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Matricial relations for polar Kerr-effect multifilm and bulk systems at oblique incidence

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A simplified matricial formalism for the polar Kerr-effect multifilm and bulk systems at oblique incidence is presented. Overall 4×4 characteristic matrices are determined for multifilms comprising both dielectric and magnetooptic thin layers having their boundaries placed into the ambient isotropic medium. Then, 2×2 extended Jones reflection matrices are obtained for polar Kerr-effect multifilm and bulk systems. Numerical examples of Kerr rotation angle and figure of merit function variations against the incident angle are given comparatively for *p*- and *s*-polarized incident light.

1. Introduction

Films of rare earth-transition metal alloys that exhibit polar Kerr-effect behaviour are considered as promising materials in erasable optical storage systems using thermomagnetic writing and magnetooptical readout [1]–[3]. Since the polar Kerr-effect is described in terms of reflection changes of the polarized incident light, determination of reflection matrix is helpful in characterization and understanding of the performances of readout systems [3]–[6].

A detailed description of the Kerr-effect in bulk and thin-film materials is given in a number of references [5]–[10]. It turns out that obtaining explicit results involves rather complicated algebra.

In this work, we present a simplified matricial formalism for the polar Kerr-effect multifilm and bulk systems at oblique incidence. We obtained a major simplification by assuming that each thin layer is embedded in between two imaginary ambient isotropic layers of zero thickness. Since both interfaces of the layer are imaginatively placed into the same isotropic medium, the final expressions are much simplified. By this procedure each layer can be seen and treated as a separate entity. Different kinds of anisotropy can be easily accounted for, as in the case of magnetooptic films coated on biaxial substrates [11].

Obviously, the theory of electromagnetic wave propagation in lossless anisotropic media makes use of unit electric and magnetic field vectors that are expressed in terms of three characteristic angles [12]: the angle of refraction (determined by the Snell's law), the polarization angle, and the walk-off angle (formed by the electric field and electric displacement vectors). Since magnetooptical media are both absorbing and anisotropic, these characteristic angles are complex and their physical meaning is lost [10], but we can still use them successfully. Rather simple expressions are obtained in terms of trigonometric functions of complex arguments.

One of the disadvantages of the Kerr-effect readout is that the Kerr rotation is small. Hence, the technique of enhancing the Kerr effect by coating dielectric thin layers on the magnetooptic films is obviously applied [4]. Reflection at oblique incidence on isotropic dielectric layers is usually described in terms of complex amplitude Fresnel reflection coefficients for p- and s-polarized light [13]. In the case of multifilm readout systems it seems worthwhile to have a framework equally well applicable to both isotropic and anisotropic layers. Therefore, the 2×2 extended Jones matrices [14] relating the reflected and transmitted amplitudes of the p and s modes are adequate to be used. In this way, we can check easily the results by comparing them to those obtained in limits of isotropy.

2. Notations and general relations

Let us consider a magnetooptic thin layer that is placed into the isotropic ambient medium of refractive index n_0. The coordinate system is chosen so that the interfaces are parallel to the x-y plane and the magnetization is in the z (polar) direction. The dielectric tensor $\tilde{\varepsilon}$ can be written as [3], [4]

$$\tilde{\varepsilon} = \varepsilon \begin{bmatrix} 1 & jq & 0 \\ -jq & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \tag{1}$$

$j = (-1)^{1/2}$, the unmagnetized index of refraction is $n = \varepsilon^{1/2}$, and q is proportional to the magnetic field.

Let a monochromatic plane wave with angular frequency ω be incident in the x-z plane at angle φ_0 with respect to the positive z axis. It propagates along the unit wave vector $\hat{k}_0^+ = (\sin\varphi_0, 0, \cos\varphi_0)^T$, where $(\ )^T$ denotes the transposed vector. At the front interface the wave is divided into a backward-reflected wave of wave vector k_0^- and two forward-propagating waves of wave vectors $\mathbf{k}_i^+$, $i = 1, 2$. At the second interface, inside the magnetooptic layer, there are two backward-propagating waves of wave vectors $\mathbf{k}_1^- = -\mathbf{k}_2^+$, and $\mathbf{k}_2^- = -\mathbf{k}_1^+$ [5]. The wave vectors $\mathbf{k}_i^\pm$, $i = 1, 2$, may be written as

