Optica Applicata, Vol. XXIX, No. 4, 1999

Teaching optics

On correction of chromatic aberration

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The elementary theory of aberrations as well as the skill to design optical systems of desired characteristics belong to canon subjects of optics teaching. However, lecturing on aberrations theory is difficult from the didactic point of view and is little attractive to the students. Besides, when designing the optical systems some specialized computer programs are used, which more and more seemingly eliminates the necessity of studying the theory of aberrations. In spite of this opinion, some acquaintance with relevant problems of aberration correction seems to be unavoidable. The aim of the present paper is to facilitate the mastering of a necessary minimum knowledge in the field of chromatic aberration correction. The problem of chromatic correction has been illustrated by typical examples. Such optical systems as classical glass doublet, hybrid lens and triplet have been considered. Three main types of chromatism correction, *i.e.*, achromatic, apochromatic and superachromatic have been analysed. Also the spherochromatic aberration correction has been demonstrated.

1. Introduction

The possibilities of chromatic aberration correction in optical systems were first considered by Newton, who – being limited to the glasses available those days – came to conclusion that achromatisation is impossible [1]. The achromatisation problem was first properly treated by DOLOND [2]–[4]. Taking advantage of the flint glass patented by Tilsone [5], he made first achromatic doublet consisting of a positive lens fabricated of crown glass and a negative one produced of flint glass. Such a system is characterized by correction of two colours, usually red and violet. This correction is often insufficient because of large secondary spectrum. A better correction has been achieved by ABBE [6], who designed optical systems of diminished secondary spectrum. The optical systems corrected for three colours (yellow, red and violet) were called achromats. In 1960 a paper by STEPHENS [7] appeared, in which the author shows that there exists a possibility of chromatic aberration correction for four colours, which means that the secondary spectrum is practically removed. The correction of this type is called superachromatic. It was Herzberger who considered the last problem in more detail in [8] and gave the conditions under which such a correction is possible.

In the contemporary optics besides the refractive optical systems (lenses) also diffractive optical elements (DOE) are used [9]–[11], which suffer from great chromatism and can operate only in monochromatic light. However, a hybrid lens being a connection of classical refractive lens with a diffraction structure (DOE) deposited on one of the refractive surfaces can create an achromatic system.

For optical systems of practical importance and corrected spherical chromatic aberration also, at least, spherical aberration must be corrected. Therefore, the course of the spherochromatic aberration can be taken as a measure of applicability of the optical systems discussed.

2. Correction of chromatism in a doublet

Let the refractive power (constringence) of a lens be φ for the basic wavelength (usually λ_D, which corresponds to the centre of the visual spectral range). For another wavelength λ the refractive power is obviously different from φ, which means that the lens suffers from chromatic aberration. When analysing the chromatism the refractive power φ is usually compared to the refractive power of the same lens but for another wavelength λ_F. Their difference amounts to [12]

$$\Delta\varphi_{F\lambda} = \frac{\varphi}{\nu_\lambda} \tag{1}$$

where ν_λ is the Abbe number defined as follows:

$$\nu_\lambda = \frac{n_D - 1}{n_F - n_\lambda}. \tag{2}$$

The simplest correction of chromatism is that of achromatic type, in which case chromatic aberration is corrected for two wavelengths, usually λ_F and λ_C. The difference of refractive powers of a lens for those two wavelengths is given by the formula

$$\Delta\varphi_{FC} = \frac{\varphi}{\nu} \tag{3}$$

where

$$\nu = \frac{n_D - 1}{n_F - n_C}. \tag{4}$$

In the system composed of two lenses, *i.e.*, in a doublet, the chromatic aberration is corrected if the conditions:

$$\frac{\varphi_1}{\nu_1} + \frac{\varphi_2}{\nu_2} = 0, \tag{5}$$

$$\varphi_1 + \varphi_2 = 1 \tag{6}$$

are fulfilled, where φ_1 and φ_2 are the refractive powers of the lenses while ν_1 and ν_2 are their respective Abbe numbers. The relation (6) defines the refractive power of the total optical system.

