1, y_1) &= A \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} t_{mn} \exp - 2\pi i\,[x_1(f_{xm} - f_{x0}) + y_1(f_{ym} - f_{y0})] \\ &= A \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} t_{mn} \exp - \frac{2\pi i}{\lambda}[x_1(\cos\alpha_m - \cos\alpha_0) + y_1(\cos\beta_n - \cos\beta_0)]. \end{aligned} \tag{10}$$

A graphical illustration of the above formula is presented in Fig. 3**a**, for the case of normal incidence (*i.e.*, when $\cos\alpha_0 = \cos\beta_0 = 0$) and in Fig. 3**b** for the general case of the illuminating plane wave incident on the object plane at an arbitrary angle to the optical axis.

It can be easily seen that this time *the structure of the object transforms the incident plane wave into the number of the partial plane waves equal again to the number of the harmonics constituting the structure of the amplitude object.* Thus, the *information about the object is here encoded in a corresponding number of partial plane waves, each of them carrying information about the respective harmonic.*

– *Concluding remarks*

The above two examples indicate that the information about the even amplitude object expressed in terms of two sets of object parameters, i.e., in terms of $\{t_{mn}\}$ *and* $\{f_{xm} = mf_{x1}, f_{yn} = nf_{y1}\}$ *reappears in the structure of the set of relevant partial waves (plane or spherical) playing there the respective roles of amplitudes* t_{mn} *and modifying suitably the spatial frequencies carried by those waves. (Note that in the case of partial spherical waves the modification of the spatial frequencies is equivalent to the changes in the positions of the virtual point sources of those waves, which follows from Eq.* (2)).

The above remark is essential since, as we shall show below, all the transformations of the object information understood in this way and occurring in the course of two fundamental effects during its transport in the optical systems, *i.e.*, the propagation in the free space and the transformation in the optical elements, consists in definite modifications of the structural parameters of the amplitude

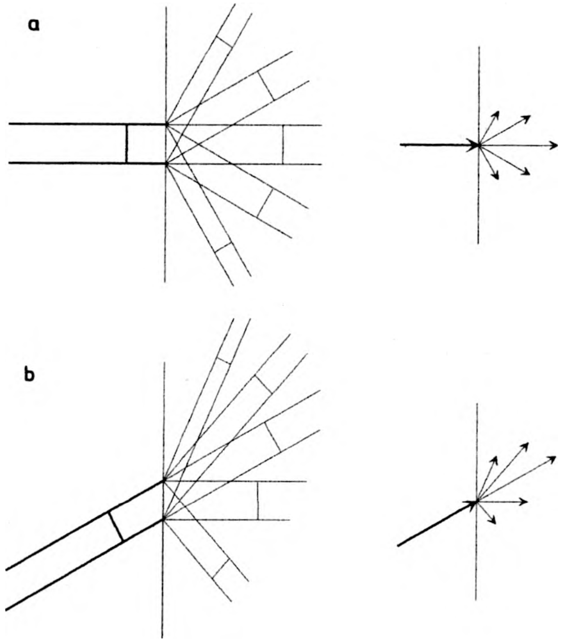

Fig. 3. Transformation of a plane wave incident on the even amplitude object structure: **a** — for a plane wave propagating along the optical axis, **b** — for the plane incident at an angle with respect to the optical axis.

object defined above. The knowledge of these modifications explains the diffraction mechanism of the information transport, on the one hand, and provides some chances of reconstruction, at least partly, of the amplitude object information. Independently, it provides a good introduction to the problem of performing some wanted transformations of this information.

3. Transformation due to free propagation

It is obvious that in the spaces between the elements of the optical system free propagation takes place. However, it is less obvious with which approximation this propagation can be described in particular cases. The decision in this case is usually a compromise between the expected accuracy of the description and the complexity of the latter (and, consequently, the time-consumption of the corresponding calculations). In our case, we are interested in diffraction description which by its nature is relatively complex, but we choose a possibly simple approach following from the Fresnel approximation. By the same means we assume that the distances between the amplitude object and the optical system represented by a substitute lens as well as that between the substitute lens and generalized image (observation plane) are sufficiently large to justify the application of the above approximation.