$$\mathbf{k}_i^\pm = \left(\frac{\omega}{c}\right)(\xi\hat{x} + \zeta_i^\pm \hat{z}), \quad i = 1, 2 \tag{2}$$

c is the vacuum velocity of light, $\hat{x}$ and $\hat{z}$ are unit vectors along the positive x and z axes, $\xi = n_0 \sin\varphi_0$ is the tangential component that is the same for all wave vectors, and $\zeta_i^\pm = n_i^\pm \cos\varphi_i^\pm$, where $n_i^\pm$ are indices of refraction, and $\varphi_i^\pm$ are the respective refraction angles. In general, for absorbing magnetooptic layers, $\zeta_i^\pm$ are complex and are determined by relations [5]:

$$\zeta_1^+ = [\zeta(\zeta - nq)]^{1/2}, \quad \zeta_2^+ = [\zeta(\zeta + nq)]^{1/2}, \quad \zeta_1^- = -\zeta_2^+, \quad \zeta_2^- = -\zeta_1^+ \tag{3}$$

where $\zeta = (\varepsilon - \xi^2)^{1/2}$. Indices of refraction are given by $n_i^\pm = [\xi^2 + (\zeta_i^\pm)^2]^{1/2}$, so that $n_1^- = n_2^+$ and $n_2^- = n_1^+$. Refraction angles $\varphi_i^\pm$ are given by $\varphi_i^\pm = \arccos(\zeta_i^\pm / n_i^\pm)$. Thus, one obtains $\varphi_1^- = \pi - \varphi_2^+$ and $\varphi_2^- = \pi - \varphi_1^+$. Then, unit wave vectors can be specified as $\hat{k}_i^\pm = (\sin\varphi_i^\pm, 0, \cos\varphi_i^\pm)^T$.

Let $\mathbf{E}_i^\sigma$ and $\mathbf{H}_i^\sigma$ ($i = 1, 2, \sigma = \pm$) be electric and magnetic field vectors of forward- and backward-propagating waves, and $\mathbf{S}_i^\sigma = \mathbf{E}_i^\sigma \times \mathbf{H}_i^\sigma$ be the respective Poynting vectors. In a lossles anisotropic medium vectors $\mathbf{E}_i^\sigma$, $\mathbf{H}_i^\sigma$, and $\mathbf{S}_i^\sigma$ ($i = 1, 2, \sigma = \pm$) form four orthogonal systems that are rotated against the (x, y, z) coordinate system as shown in Fig. 1 [12], [15]. The relationship between $(\mathbf{E}_i^\sigma, \mathbf{H}_i^\sigma, \mathbf{S}_i^\sigma)$ and (x, y, z) systems is specified by the refraction angle φ_i^σ, the polarization angle α_i^σ, and the walk-off angle δ_i^σ [12]

$$\begin{bmatrix} \hat{E}_i^\sigma \\ \hat{H}_i^\sigma \\ \hat{S}_i^\sigma \end{bmatrix} = \begin{bmatrix} \cos\delta_i^\sigma & 0 & -\sin\delta_i^\sigma \\ 0 & 1 & 0 \\ \sin\delta_i^\sigma & 0 & \cos\delta_i^\sigma \end{bmatrix} \begin{bmatrix} \cos\alpha_i^\sigma & \sin\alpha_i^\sigma & 0 \\ -\sin\alpha_i^\sigma & \cos\alpha_i^\sigma & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\varphi_i^\sigma & 0 & -\sin\varphi_i^\sigma \\ 0 & 1 & 0 \\ \sin\varphi_i^\sigma & 0 & \cos\varphi_i^\sigma \end{bmatrix} \begin{bmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \end{bmatrix}. \tag{4}$$