The conditions (5) and (6) allow us to design an achromatic system composed of two sorts of glasses. (Here, we do not intend to discuss the doublet made of the same sort of glass but with an air spacing which also allows an achromatic correction to be achieved because it has no greater practical significance.)

For the wavelength λ other than those for which the correction has been performed the refractive power deviates from φ while this difference is given by formula (1) which can be rewritten in the form

$$\Delta\varphi_{F\lambda} = \frac{\varphi}{\nu} P_\lambda \tag{7}$$

where P_λ is the partial dispersion of glass understood as

$$P_\lambda = \frac{n_F - n_\lambda}{n_F - n_C}. \tag{8}$$

Taking advantage of Eqs. (5)–(7) the refractive power differences can be found for the wavelengths λ and λ_F

$$\Delta\varphi_{F\lambda} = \frac{1}{\nu_1 - \nu_2}(P_{1\lambda} - P_{2\lambda}). \tag{9}$$

For typical optical glasses there exists a relation between the partial dispersion and the Abbe number

$$P_\lambda = A_\lambda \nu + B_\lambda \tag{10}$$

where A and B are constants independent of the glass sorts but dependent only on the wavelength λ. Inserting (10) to (9) we have

$$\Delta\varphi_{F,\lambda} = \frac{1}{\nu_1 - \nu_2} A_\lambda, \tag{11}$$

and, thus, the doublets built from the classic glasses will always suffer from secondary spectrum. The latter is smaller in an optical system corrected for three wavelengths (usually yellow, red and violet). Then:

$$\frac{\varphi_1}{\nu_1} + \frac{\varphi_2}{\nu_2} = 0, \tag{12}$$

$$\frac{\varphi_1}{\nu_1} P_{1,D} + \frac{\varphi_2}{\nu_2} P_{2,D} = 0, \tag{13}$$

$$\varphi_1 + \varphi_2 = \varphi = 1. \tag{14}$$

The following relation results from these equations:

$$\frac{\varphi_1}{\nu_1}(P_{1,D} - P_{2,D}) = 0, \tag{15}$$

which means that the partial dispersion of the glasses, of which an apochromat doublet can be produced, should fulfil the condition

$$P_{1,D} = P_{2,D}. \tag{16}$$

This is possible if one of the lenses is made of a special glass or of the fluorite.

The secondary spectrum disappears practically along the whole interval of wavelength 0.365 μm $\leqslant \lambda \leqslant$ 1.014 μm first in superachromat, *i.e.*, in an optical system corrected for four wavelengths λ_F, λ_C, $\lambda_* = 1.014$ μm and $\lambda_{**} = 0.365$ μm). The conditions assuring such correction are of the form:

$$\frac{\varphi_1}{\nu_2}+\frac{\varphi_2}{\nu_2} = 0, \tag{17}$$

$$\frac{\varphi_1}{\nu_1}P_{1,*}+\frac{\varphi_2}{\nu_2}P_{2,*} = 0, \tag{18}$$

$$\frac{\varphi_1}{\nu_1}P_{1,**}+\frac{\varphi_2}{\nu_2}P_{2,**} = 0, \tag{19}$$

$$\varphi_1+\varphi_2 = 1. \tag{20}$$

From these equations it follows that:

$$\frac{\varphi_1}{\nu_1}(P_{1*}-P_{2*}) = 0, \tag{21}$$

$$\frac{\varphi_1}{\nu_1}(P_{1**}-P_{2**}) = 0. \tag{22}$$

The glasses creating a superachromat doublet must fulfil the conditions:

$$P_{1,*} = P_{2,*}, \tag{23}$$

$$P_{1,**} = P_{2,**}. \tag{24}$$

Dividing (24) by (23) and taking advantage of (8) we obtain the condition necessary for superachromatic correction

$$\left(\frac{n_F-n_*)}{n_F-n_{**}}\right)_1 = \left(\frac{n_F-n_*}{n_F-n_{**}}\right)_2. \tag{25}$$

A magnitude which can be defined for each glass is

$$C = \frac{n_F-n_*}{n_F-n_{**}} \tag{26}$$

being a useful parameter, when designing a superachromat doublet, which facilitates the choice of the proper glasses.