Let an optical system of the scheme shown in Fig. 4 be represented by its entrance pupil P_{in} and the exit pupil P_{out}. In accordance with the above discussion

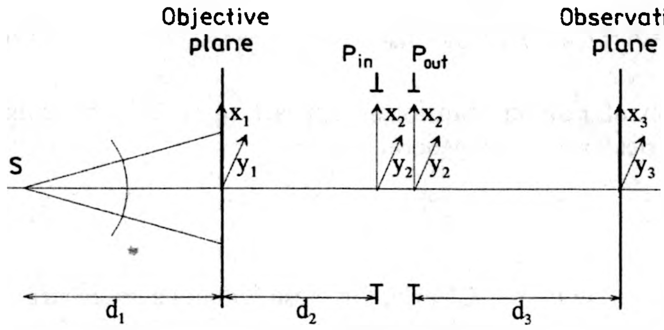

Fig. 4. Pupil representation of the simplified optical system.

the optical field incident on the entrance pupil, after having passed the distance d_2 in the free object space, can be described by the following equation:

$$\begin{aligned}
u_{\text{in}}(x_2, y_2) &= \frac{\exp(ikd_2)}{i\lambda d_2} \iint\limits_{-\infty}^{+\infty} u_{\text{out}}(x_1, y_1) \exp \frac{i\pi}{\lambda d_2} [(x_2 - x_1)^2 + (y_2 - y_1)^2] \mathrm{d}x_1 \mathrm{d}y_1 \\
&= A \frac{\exp(ikd_2)}{i\lambda d_2} \exp\left[\frac{i\pi}{\lambda d_2}(x_2^2 + y_2^2)\right] \\
&\times \iint\limits_{-\infty}^{+\infty} u_{\text{out}}(x_1, y_1) \exp\left[\frac{i\pi}{\lambda d_2}(x_1^2 + y_1^2)\right] \exp\{-2\pi i(x_1 f_x + y_1 f_y)\} \mathrm{d}x_1 \mathrm{d}y_1
\end{aligned} \tag{11}$$

where $f_x = x_2/\lambda d_2$, $f_y = y_2/\lambda d_2$. Substituting Eq. (8) into Eq. (11) we obtain

$$\begin{aligned}
u_{\text{in}}(x_2, y_2) &= \frac{\exp(ikd_2)}{i\lambda d_2} \exp\left[\frac{i\pi}{\lambda d_2}(x_2^2 + y_2^2)\right] \\
&\times A \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} t_{mn} \iint\limits_{-\infty}^{+\infty} \exp\left\{\frac{i\pi}{\lambda}\left[x_1^2\left(\frac{1}{d_1} + \frac{1}{d_2}\right) + y_1^2\left(\frac{1}{d_1} + \frac{1}{d_2}\right)\right]\right\} \\
&\times \exp\{-2\pi i[x_1(f_x + f_{xm}) + y_1(f_y + f_{yn})]\} \mathrm{d}x_1 \mathrm{d}y_1 \\
&= \frac{\exp(ikd_2)}{i\lambda d_2} \exp\left[\frac{i\pi}{\lambda d_2}(x_2^2 + y_2^2)\right] \\
&\times A \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} t_{mn} \iint\limits_{-\infty}^{+\infty} \exp\left\{\frac{i\pi}{\lambda D}[x_1^2 + y_1^2]\right\} \exp -2\pi i[x_1 \tilde{f}_{xm} + y_1 \tilde{f}_{yn}] \mathrm{d}x_1 \mathrm{d}y_1
\end{aligned} \tag{12}$$

where:

$$D=\frac{d_1 d_2}{d_1+d_2} \quad \text{and} \quad f_{xm}^{\sim}=f_x+f_{xm}, \quad f_{yn}^{\sim}=f_y+f_{yn}. \tag{13}$$