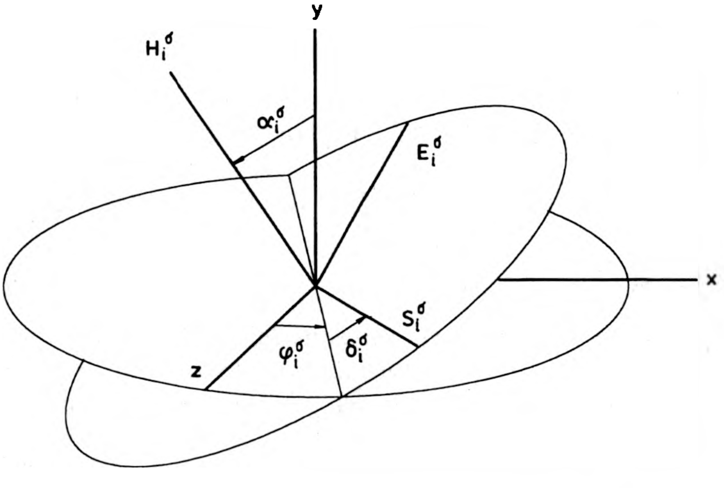

Fig. 1. Relationship between the $(E_i^\sigma, H_i^\sigma, S_i^\sigma)$ orthogonal triplet and the (x, y, z) laboratory coordinate system. The relationship is specified by angles α_i^σ, φ_i^σ, and δ_i^σ.

This general relation can be applied also to absorbing magnetooptic media. The polarization angles are determined by

$$\begin{aligned} \alpha_1^+ &= \arctan[jn_1^+\zeta/(n\zeta_1^+)], \quad \alpha_1^- = -\alpha_2^+, \\ \alpha_2^+ &= \arctan[-jn_2^+\zeta/(n\zeta_2^-)], \quad \alpha_2^- = -\alpha_1^+. \end{aligned} \tag{5}$$

The walk-off angles are given by relations:

$$\begin{aligned} \delta_i^+ &= \arctan(jq\xi\sin\alpha_i^+/n_i^+), \quad i = 1, 2, \\ \delta_1^- &= -\delta_2^+, \quad \delta_2^- = -\delta_1^+. \end{aligned} \tag{6}$$

These relations result from Maxwell's equations. Further, one obtains $n_i^\sigma \cos\delta_i^\sigma = n$, where $i = 1, 2$ and $\sigma = \pm$.

Although this procedure may seem complicated, it presents some advantages: it leads to simpler algebraic expressions, it allows determination and checking of the unit vectors separately before using them to calculate reflection matrices, and finally, it allows simple general programs accounting for different kinds of anisotropy to be developed.

3. 4 × 4 characteristic matrix of a magnetooptic thin layer having both boundaries into the ambient medium

Let us consider a magnetooptic thin layer having both boundaries imaginatively placed into the ambient medium. Let $E_{01p}^{\pm}$, $E_{01s}^{\pm}$ and $E_{02p}^{\pm}$, $E_{02s}^{\pm}$ be the complex amplitudes of the p and s modes of forward- and backward-propagating waves into the incident (index 01) and emergent (index 02) ambient medium, and let $E_i^{\pm}$ ($i = 1, 2$) be the complex amplitudes of electric fields for waves propagating into the magnetooptic layer. Electric field vectors of forward- and backward-travelling waves into the incident and emergent ambient regions are given by [14]

$$\mathbf{E}_{0i}^\sigma = (E_{0is}^\sigma \hat{s} + E_{0ip}^\sigma \hat{p}^\sigma) \exp[j(\omega t - \mathbf{k}_0^\sigma \mathbf{r})] \tag{7}$$

where: $i = 1, 2$, $\sigma = \pm$, $\hat{s} = (0, 1, 0)^T$ is a unit vector perpendicular to the incident plane, $\hat{p}^\sigma$ are unit vectors parallel to the incident plane that are given by $\hat{p}^\sigma = \hat{k}_0^\sigma \times \hat{s} = (-\sigma \cos\varphi_0, 0, \sin\varphi_0)^T$. The time dependence is specified by $\exp(j\omega t)$. The respective magnetic-field vectors are given by $\mathbf{H}_{0i}^\sigma = \mathbf{k}_0^\sigma \times \mathbf{E}_{0i}^\sigma/\omega$. Using Eq. (7) gives [14]

$$\mathbf{H}_{0i}^\sigma = (n_0/c)(E_{0is}^\sigma \hat{p}^\sigma - E_{0ip}^\sigma \hat{s}) \exp[j(\omega t - \mathbf{k}_0^\sigma \mathbf{r})]. \tag{8}$$