A specific kind of doublet is a hybrid lens. The corresponding values of the Abbe number and the partial dispersion for the diffractive component (DOE) of the hybrid lens are defined as follows [14]:

$$\tilde{\nu} = \frac{\lambda_D}{\lambda_F - \lambda_C}, \tag{27}$$

$$\tilde{P}_\lambda = \frac{\lambda_F - \lambda}{\lambda_F - \lambda_C}. \tag{28}$$

The condition for achromatic correction of the hybrid lens is defined by formula (5), while the relative dispersion is defined by formula (27). This dispersion takes a negative value $\tilde{\nu} = -3.46$ and thus the doublet is composed of two components, *i.e.*, refractive and diffractive ones each of which is of positive refractive power. As far as the correction of chromatic aberration is concerned this situation is more advantageous than that in the case of a classical doublet.

Table 1. Abbe number and partial dispersion for selected optical materials and DOE.

	ν	P_D
DOE	−3.462	0.6063
BK7	64.188	0.7019
SF2	33.866	0.7165
LaK1	57.485	0.7039
Fluorite	95.550	0.7071

In Table 1, the values of Abbe number are collected for selected standard and special glasses as well as for fluorite and DOE. It may be seen that the value of partial dispersion for a diffractive element deviates from the corresponding values for optical glasses and fluorite which causes that the hybrid lens is characterized by greater secondary spectrum than classical doublets. This can be easily seen when inspecting $\Delta\varphi_{FD}$ for selected achromatic doublets given in Tab. 2.

Table 2. Secondary spectrum for selected achromatic doublets.

Kind of doublet	$\Delta\varphi_{F,D}$
BK3 SF5	-5.5×10^{-4}
BK7 DOE	1.4×10^{-3}
SF2 DOE	3.0×10^{-3}
LaK1 DOE	1.6×10^{-3}
Fluorite DOE	1.0×10^{-3}

For more complete illustration of the problem the values of differences between the image distance for different wavelengths and for the wavelength λ_F under different corrections are presented in Tab. 3 (systems 1−4). The focal lengths of all the systems equal $f = 100$ mm.

Table 3. Refractive powers of lenses and their image distances for selected types of chromatic correction.

Kind of correction	No.	Kind of glass	φ	$s'_F - s'$ [mm]					
				*	A'	C	D	F	**
Achromatic	1	BK3 SF5	0.01994 −0.00994	−0.61	−0.16	0	0	0.5	−1.02
Achromatic	2	BK7 DOE		1.8	0.45	0.01	−0.13	0	1.46
Apochromatic	3	K2 KZF2	0.11444 −0.10444	−0.07	0.01	−0.02	0.02	0	−0.60
Superachromatic	4	FK50 SK20	0.04022 −0.03022	−0.01	−0.06	−0.02	−0.03	0	−0.02
Apochromatic	5	F2 KZFSN5 FK51	0.02738 −0.04148 0.02415	0.21	0.09	0	0	0	0.04
Superachromatic	6	Fluorite BaSFS1 SF2	0.02096 −0.01990 0.00894	−0.03	−0.03	−0.02	0.02	0	−0.08

When analysing the above results it should be stated that the achromatic hybrid lens has the largest secondary spectrum and, therefore, is usually exploited only in monochromatic light. The apochromatic and superachromatic correction of a doublet and especially the first one requires high refractive powers, which makes the correction of spherical aberration more difficult and restricts the size of the aperture. In order to solve this problem a three element system, *i.e.* triplet, appears to be necessary.

