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Boundary diffraction wave as a phase filter correction tool

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A filter which transforms Gaussian intensity distribution into the uniform one has been considered. Two examples of the modification of the filter function near its edge, making the intensity distribution smoother, are shown.

1. Introduction

The Gaussian function is a good approximation of the transversal intensity distribution of the beam leaving the laser. Depending on the intended application, the beam is transformed into, *e.g.*, the one of uniform, Bessel, or other intensity distribution, as required. However, the energy economy is always desired. To achieve a high diffraction efficiency it is indispensable to know how to redistribute the energy in order to satisfy expectations. The filter's function shapes are designed in many ways. The common methods are the Fourier transform, the stationary phase method and the combination of those, included in the Saxton–Gerchberg algorithm [1]–[3]. The stationary phase method has been perfectly worked out and now offers much more possibilities than its preliminary form which expresses the geometrical optics. It permits, for example, evaluation of the diffraction effects and then the finding of the field shape, since the boundary diffraction wave has been included in the method.

2. Basic relations

Let us consider two parallel planes r and q separated by z. The first one is a filter plane and the second one the observation plane. The design of the phase filter, which should transform the beam, is based on the geometrical optics. Required intensity and phase distributions are calculated under the assumption of straight-linear light propagation and with the application of the energy conservation principle. The condition of the phase stationarity has the form

$$\Phi_{\rm inc}+\Phi_f+\Phi_{\rm prop}=\text{const} \tag{1}$$

where $\Phi_{\rm inc}$, Φ_f, $\Phi_{\rm prop}$ are the phase of the incident beam in the filter plane, the phase change introduced by the filter, the phase change resulting from the propagation in the free space between the filter and the observation plane, respectively.

For the circularly symmetric filter with the incident collimated Gaussian beam (such that $\Phi_{\rm inc} = \text{const}$) the stationarity condition (1) leads to the equation for the filter function

$$\frac{r-q(r)}{\sqrt{(r-q(r))^2+z^2}} = \frac{\mathrm{d}f_f}{\mathrm{d}r}. \tag{2}$$

The quantities r and q are related to each other via the relationship which follows from the energy conservation law and the required intensity distribution in the observation plane. For example, the filter assuring uniform intensity distribution is described in [1], [2].

The limited size of the filter causes the real intensity distribution to be somewhat different from that assumed during the designing. The oscillations resulting from the diffraction affect the distribution. The amplitudes of these oscillations are quite high compared with the amplitude corresponding to constant intensity. The amplitude distribution has been evaluated taking advantage of the possibility of separating the wave disturbance described by the Kirchhoff integral into the geometrical and the boundary diffraction waves (BDW). Both waves represent the action of the Fresnel's zones associated with the light originating from the interior of the integration area and from the narrow strip touching the integration limits (*i.e.*, the edge of the aperture) [4]. The separation has been introducd for empty apertures, however, it is correct in the cases when the opening itself is also filled up with the diffraction structure (assumptions of the stationary phase method are fulfilled).

Among many formulae describing the boundary wave, the one of Fedoryuk [5]

$$\hat{U}_B = \frac{1}{ik}\int_{\Gamma} g(\xi,\eta)\,\frac{\partial f(\xi,\eta)/\partial n}{|\nabla f(\xi,\eta)|^2}\,\mathrm{d}\sigma \tag{3}$$

(where in our case: $f(\xi,\eta)$ is the phase function of the integrand *, in this case the filter function; $\partial f(\xi,\eta)/\partial n$ means its derivative along the outer normal to the line limiting the integration area, Γ is the line limiting the aperture, $\mathrm{d}\sigma$ is the element of the curve) seems to be particularly convenient for the discussion and for the use in filter designing. It follows from the above formula that one can modify the filter properties by changing the filter function near the edge as well as by changing the shape of the boundary line. Both ways of shaping the aperture field are known and have been used in the past, though without mathematical support.