As far as the magnetooptic layer is concerned, we consider that the resultant electric-field vector of all the forward-travelling waves adds up to $\mathbf{E}_m^+$ and those of backward-travelling to $\mathbf{E}_m^-$. These resultant electric-field vectors are given by

$$\mathbf{E}_m^\sigma = \left[\sum_{i=1}^{2} E_i^\sigma \hat{E}_i^\sigma \exp(-j\mathbf{k}_i^\sigma \mathbf{r})\right] \exp(j\omega t), \quad \sigma = \pm. \tag{9}$$

Taking into account that $\mathbf{k}_i^\sigma \times \hat{E}_i^\sigma = (\omega/c)\hat{H}_i^\sigma n_i^\sigma \cos\delta_i^\sigma = \hat{H}_i^\sigma n\omega/c$, one obtains the resultant magnetic-filed vectors

$$\mathbf{H}_m^\sigma = \left[\sum_{i=1}^{2} E_i^\sigma \hat{H}_i^\sigma \exp(-j\mathbf{k}_i^\sigma \mathbf{r})\right] (n/c) \exp(j\omega t), \quad \sigma = \pm. \tag{10}$$

Let us denote $\bar{E}_{0i}^\sigma = (E_{0is}^\sigma, E_{0ip}^\sigma)^T$ and $\bar{E}^\sigma = (E_1^\sigma, E_2^\sigma)^T$ with $i = 1, 2$ and $\sigma = \pm$. Then, by applying the standard boundary conditions to the resultant electric- and magnetic-field vectors at the two interfaces, one obtains four matricial relations for reflected and transmitted electric-field amplitudes [16]

$$\bar{\tau}^+ \bar{E}_{01}^+ = \bar{E}^+ + \bar{\rho}^+ \bar{E}^-, \tag{11a}$$

$$\tilde{\tau}^+\bar{E}_{01}^- = \tilde{\rho}^+ - \bar{E}^+ + \bar{E}^-, \tag{11b}$$

$$\tilde{\chi}^+\bar{E}^+ + \tilde{\rho}^+\tilde{\chi}^-\bar{E}^- = X_0^+\tilde{\tau}^+\bar{E}_{02}^+, \tag{11c}$$

$$\tilde{\chi}^-\bar{E}^- + \tilde{\rho}^-\tilde{\chi}^+\bar{E}^+ = X_0^-\tilde{\tau}^-\bar{E}_{02}^-, \tag{11d}$$

$\tilde{\tau}^\sigma$, $\tilde{\rho}^\sigma$ and $\tilde{\chi}^\sigma(\sigma = \pm)$ are 2×2 matrices. The elements of the matrix $\tilde{\tau}^+$ are determined by relations:

$$\tau_{11}^+ = -j\zeta_0 a_2/(g\zeta_2^+ \cos\alpha_1^+), \tag{12a}$$

$$\tau_{12}^+ = -n_0 n\zeta_0\zeta(\zeta_0+\zeta_2^+)/(g\zeta_2^+ \cos\alpha_1^+), \tag{12b}$$

$$\tau_{21}^+ = j\zeta_0 a_1/(g\zeta_1^+ \cos\alpha_2^+), \tag{12c}$$

$$\tau_{22}^+ = -n_0 n\zeta_0\zeta(\zeta_0+\zeta_1^+)/(g\zeta_1^+ \cos\alpha_2^+) \tag{12d}$$

where $\zeta_0 = (n_0^2-\xi^2)^{1/2}$, $a_i = (n_0^2\zeta^2+\varepsilon\zeta_0\zeta_i^+)$ with $i = 1, 2$, and $g = (\zeta_0+\zeta)(n_0^2\zeta+\varepsilon\zeta_0)$. The elements of the matrix $\tilde{\rho}^+$ are determined by relations:

$$\rho_{11}^+ = -j\xi^2 n\beta\sin\alpha_2^+/(2n_2^+\zeta_0\zeta\cos\alpha_1^+), \tag{13a}$$

$$\rho_{22}^+ = j\xi^2 n\beta\sin\alpha_1^+/(2n_1^+\zeta_0\zeta\cos\alpha_2^+), \tag{13b}$$