As can be seen, a double sum of Fourier transforms appears in (12), each being applied to unity partial spherical waves of general form

$$\exp\left[\frac{i\pi}{\lambda D}(x_1^2+y_1^2)\right] \tag{14}$$

and, thus, of the form of Gaussian functions. Clearly, for these functions an accurate Fourier function is known.* By performing these transforms for $c=-i/(\lambda D)$ we obtain

$$u_{\text{in}}(x_2,y_2)=A\,\frac{\exp(ikd_2)}{i\lambda d_2}\exp\left[\frac{i\pi}{\lambda d_2}(x_2^2+y_2^2)\right]\left(\frac{\lambda D}{-i}\right)$$
$$\times\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}t_{mn}\exp[i\pi\lambda D(f_{xm}^{\sim 2}+f_{yn}^{\sim 2})]. \tag{15}$$

However, when substituting to (15) the corresponding values from formula (13), instead of $f_{xm}^{\sim}$ and $f_{yn}^{\sim}$, we get

$$u_{\text{in}}(x_2,y_2)=W(x_2,y_2,d_1,d_2)$$
$$\times\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}t_{mn}\exp[i\pi\lambda D(f_{xm}^2+f_{yn}^2)]$$
$$\times\exp\{i\pi\lambda D[f_x^2+f_y^2+2f_xf_{xm}+2f_yf_{yn}]\}$$
$$=W(x_2,y_2,d_1,d_2)\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}t'_{mn}\exp\{i\pi\lambda D[f_x^2+f_y^2+2f_xf_{xm}+2f_yf_{yn}]\} \tag{16}$$

where

$$t'_{mn}=t_{mn}\exp[i\pi\lambda D(f_{xm}^2+f_{yn}^2)], \tag{17}$$

and

$$W(x_2,y_2,d_1,d_2)=\frac{D}{d_2}\exp(ikd_2)\exp\left[\frac{i\pi}{\lambda d_2}(x_2^2+y_2^2)\right]=W(d_1,d_2)\exp\left[\frac{i\pi}{\lambda d_2}(x_2^2+y_2^2)\right], \tag{18}$$

---

* This transform has the form

$$\mathscr{F}\{\exp-\pi c(x^2+y^2)\}=\frac{1}{c}\exp\left[-\pi\frac{f_x^2+f_y^2}{c}\right], \tag{14a}$$

for c being an arbitrary constant.

for

$$W(d_1, d_2) = \frac{D}{d_2} \exp ikd_2 = \frac{d_1}{d_1 + d_2} \exp ikd_2.$$

Formula (16) expresses the superposition of the set of partial spherical waves, this time described in the space of spatial frequencies. Representing this superposition in terms of (x_2, y_2) variables prescribed to the entrance pupil of the optical system is reduced to performing the substitution $x_2 = f_x \lambda d_2$, $y_2 = f_y \lambda d_2$ and $x_m = f_{xm} \lambda d_2$, $y_n = f_{yn} \lambda d_2$, respectively, which leads to the expression of the type

$$\begin{aligned} u_{\text{in}}(x_2, y_2) &= W(x_2, y_2, d_1, d_2) \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} t'_{mn} \exp\left[\frac{i\pi D}{\lambda d_2^2}(x_2^2 + y_2^2 + 2x_2 x_m + 2y_2 y_n)\right] \\ &= W(d_1, d_2) \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} t'_{mn} \exp\left[\frac{i\pi}{\lambda B}\left(x_2^2 + y_2^2 + 2x_2\left(\frac{DB}{d_2^2}\right)x_m + 2y_2\left(\frac{DB}{d_2^2}\right)y_n\right)\right] \\ &= W(d_1, d_2) \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} t'_{mn} \exp \frac{i\pi}{\lambda B}(x_2^2 + y_2^2 + 2x_2 x'_m + 2y_2 y'_n) \end{aligned} \tag{19}$$

where:

$$1/B = \frac{d_2 + D}{d_2^2} = \frac{2d_1 + d_2}{d_2(d_1 + d_1)} \quad \text{and} \quad x'_m = \frac{DB}{d_2^2} x_m, \quad y'_m = \frac{DB}{d_2^2} y_n. \tag{20}$$