If the filter is the circle of the radius r_0 and the incident collimated wave is impinging on the edge at the point where its amplitude reaches $1/e$ of maximum

* In the original Fedoryuk's work, multidimensional integral $\int g(x)e^{ikf(x)}\mathrm{d}x$, $x = x_1, x_2, \ldots x_m$ is evaluated, relation (3) corresponds to this integral. In the present work, the double integral of the form $k\int g(x)e^{ikf(x)}\mathrm{d}x$, $x = x_1, x_2$, is evaluated, next all the further relations for $\hat{U}_3$ are adapted to this form of integral.

value (waist of the beam), then the complex amplitude which is due to the presence of the edge can be described by

$$\hat{U}_B = \frac{2r_0}{e} e^{-i\frac{\pi}{2}} \int_0^{\pi} e^{ik(f_f(r_0,\alpha)+p(r_0,q,\alpha))} Q(r_0, q, \alpha) \mathrm{d}\alpha \tag{4}$$

where: $f_f(r_0, q, \alpha)$ is the filter function, k is the wave number,

$$p(r_0, q, \alpha) = (r^2 + q^2 - 2rq\cos\alpha + z^2)^{1/2},$$

$$Q(r_0, q, \alpha) = \frac{A}{A^2 + B^2},$$

$$A = p(r_0, q, \alpha)\left(\frac{\partial f_f}{\partial r}\right)_{r_0} + r_0 - q\cos\alpha,$$

$$B = \frac{p(r_0, q, \alpha)}{r_0}\left(\frac{\partial f_f}{\partial \alpha}\right)_{r_0} + q\sin\alpha.$$

The integration limits in formula (4) (from 0 to π) have been chosen to reduce the time of calculations. Even if the filter function is of radial symmetry only, an increase of the distance of the observation point from the axis causes an increase in the Fresnel's zones number associated with this point. In this case, the evaluation of the BDW can be done easily with great accuracy by adding contributions originating from the two, well determined critical points of the second kind [4]. But in the case where the filter function with angular derivative is introduced, searching for the positions of the numerous critical points could be as much time-consuming as the integration along the edge. Thus, by choosing the even filter function, with respect to the observation point position, we can carry out our calculations in a reasonable time. Obviously, the integration limits must be changed if the observation point is beyond (r, z) plane, where the filter function is no longer even.

3. Calculations — examples

To illustrate some of the possibilities described above, we present (Fig. 1) the boundary wave amplitude for the case where the filter described by Eq. (2) is placed in the opening of the radius $r_0 = 4$ mm, with additional condition

$$q(r) = \left[\frac{r_0^2}{1 - 1/e^2}\left(1 - e^{-2(r/r_0)^2}\right)\right]^{1/2}, \tag{5}$$

assuring the requirement of the uniform intensity distribution in the observation plane [1].

The filter function derivative with respect to r-coordinate is equal to zero at the edge. As can be seen, the mean value of the amplitude around which the BDW amplitude oscillates grows both in the central part of the observation plane as well

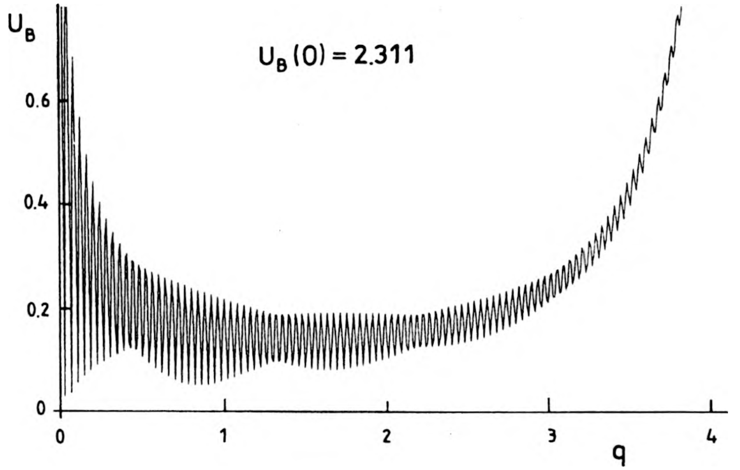

Fig. 1. Boundary diffraction wave amplitude vs. radial distance in observation plane for the filter designed according to relation (2). Filter function has radial symmetry and its derivative at the edge equals zero.

as near the edge. Such a BDW amplitude distribution affects the total disturbance distribution. The amplitude of the BDW becomes comparable to that of the geometrical wave.