$$\rho_{12}^+ = \beta/2+G, \quad \rho_{21}^+ = \beta/2-G \tag{13c}$$

where: $\beta = 2\zeta_0\zeta(\varepsilon-n_0^2)/g$ and $G = (\zeta_1^+-\zeta_2^+)(\varepsilon\zeta_0^2+n_0^2\zeta^2)/(2g\zeta)$. The matrices $\tilde{\tau}^-$ and $\tilde{\rho}^-$ are given by

$$\tilde{\tau}^- = \begin{bmatrix} -\tau_{21}^+ & \tau_{22}^+ \\ -\tau_{11}^+ & \tau_{12}^+ \end{bmatrix}, \quad \tilde{\rho}^- = \begin{bmatrix} \rho_{22}^+ & \rho_{21}^+ \\ \rho_{12}^+ & \rho_{11}^+ \end{bmatrix}, \tag{14}$$

$\tilde{\chi}^\sigma$ $(\sigma = \pm)$ are diagonal matrices with elements given by

$$\chi_{ii}^\sigma = \exp[-j(\omega/c)h_m\zeta_i^\sigma], \quad i = 1, 2, \quad \sigma = \pm \tag{15}$$

where h_m is the thickness of the magnetooptic layer. In Equations (11c) and (11d), $X_0^\sigma = \exp[-j\sigma(\omega/c)h_m\zeta_0]$, with $\sigma = \pm$.

Let us define the 4×4 matrices

$$\tilde{\tilde{\tau}} = \begin{bmatrix} \tilde{\tau}^+ & \tilde{O}_2 \\ \tilde{O}_2 & \tilde{\tau}^- \end{bmatrix}, \quad \tilde{\tilde{\rho}} = \begin{bmatrix} \tilde{\rho}^+ & \tilde{I}_2 \\ \tilde{I}_2 & \tilde{\rho}^- \end{bmatrix}, \quad \tilde{\tilde{\chi}} = \begin{bmatrix} \tilde{\chi}^- & \tilde{O}_2 \\ \tilde{O}_2 & \tilde{\chi}^+ \end{bmatrix} \tag{16}$$

where $\tilde{I}_2$ is the 2×2 identity matrix, and $\tilde{O}_2$ the 2×2 zero matrix. Let us also define the vectors with four components:

$$\bar{E}_{01} = (E_{01s}^+, E_{01p}^+, E_{01s}^-, E_{01p}^-)^T, \tag{17a}$$

$$\bar{E} = (E_1^-, E_2^-, E_1^+, E_2^+)^T, \tag{17b}$$

$$\bar{E}_{02} = (X_0^+E_{02s}^+, X_0^+E_{02p}^+, X_0^-E_{02s}^-, X_0^-E_{02p}^-)^T, \tag{17c}$$

Then, Eqs. (11) can be rewritten in the compact form:

$$\tilde{\tau}\bar{E}_{01} = \tilde{\rho}\bar{E}, \tag{18a}$$

$$\tilde{\tau}\bar{E}_{02} = \tilde{\rho}\tilde{\chi}\bar{E}. \tag{18b}$$

Eliminating $\bar{E}$ from Eqs. (18) gives

$$\bar{E}_{01} = \tilde{M}\bar{E}_{02}. \tag{19}$$

The matrix $\tilde{M}$ is the 4×4 characteristic matrix of the magnetooptic layer having both interfaces into the ambient medium. It is determined by relation

$$\tilde{M} = (\tilde{\tau})^{-1}\tilde{\rho}(\tilde{\chi})^{-1}(\tilde{\rho})^{-1}\tilde{\tau}. \tag{20}$$

This 4×4 characteristic matrix can be written in the form

$$\tilde{M} = \begin{bmatrix} \tilde{A}_m & \tilde{B}_m \\ \tilde{C}_m & \tilde{D}_m \end{bmatrix} \tag{21}$$

where: $\tilde{A}_m$, $\tilde{B}_m$, $\tilde{C}_m$ and $\tilde{D}_m$ are 2×2 matrices.