Thus, as a result of the free propagation in the object space (along the distance d_2) we can see that the state of the optical field in the entrance pupil can be described with the accuracy to the coefficient $W(x_2, y_2, d_1, d_2)$, as a coherent superposition of the partial spherical waves of general form

$$t'_{mn} = \exp \frac{i\pi}{\lambda B}(x_2^2 + y_2^2 + 2x_2 x'_m + 2y_2 y'_n). \tag{21}$$

Comparing the above with the state of optical field at the exit from the amplitude object we see that the new partial spherical waves are seemingly generated by the virtual point sources of new positions, this time described by the respective coordinates $(x'_m, y'_n, -B)$. In turn, their complex amplitudes t'_{mn} are perturbed by the quadratic phase factors $\exp[i\pi\lambda D(f_{xn}^2 + f_{yn}^2)]$; they are, however, connected with the amplitudes t_{mn} of the relevant harmonics in the amplitude object by formula (17), which results in an obvious equality of the modules $|t_{mn}| = |t'_{mn}|$. The phase perturbation in the form of the expression $\exp[i\pi\lambda D(f_{xm}^2 + f_{yn}^2)]$ has, as mentioned before, the character of quadratic phase factor and is different for each amplitude t_{mn} of particular harmonics of the amplitude object. This situation can be alternatively interpreted, for example, as an appearance (due to propagation along the distance d_2) of diversified phase shifts in the virtual point sources of the partial object waves, *i.e.*, in the points of co-ordinates $(x_m = \lambda d_1 f_{xm}, y_n = \lambda d_1 f_{yn})$. Note additionally that the appearing phase disturbance changes also with the parameter D and thus with the distances d_1 and d_2 characterising the geometry of the part of the optical system travelled so far by the corresponding waves (see Fig. 1 or Fig. 4), *i.e.*, the illuminator and the object space.

4. Transformation in the optical system

As mentioned above, we assume the optical system to be reduced to a single substitution lens which for the sake of highest simplicity is considered as thin and infinitely extended one.* Let us assume additionally that this lens is idealised in the sense that there are no aberrations in its entrance and exit pupils. Under these circumstances the optical system can be treated as a simple phase transform of amplitude transmittance $t(x_2, y_2)$ of the type

$$t(x_2, y_2) = \exp \frac{-i\pi}{\lambda f}(x_2^2 + y_2^2) \tag{22}$$

where f denotes the *substituting focal length*, while the whole expression defines change of the phase suffered by all the partial waves when passing from the entrance to the exit pupils.

Thus, the transformation of the optical field by the substitution lens is of the form

$$\begin{aligned} u_{\mathrm{out}}(x_2, y_2) &= \exp\left[\frac{-i\pi}{\lambda f}(x_2^2 + y_2^2)\right] u_{\mathrm{in}}(x_2, y_2) = W(d_1, d_2) \\ &\times \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} t'_{mn} \exp\left\{\frac{i\pi}{\lambda}\left[x_2^2\left(\frac{1}{B} - \frac{1}{f}\right) + y_2^2\left(\frac{1}{B} - \frac{1}{f}\right) + 2x_2\frac{x_m}{B} + 2y_2\frac{y_n}{B}\right]\right\} \\ &= W(d_1, d_2) \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} t'_{mn} \exp\left[\frac{i\pi}{\lambda C}(x_2^2 + y_2^2 + 2x_2 x''_m + 2y_2 y''_n)\right] \end{aligned} \tag{23}$$

where:

$$C = \frac{Bf}{f - B}, \tag{24}$$

and

$$x''_m = (C/B)x_m, \quad y''_n = (C/B)y_n. \tag{25}$$

As we see, the positions of the virtual point sources of the partial waves given originally by the co-ordinates $(x_m, y_n, -d_1)$ are changed to the positions defined by $(x''_m, y''_n, -C)$. Let us note again that Eq. (23) describing the state of the optical field at the exit pupil of the optical system has the form of a coherent superposition of the suitably modified partial spherical waves, this time of the type

* As mentioned earlier, this assumption allows us to avoid complications following from diffraction on the rims of the lens.