3. Chromatism correction in a triplet

In order to achieve an apochromatic correction in a triplet the refractive powers of the component lenses must satisfy the following set of equations:

$$\frac{\varphi_1}{\nu_1}+\frac{\varphi_2}{\nu_2}+\frac{\varphi_3}{\nu_3}=0, \tag{29}$$

$$\frac{\varphi_1}{\nu_1}P_{1,D}+\frac{\varphi_2}{\nu_2}P_{2,D}+\frac{\varphi_3}{\nu_3}P_{3,D}=0, \tag{30}$$

$$\varphi_1+\varphi_2+\varphi_3=1. \tag{31}$$

This set is solvable if the condition

$$\begin{vmatrix} 1 & 1 & 1 \\ P_{1,D} & P_{2,D} & P_{3,D} \\ \nu_1 & \nu_2 & \nu_3 \end{vmatrix} \neq 0 \tag{32}$$

is fulfilled. This means that if the particular sorts of glass are presented as points in the plot made in coordinates P and ν, the glasses creating an apochromat cannot lie on a straight line. At least one glass should be a special one.

From Eqs. (29)–(31) the refractive powers of particular lenses can be determined. We obtain

$$\varphi_1=-\frac{C_D\nu_1}{C_D(\nu_3-\nu_1)+\nu_2-\nu_3}, \tag{33}$$

$$\varphi_2=\frac{\nu_2}{C_D(\nu_3-\nu_1)+\nu_2-\nu_3}, \tag{34}$$

$$\varphi_3=\frac{\nu_3(C_D-1)}{C_D(\nu_3-\nu_1)+\nu_2-\nu_3} \tag{35}$$

where

$$C_D=\frac{P_{2,D}-P_{3,D}}{P_{1,D}-P_{3,D}}. \tag{36}$$

The positions of foci of such an optical system for three wavelengths (usually for λ_D, λ_F and λ_C) cover each other. Due to the requirement that the spherical aberration should be corrected, the glasses should be chosen in such a way as to achieve the possibly small refractive powers of particular components of the optical system.

HERZBERGER [8] showed hat a good superachromatic correction can be achieved first in a triplet. The equations defining the suitable conditions have the forms:

$$\frac{\varphi_1}{\nu_1}+\frac{\varphi_2}{\nu_2}+\frac{\varphi_3}{\nu_3}=0, \tag{37}$$

$$\frac{\varphi_1}{\nu_1}P_{1,*}+\frac{\varphi_2}{\nu_2}P_{2,*}+\frac{\varphi_3}{\nu_3}P_{3,*}=0, \tag{38}$$

$$\frac{\varphi_1}{\nu_1}P_{1,**}+\frac{\varphi_2}{\nu_2}P_{2,**}+\frac{\varphi_3}{\nu_3}P_{3,**}=0, \tag{39}$$

$$\varphi_1+\varphi_2+\varphi_3=1. \tag{40}$$

Since the number of unknowns is less than the number of equations one of the unknowns must be dependent on the others. The corresponding determinant must be equal to zero and thus the glasses creating the triplet must satisfy the condition

$$\begin{vmatrix} 1 & 1 & 1 \\ P_{1,*} & P_{2,*} & P_{3,*} \\ P_{1,**} & P_{2,**} & P_{3,**} \end{vmatrix}=0. \tag{41}$$

Taking advantage of Eq. (36), the relation (41) can be written either in the form

$$C_*=C_{**} \tag{42}$$

or the following geometric interpretation can be formulated. If the particular glasses are presented as points in coordinates P_* and P_{**} the glasses creating superachromat should lie on one straight line. Due to the requirement that spherical aberration should be corrected the refractive powers of the particular components of the triplet must be small. According to Herzberger the partial dispersion of the glasses of which the superachromatic triplet must be produced should satisfy the conditions:

$$P_{1,*}-P_{2,*}\geqslant 0.07, \tag{43}$$

$$P_{2,*}-P_{3,*}\geqslant 0.07. \tag{44}$$

In Table 3, two examples of triplets of an apochromatic correction [15] and of superachromatic correction [16] are presented (Nos. 5 and 6).