4. 4×4 characteristic matrix of a dielectric thin layer having both boundaries into the ambient medium

Since the polar Kerr-effect multifilm systems may comprise isotropic dielectric layers, we have to apply the same framework to both isotropic and anisotropic layers. Thus, let us consider a dielectric film of thickness h_d and index of refraction n_d, having both surfaces in the ambient medium. Since the problem of plane-wave propagation in isotropic media is well known [13], we will present further only relations for the 2×2 matrices $\tilde{A}$, $\tilde{B}$, $\tilde{C}$ and $\tilde{D}$. All of them are diagonal matrices. Let A_s and A_p be the elements on the main diagonal of matrix $\tilde{A}$ ($A_{11} = A_s$ and $A_{22} = A_p$). They are given by

$$A_v = X_d^{-1}(1 - r_v^2 X_d^2)/(1 - r_v^2), \quad v = p, s \tag{22a}$$

where: $X_d = \exp[-j(\omega/c)h_d\zeta_d]$, $\zeta_d = (n_d^2 - \xi^2)^{1/2}$ and r_v, with $v = p, s$, are Fresnel reflection coefficients, $r_s = (\zeta_0 - \zeta_d)/(\zeta_0 + \zeta_d)$ and $r_p = (n_d^2\zeta_0 - n_0^2\zeta_d)/(n_d^2\zeta_0 + n_0^2\zeta_d)$.

Similarly, we denote by B_v, C_v and D_v, with $v = p, s$, the elements on the main diagonal of matrices $\tilde{B}$, $\tilde{C}$ and $\tilde{D}$. They are determined by relations:

$$B_v = -X_d^{-1} r_v (1 - X_d^2)/(1 - r_v^2), \tag{22b}$$

$$C_v = -B_v, \tag{22c}$$

$$D_v = X_d^{-1}(X_d^2 - r_v^2)/(1 - r_v^2). \tag{22d}$$

Note that elements of the main diagonal of matrices $\tilde{A}_m$, $\tilde{B}_m$, $\tilde{C}_m$ and $\tilde{D}_m$ for the magnetooptic layer are well approximated by relations similar to Eqs. (22). For example, elements A_{mii} ($i = 1, 2$) on the main diagonal of the matrix $\tilde{A}_m$ are given approximately by relations similar to Eq. (22a)

$$A_{m11} \simeq (1/2)\sum_{i=1}^{2} X_i^{-1}(1 - r_{is}^2 X_i^2)/(1 - r_{is}^2), \tag{23a}$$

$$A_{m22} \simeq (1/2)\sum_{i=1}^{2} X_i^{-1}(1-r_{ip}^2 X_i^2)/(1-r_{ip}^2) \tag{23b}$$

where r_{is} and r_{ip} ($i = 1, 2$) are Fresnel reflection coefficients at the ambient/magnetooptic interface, $r_{is} = (\zeta_0-\zeta_i^+)/(\zeta_0+\zeta_i^+)$, $r_{ip} = [(n_i^+)^2\zeta_0-n_0^2\zeta_i^+]/[(n_i^+)^2\zeta_0+n_0^2\zeta_i^+]$, and $X_i = \chi_{ii}^+$ that is given by Eq. (15). The approximate relations are useful to check the respective, exactly determined elements of the 4×4 characteristic matrix for the magnetooptic thin film.

5. 4×4 characteristic matrix of the polar Kerr-effect multifilm placed into the ambient medium

Once the 4×4 characteristic matrices are determined for both isotropic dielectric and anisotropic magnetooptic films, one can determine the 4×4 characteristic matrix of a multifilm placed into the ambient medium. Thus, for a succession of N films, each of them being embedded in between two imaginary ambient layers of zero thickness, the 4×4 characteristic matrix is

$$\tilde{\tilde{M}} = \tilde{\tilde{M}}_1\tilde{\tilde{M}}_2 \;\ldots\; \tilde{\tilde{M}}_{N-1}\tilde{\tilde{M}}_N. \tag{24}$$

The layers that are of either dielectric or magnetooptic materials are numbered starting from the incident ambient medium.