$$\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} t'_{mn}\exp\left[\frac{i\pi}{\lambda C}(x_2^2+y_2^2+2x_2x''_m+2y_2y''_n)\right]. \tag{26}$$

It is worth noting that the parameter C common for all the partial waves contains the information about the focusing properties of the optical system and the influence of the geometry of this part of the optical system which has been passed so far by each of the partial waves and by the same means on the partial information about the corresponding harmonics of the object structure. As can be easily noticed at this stage, the positions of the point sources of the partial waves superposing in the exit pupil are changed again from these defined by the co-ordinates $(x'_m, y'_n, -B)$ to those defined by co-ordinates $(x'_m, y'_n, -C)$. This time, however, the point sources of the partial spherical waves are not necessarily virtual. All depends on the sign of the parameter C which can be either negative or positive depending on which of the inequalities is fulfilled

$$B > f \quad \text{or} \quad B < \text{f}. \tag{27}$$

In the first case $C < 0$, which means that all the partial spherical waves become convergent, while for the case of $C > 0$ the partial spherical waves remain divergent. This means that the images of the sources become real in the first case, while they remain virtual in the other.

5. Transport of information in the image space

The last stage of the object information transport in the simplified optical system considered is the free propagation from the exit pupil to the plane of observation. Here also the Fresnel approximation will be used, in the framework of which the optical field in the observation plane, after some calculations analogous to those applied for the free propagation in the object space (compare the passage from Eqs. (12) to (13)), takes the form

$$U(x_3,y_3) = \frac{\exp ikd_3}{i\lambda d_3}\exp\frac{i\pi}{\lambda d_3}(x_3^2+y_3^2)\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} U_{\text{out}}(x_2,y_2)$$
$$\times\exp\frac{i\pi}{\lambda d_3}(x_2^2+y_2^2)\exp -2\pi i(x_2f'_x+y_2f'_y)\mathrm{d}x_2\mathrm{d}y_2 \tag{28}$$

where: $f'_x = x_3/(\lambda d_3), f'_y = y_3/(\lambda d_3)$. Substituting to the above equation the expression for $U_{\text{out}}(x_2, y_2)$ defined by Eq. (23) and denoting

$$W(d_1,d_2,d_3) = W(d_1,d_2)\,\frac{\exp ikd_3}{i\lambda d_3}$$

we obtain

$$U(x_3,y_3) = W(d_1,d_2,d_3)\exp\left[\frac{i\pi}{\lambda d_3}(x_3^2+y_3^2)\right]\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} t'_{mn}$$

$$\times \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \exp\left[\frac{i\pi}{\lambda C}(x_2^2+y_2^2+2x_2x_m''+2y_2y_n'')\right]$$

$$\times \exp\left[\frac{i\pi}{\lambda d_3}(x_2^2+y_2^2)\exp[-2\pi i(x_2f_x'+y_2f_y')]\mathrm{d}x_2\mathrm{d}y_2\right.$$

$$= W(d_1,d_2,d_3)\exp\left[\frac{i\pi}{\lambda d_3}(x_3^2+y_3^2\right]\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} t_{mn}$$

$$\times \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \exp\left[\frac{i\pi}{\lambda}(x_2^2+y_2^2)\left(\frac{1}{C}+\frac{1}{d_3}\right)\right]$$

$$\times \exp\left\{-2\pi i\left[x_2\left(f_x'-\frac{x_m''}{\lambda C}\right)+y_2\left(f_y'-\frac{y_n''}{\lambda C}\right)\right]\right\}\mathrm{d}x_2\mathrm{d}y_2. \tag{29}$$

Substituting

$$f_{xm}''=\frac{x_m''}{\lambda C},\quad f_{yn}''=\frac{x_n''}{\lambda C}\quad\text{and}\quad \frac{1}{E}=\frac{1}{C}+\frac{1}{d_3} \tag{30}$$

we get

$$U(x_3,y_3)=W(d_1,d_2,d_3)\exp\left[\frac{i\pi}{\lambda d_3}(x_3^2+y_3^2)\right]$$

$$\times \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} t_{mn}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\exp\left[\frac{i\pi}{\lambda E}(x_2^2+y_2^2)\right]$$

$$\times\exp\{-2\pi i[x_2(f_x'-f_{xm}'')+y_2(f_y'-f_{yn}'')]\}dx_2\mathrm{d}y_2.$$