In order to make the comparison of the results given in Tab. 3 more convenient they are illustrated graphically in Fig. 1. There are shown the secondary spectra for four selected optical systems, *i.e.*, classical achromatic doublet (system 1), achromatic hybrid lens (system 2), apochromatic system (system 5), and superachromatic triplet (system 6).

When analysing the results presented in Tab. 3 and in Fig. 1 it is easy to notice distinct differences between the particular types of correction. It can be stated that the achromatic hybrid lens has the largest secondary spectrum and should not

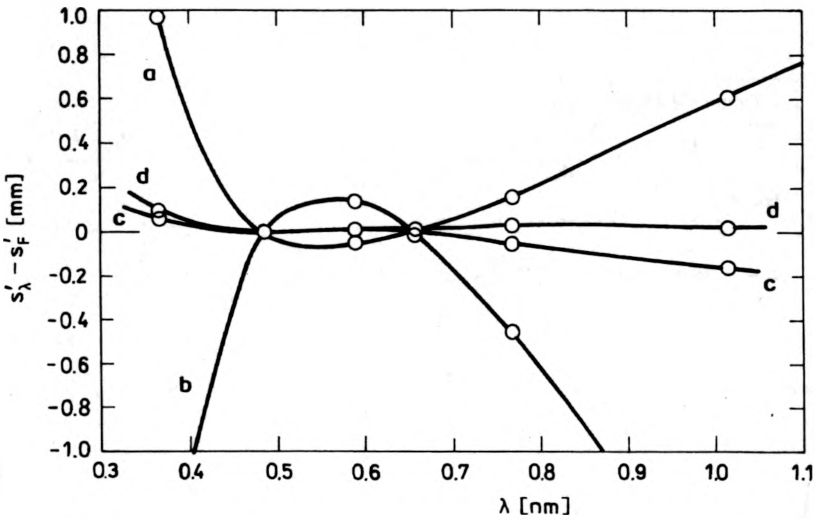
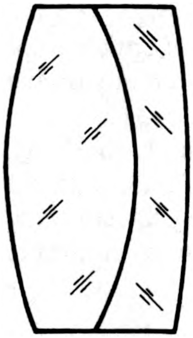


Fig. 1. Secondary spectrum for the selected optical systems: achromatic lens — glass doublet (curve *a*), achromatic lens — hybrid doublet (curve *b*), apochromatic lens — glass triplet (curve *c*), superachromatic lens — glass triplet (curve *d*).

operate in the white light. The correction of chromatism in the apochromate is ideal. The superachromatic system is characterized by a relatively good correction along the whole range of wavelengths. It is also visible that the superachromatic and apochromatic corrections (especially the second one) in a doublet requires high refractive powers, which makes the correction of spherical aberration difficult and restricts the aperture. For such a correction the triplets are to be used which show an obvious superiority to doublets.

4. Spherochromatic aberration correction

When designing optical systems working in the white light in addition to the chromatic correction also spherical aberration must be corrected. This can be done either based on the third order theory of aberration or by taking advantage of



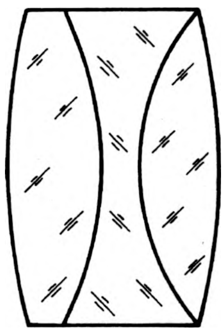
r [mm]	*d* [mm]	Glass
52.946		
	3	BK3
−47.6		
	1.5	SF5
−159.52		