To achieve the diminution of the BDW amplitude we change the filter function near the edge, keeping the radial symmetry in the first approach. The results of such a modification are in the focus of our interest. The filter function has been extended near the edge to satisfy evenness condition (equality of the 1st and the 2nd derivatives), to avoid the necessity of dividing the filter area into zones at which the filter function is monotonously increasing.

If the result is to be perceptible, the term containing the filter function derivative in (4) should be of the same order as that resulting from the derivative of the part of the phase function associated with the propagation in free space. Our extension

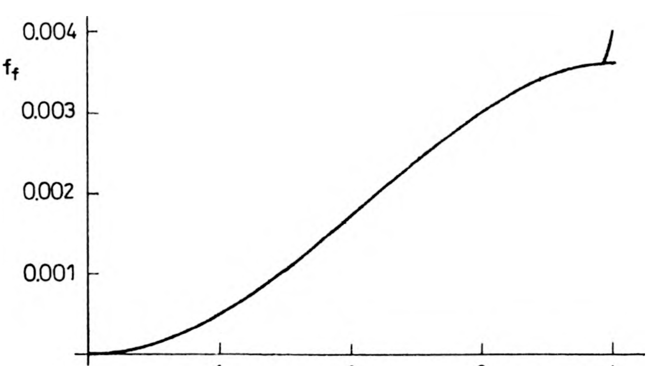

Fig. 2. Filter function obtained from relation (2) with the condition (5) which should assure a uniform intensity distribution and a filter extension according to relation (6).

has the form (see Fig. 2)

$$f_f = f(r)h[r_1 - r] + (ar^2 + br + c)h[r - r_1] \tag{6}$$

where: $r_1 = 3.9$ mm, $a = 0.0393262306$, $b = -0.3066098452$, $c = 0.601238$, $h[\ ]$ is the cut-off function.

With this extension the radial derivative is equal to 0.008 at the filter edge. The result is shown in Fig. 3. As one can see, the average value of BDW as well as its amplitude have decreased by one order of magnitude. In the vicinity of $r = 0$, the BDW remains high although decreased due to the contributions coming from the edge which are in phase. Much greater diminution is possible if other filter function extensions, with higher radial derivative at r_0, are used, or by introducing the functions with angular derivatives.

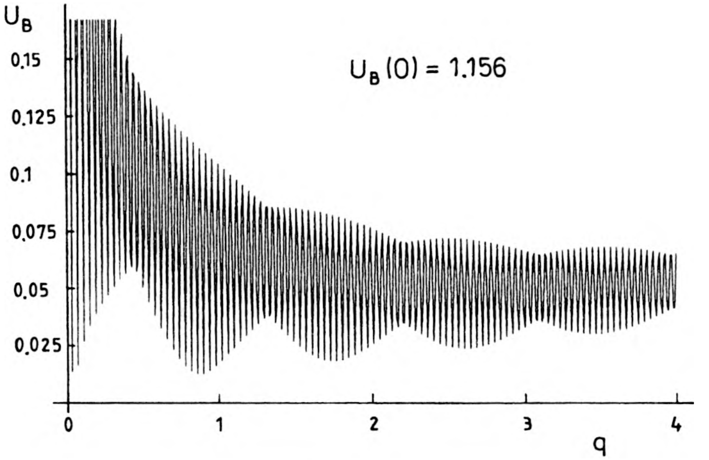

Fig. 3. Boundary diffraction wave amplitude vs. observation point position for the filter which behaving radial symmetry is modified at the edge according to relation (6). We observe diminution of the amplitude with respect to that of Fig. 1, but in the central part of the observation plane the amplitude, although gets reduced, still remains high.