Thus, we have again to do with a Fourier transform, this time, of the Gaussian function: $\exp\left[\frac{i\pi}{\lambda E}(x_2^2+y_2^2)\right]$, from the plane x_2, y_2 associated with the exit pupil to the space of differences of spatial frequencies

$$f_x^{\approx}=f_x'-f_{xm}'',\quad f_y^{\approx}=f_y'-f_{yn}''. \tag{31}$$

Performing the Fourier transformation, according to (14a), for $c = -i/\lambda E$ we obtain

$$U(x_3,y_3)=W(d_1,d_2,d_3)\exp\left[\frac{i\pi}{\lambda d_3}(x_3^2+y_3^2)\right]\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} t_{mn}'\exp\left[i\pi\lambda E(f_x^{\approx 2}+f_y^{\approx 2})\right]. \tag{32}$$

Passing to the co-ordinates x_3, y_3 in the observation plane finally gives

$$U(x_3,y_3)=W(d_1,d_2,d_3)\exp\left[\frac{i\pi}{\lambda d_3}(x_3^2+y_3^2)\right]$$

$$\times \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} t'_{mn} \exp\left[\frac{i\Delta}{\lambda F} \lambda E (x_3^2 + y_3^2 - 2x_3 x''_m - 2y_3 y''_n)\right] \tag{33}$$

where:

$$t''_{mn} = t'_{mn} \exp i\pi\lambda E(f_{xm}^2 + f_{yn}^2), \quad x''_m = x''_m \frac{ED}{cd_3}, \quad y''_m = y''_m \frac{ED}{cd_3}, \quad \frac{1}{F} = \frac{1}{d_3} + \frac{\lambda^2 E}{d_3}. \tag{34}$$

Comparing the final formula (33) with both the state of the optical field (8) emerging from the object plane and given in the form

$$u_{\text{out}}(x_1, y_1) = u_{\text{in}}(x_1, y_1) \; t(x_1, y_1)$$
$$= A \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} t_{mn} \exp \frac{i\pi}{\lambda d_1} [x_1^2 + y_1^2 - 2x_1 x_m - 2y_1 y_n],$$

and the structure of the even amplitude object (1) of the amplitude transmittance in the form

$$t(x_1, y_1) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} t_{mn} \exp \frac{-2i\pi}{\lambda d_1} [x_1 x_{1m} + y_1 y_{1n}]$$

some striking formal similarities appear visible (apart from the functional factor $W(d_1, d_2, d_3) \times \exp[i\pi/\lambda d_3 (x_3^2 + y_3^2)]$) which allow us to formulate the following final conclusions.

6. Final conclusions

The above considerations allowed a relatively simple presentation of the diffraction mechanism of the transport of optical information in simplified optical systems visualising their following features:

– The object information for even amplitude objects is encoded in two sets of data, *i.e.*, the set $\{t_{mn}\}$ of amplitudes of the harmonics of the amplitude transmittance distribution and the set $\{f_{xm}, f_{yn}\}$ of spatial frequencies in the object, which are all transported by a set of corresponding partial spherical waves generated by the object if illuminated by a spherical wave.

– In the course of the transportation, these waves are subject to the relevant transformations and in particular both their amplitudes t_{mn} and their spatial frequencies f_{xm} and f_{yn} suffer from the following modifications:

$$t''_{mn} = t_{mn} \exp[i\pi\lambda D (f_{xm}^2 + f_{yn}^2) \exp[i\pi\lambda F (f_{xm}^2 + f_{yn}^2)]$$
$$= t_{mn} \exp[i\pi\lambda (D + F)(f_{xm}^2 + f_{yn}^2)]$$

and

$$f_{xm}^{4''} = K f_{xm} \quad \text{and} \quad f''_{yn} = K f_{yn},$$

respectively.