Fig. 2a

r [mm]	d [mm]	Glass
50.38		
	3	BK7
660.38		

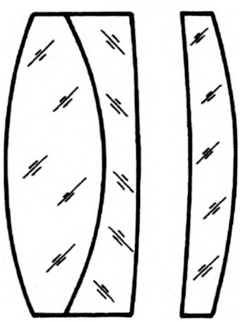
$z_\alpha = -19.33$ mm, $z_\beta = 19.54$ mm

Fig. 2b



r [mm]	d [mm]	Glass
63.434		
	2.5	F2
−40.000		
	2.0	KzFSN
5		
28.540		
	2.8	FK51
−68.500		

Fig. 2c



r [mm]	d [mm]	Glass
48.292		
	3.0	Fluorite
−36.538		
	1.5	BaSFS1
1828.8		
	1.0	
−301.75		
	3.0	SF2
−59.404		

Fig. 2d

Fig. 2. Construction data for analysed optical systems: **a** — achromatic doublet, **b** — achromate hybride lens, **c** — apochromatic triplet, **d** — superachromatic triplet.

the commercially available numerical programs offered by a number of optical firms in order to design simple optical systems operating in white light without any deeper knowledge about the aberrations of optical systems.

Four optical systems considered above assure such a correction. Their design parameters are shown in Fig. 2. In the case of hybrid doublet the diffractive part is defined by specifying the distances z_α and z_β of the virtual sources of the spherical waves the interference of which on the surface of the lens produces a proper diffraction structure of the DOE.

Figures 3**a**–**d** show the spherical aberration curves for each of the said lenses for the wavelengths λ_*, λ_C, λ_D, λ_F and λ_{**} under the assumption of maximal relative aperture 1:5. These curves confirm the conclusions formulated in the previous section. Unfortunately, also the triplets not always offer the possibility of achieving sufficiently large relative aperture. In order to achieve a bigger relative aperture more complex optical systems should be designed the components of which could be the relatively simple systems considered above.