6. 2×2 extended Jones reflection matrix for the polar Kerr-effect multifilm coated on a substrate

Let us consider that the polar Kerr-effect multifilm is coated on a substrate. We assume that there is also an imaginary ambient layer of zero thickness between the multifilm and the substrate. By applying standard boundary conditions [14], one can determine 2×2 extended Jones matrices of reflection, $\tilde{r}_g$ and transmission $\tilde{t}_g$ at the ambient/substrate interface. For an isotropic substrate of refractive index n_g, $\tilde{r}_g$ and $\tilde{t}_g$ are diagonal matrices with Fresnel reflection and transmission coefficients r_{gv} and t_{gv} ($v = s, p$) as elements on the main diagonals [$r_{g11} = r_{gs}$, $r_{g22} = r_{gp}$, and similarly for t_{gii}, $i = 1, 2$, where $r_{gs} = (\zeta_0-\zeta_g)/(\zeta_0+\zeta_g)$, $r_{gp} = (n_g^2\zeta_0-n_0^2\zeta_g)/(n_g^2\zeta_0+n_0^2\zeta_g)$, $t_{gs} = 2\zeta_0/(\zeta_0+\zeta_g)$, $t_{gp} = 2n_g^2\zeta_0/(n_g^2\zeta_0+n_0^2\zeta_g)$, with $\zeta_g = (n_g^2-\xi^2)^{1/2}$]. Then, we obtain

$$\bar{E}_{01} = \begin{pmatrix} \tilde{A} & \tilde{B} \\ \tilde{C} & \tilde{D} \end{pmatrix}\begin{pmatrix} \tilde{I}_2 \\ \tilde{r}_g \end{pmatrix}(\tilde{t}_g)^{-1}\bar{E}_g^+ \tag{25}$$

where: $\bar{E}_{01}$ is defined by Eq. (17a), $\tilde{A}$, $\tilde{B}$, $\tilde{C}$ and $\tilde{D}$ are 2×2 matrices forming the overall 4×4 characteristic matrix $\tilde{\tilde{M}}$ of the multifilm that was defined by Eq. (24), $\tilde{I}_2$ is the 2×2 identity matrix, and $\bar{E}_g^+ = (E_{gs}^+, E_{gp}^+)^T$ with E_{gv}^+, $v = s, p$, the complex amplitudes of the s and p modes of waves transmitted into the substrate. From Eq. (25) we get the 2×2 extended Jones reflection matrix $\tilde{r}$ of the polar Kerr-effect multifilm coated on a substrate

$$\tilde{r} = (\tilde{C}+\tilde{D}\tilde{r}_g)(\tilde{A}+\tilde{B}\tilde{r}_g)^{-1}. \tag{26}$$

Note that explicit matricial relations presented in this work for polar Kerr-effect multifilm systems are simpler than those given in [10].

7. 2 × 2 extended Jones reflection matrix for a polar Kerr-effect bulk system

Let us consider a simple bulk magnetooptic material inserted into the ambient medium. Using Eqs. (11)–(15) gives the 2 × 2 extended Jones reflection matrix of the bulk magnetooptic material, $\tilde{r}_b$ with elements given by relations:

$$r_{b11} = (r_{1s}+r_{2s})/2, \quad r_{b22} = (r_{1p}+r_{2p})/2,$$
$$r_{b12} = r_{b21} = -jq\varepsilon n_o\zeta_0/[(\zeta_0+\zeta)(n_0^2\zeta+\varepsilon\zeta_0)] \tag{27}$$

where r_{is} and r_{ip} $(i = 1, 2)$ are Fresnel reflection coefficients at the ambient/magnetooptic interface. These relations are considerably simpler than those presented in [3].

8. Numerical example

Let us consider a polar Kerr-effect bulk system with $n = 2.96-j3.4$, $q = 0.001 -j0.025$ [5], in air ($n_0 = 1$). Let us define the ratio $\gamma_s = r_{b12}/r_{b11}$ in the case of s-polarized incident waves, and $\gamma_p = r_{b21}/r_{b22}$ for p-polarized incident waves. Then, Kerr rotation angles θ_{Ks} and θ_{Kp} are given approximately by [8]