– From the above it follows that: **a**) $|t_{xm}| = |t''_{xm}|$ and $|t_{yn}| = |t''_{yn}|$ and consequently these modules are invariants of the transformations of the partial spherical waves in the optical system, and **b**) the spatial frequencies satisfy the equations: $f''_{xm} = mf''_{x1} = mKf_{x1}$ and $f''_{yn} = nf''_{y1} = nKf_{y1}$ and, consequently, the spatial frequencies preserve their mutual relations in the sense that they are entire multiples of the basic spatial frequencies f''_{x1} and f''_{y1} which, in turn, are proportional to the basic spatial frequencies f_{x1} and f_{y1} determining the spatial frequency spectrum constituting the frequency structure of the even amplitude object.

This state of affairs allows us to notice that the considered transport of information about an even amplitude object occurring in an extremely simplified model of an optical system preserves (in the face of the arbitrariness of the parameters d_1, d_2 and d_3 determining all the transport constants D, B, C, E, F and, consequently, also K) the whole information about the even amplitude object encoded in $\{t_{mn}\}$ and $\{f_{xm}, f_{yn}\}$ during all the stages of transportation, though modifying them in a simple way just explained.

The above considerations provide a basis for formulation of a kind of **law of information conservation**, which says that *the optical information about an amplitude object is preserved in all the stages of its transportation* (*and by the same means is valid for all its possible transformations occurring on the way*), *though its form usually suffers from the corresponding modifications*. This law is true, at least, for the case of simplified coherent optical system discussed above.

Let us add that the said law of information conservation is of not only theoretical importance providing an insight into the diffraction aspect of information transport in optical systems. It can have a practical significance as well following from the consciousness that in principle this information could be extracted from an arbitrary observation plane (located perpendicularly to the optical axis), though the comfort of this extraction can be highly differentiated due to the phase recovery problem which appears in spite of the real character of the amplitude object in all the cases different from conventional object-image relation.

Different degree of complexity when extracting the optical information follows from different results of superposition of differently modified partial spherical waves at different observation planes. The simplest cases are: i) identity transformation which occurs for such a system when the lens formula holds, and ii) when the Fourier transform of the optical information is needed, which is achievable if the known conditions for the parameters d_1, d_2, d_3 and f are satisfied [2]. The derivations of the respective formulas from the general expressions given in this paper are simple and can be suggested as a problem to be solved by the students.

7. Critical remarks

The relative high simplicity of the above considerations leading in a natural way to formulation of the law of optical information conservation has been achieved only because we have assumed some far going approximations facilitating the diffraction description of the propagation phenomenon in so much reduced optical system. The most radical simplifications were:

– assumption of the monochromatic point source generating coherent spherical wave which consequently limits the considerations to the coherent optical system;

– assumption of the spherical wave illuminating the amplitude object in the form $\exp\left[\frac{i\pi}{\lambda d_1}(x_1^2+y_1^2)\right]$, in spite of the fact that the object is infinitely extended;

– assumption that the optical system is represented by a single thin but infinitely extended substitution lens;

– assumption that the pupils of the optical system are aberration-free;

– assumption of the Fresnel approximation to describe the mechanism of propagation despite infinite sizes of both optical system and the observation plane;

– neglecting the influence of the detection stage.

Obviously, the real optical systems do not meet those assumptions. Therefore, the real transport of optical information is realised in a way much more complicated which results in relevant losses of the information both during its transport and detection. Consequently, the real optical systems satisfy the conservation law for optical information only approximately. However, even in its approximate version this law can play an essential role offering a kind of "ideal reference level" for the real optical systems, on the one hand, and providing a deepened insight into the essence of the diffraction transport of optical information

References

[1] Wilk I., Wilk P., *Optyka fizyczna*, Part I *Dyfrakcja światla* (in Polish), Oficyna Wydawnicza PWr., Wrocław 1995.

[2] Goodman J. W., *Introduction to Fourier Optics*. McGraw-Hill Book Co., New York 1968.

Received July 26, 1999