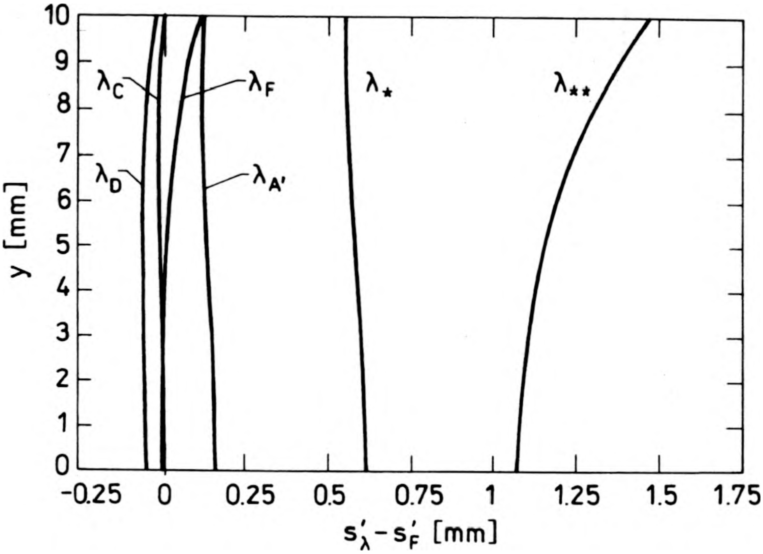


Fig. 3**a**

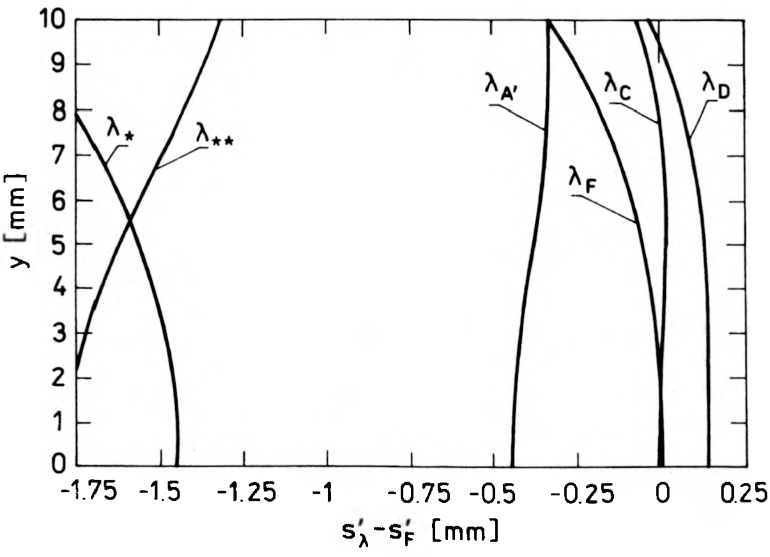


Fig. 3**b**

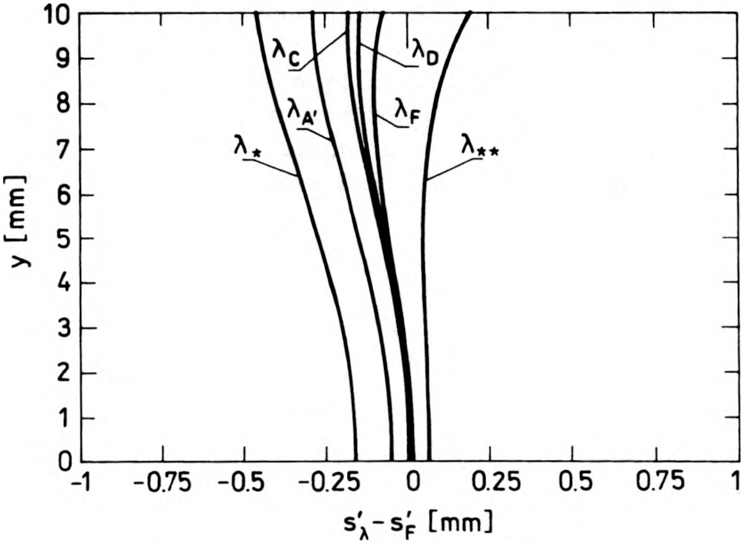


Fig. 3c

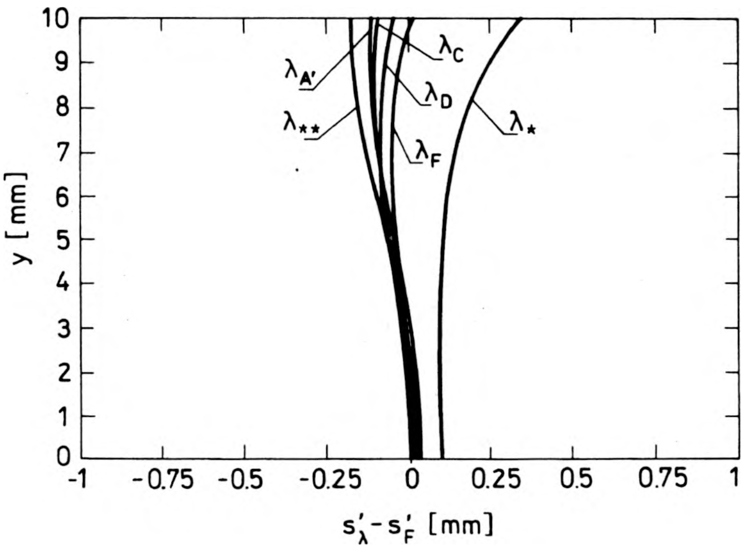


Fig. 3d

Fig. 3. Spherochromatic aberration of the selected optical systems: **a** — achromatic doublet, **b** — achromatic hybrid lens, **c** — apochromatic triplet, **d** — superachromatic triplet.

Acknowledgements — This work was supported by Ministry of Education, Poland, grant 33154-0.

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Received April 16, 1999