$$\theta_{K\nu} = \text{Real}(\gamma_\nu), \quad \nu = p, s. \tag{28}$$

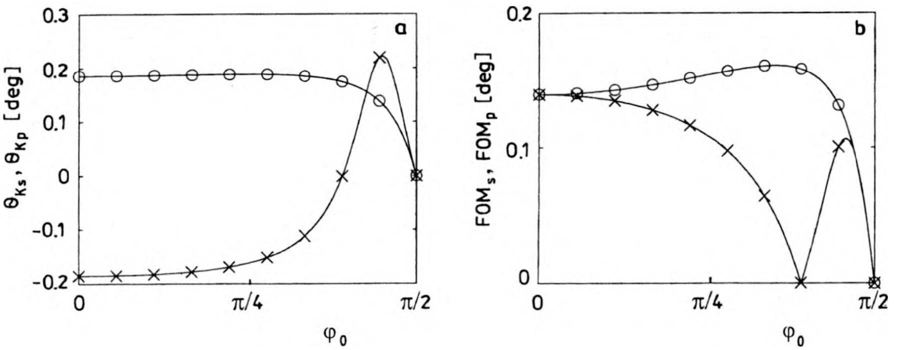

Fig. 2. Variation of Kerr rotation angles θ_{Ks} (o) and θ_{Kp} (×) against incident angle φ_0 (a). Variation of respective figure of merit functions FOM$_s$ (o) and FOM$_p$ (×) against φ_0 (b). A polar Kerr-effect bulk system is considered with $n = 2.96-j3.4$ and $q = 0.001-j0.025$, in air ($n_0 = 1$). On either curve the signs are marked in steps of 10°.

Similarly to the case of normally incident waves [17], we define the figure of merit functions FOM$_s$ and FOM$_p$ for s- and p-polarized incident waves

$$\mathrm{FOM}_\nu = (R_\nu \theta_{K\nu}^2)^{1/2}, \quad \nu = p, s \tag{29}$$

where $R_s = |r_{b11}|^2 + |r_{b12}|^2$ and $R_p = |r_{b21}|^2 + |r_{b22}|^2$. Variations of Kerr rotation angle and figure of merit function against incident angle φ_0 are shown in Fig. 2, comparatively for *s*- and *p*-polarized incident waves. One can see that variations of θ_{Ks} and FOM_s against φ_0 are smoother than the respective variations of θ_{Kp} and FOM_p.

9. Summary

In this work, we present a simplified matricial formalism for the polar Kerr-effect multifilm and bulk systems at oblique incidence. Simpler final expressions result for both multifilm and bulk systems in comparison to those presented in other works [3], [10].

Three simplifications are introduced:

1. We consider that either layer of the system is imaginatively embedded in between two ambient isotropic layers of zero thickness. Since both boundaries of the layer are placed into the same isotropic medium, the final expressions are much simplified.

2. We extended the procedure of unit vector representation in terms of three characteristic angles to the absorbing and anisotropic magnetooptic media. Although the characteristic angles (that are complex in this case) lose their physical meaning, the procedure is still applicable by using trigonometric functions of complex arguments.

3. Since the polar Kerr-effect multifilm systems may comprise both isotropic and anisotropic layers, and for isotropic layers one obviously uses Fresnel coefficients for *p* and *s* polarizations, we used in this work 2×2 extended Jones matrices relating the reflected and transmitted amplitudes of the *p* and *s* modes. Simpler final expressions result in comparison to those presented in other works in which one uses 2×2 Jones matrices relating the *x* and *y* components of reflected and transmitted waves.

In the case of a magnetooptic thin layer having both boundaries in the ambient medium we obtained four matricial relations for reflected and transmitted electric field amplitudes at the two interfaces (Eqs. (11)) that are written in terms of 2×2 matrices given by Eqs. (12)–(15). The four matricial relations are rewritten in the form of two simpler matricial relations (Eq. (18)) in terms of 4×4 matrices. A 4×4 characteristic matrix $\tilde{M}$ is defined by Eqs. (19)–(21). The respective 4×4 characteristic matrix for an isotropic dielectric thin layer having both boundaries into the ambient medium is given by Eqs. (22).

Once having the 4×4 characteristic matrices defined for both dielectric and magnetooptic thin layers, we can determine by Eq. (24) the overall 4×4 characteristic matrix of a multifilm placed into the ambient medium.

The 2×2 extended Jones reflection matrix is given by Eq. (26) for the polar Kerr-effect multifilm coated on a substrate and by Eq. (27) for a simple bulk magnetooptic system.

Numerical examples of Kerr rotation angle and figure of merit function variations against the incident angle are given in Fig. 2, comparatively for p- and s-polarized incident waves.

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