An example of such procedure is shown in Fig. 4. The filter function near the edge was extended according to

$$f = \frac{f_1(r) + f_2(r)}{2} + \frac{f_1(r) - f_2(r)}{2}\cos(n\alpha) \tag{7}$$

where: $f_1(r)$ is given by extension (6), $f_2(r)$ is given by the integral equation (2) with the additional condition (5), n — is an integer.

The filter function (7) has been chosen to satisfy the Fedoryuk's assumption that the function must be analytical, and with the intention to avoid the time consuming angular integration.

The result presented in Fig. 4 corresponds to $n = 100$. The calculation was limited to the $0-1$ mm division because of the very long time needed for the

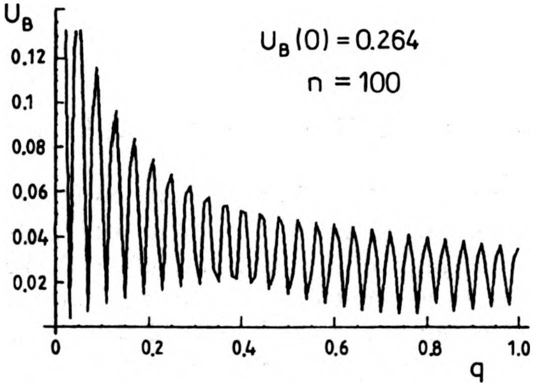

Fig. 4. Boundary diffraction wave amplitude vs. observation point position lying in the meridional plane, for the filter whose function was modified at the edge introducing angular derivative according to relation (7) in order to diminish the amplitude value in the central part of the observation plane. The quoted results are limited to $0 \leqslant r \leqslant 1$ for the reason of time-consuming calculations.

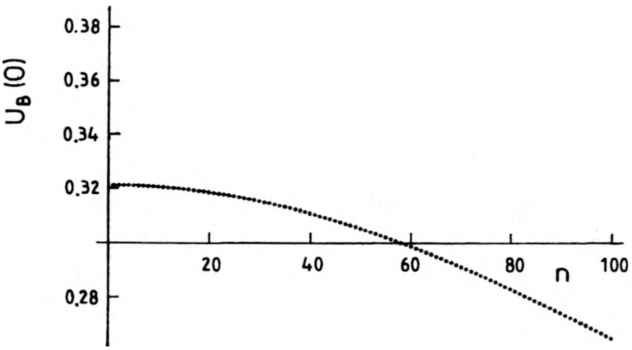

Fig. 5. Dependence of the boundary wave amplitude on the number n from relation (7) at observation point $q = 0$.

calculation. The dependence of $\hat{U}_B$ at $q = 0$ vs. n is shown in Fig. 5. All the calculations have been done for $\lambda = 632.8$ nm. They were carried out with the help of *Mathematica* program [6].

5. Conclusions

Although the quoted results merely show the possibility of the BDW having influence on the field shaping, they illustrate the possible procedure and help us understand the phenomenon. If the radial extension of the filter function contains the increasing derivative, then the "horizon" is artificially widened by introducing higher spatial frequencies. In the case of an empty aperture the edge cuts off the higher frequencies. Such an extension, however, changes the situation. The rays associated with the geometrical wave originating from this region are directed outside the area of the radius $r_0 = 4$ mm (*i.e.*, radius of previously designed filter with the uniform illumination distribution). Thus, the procedure leading to the more uniform intensity distribution, using a single simple filter, results in a decrease of the filter efficiency.

One can imagine the best case where the filter function lifts rapidly by $\lambda/2$ with respect to the value obtained from (1). The action of the BDW can be interpreted in terms of the first Fresnel zone associated with the edge [4]. At this point the question of feasibility of such a filter in the binary technique arises. Although modern technologies of spatial frequency recording are very high, the question is whether a single fringe, not necessarily the whole one (it can be cut off by the edge), will satisfy our requirements. How many levels of greyness should be used?

The observation point lying very close to r_0, at a distance of the order of λ, has been excluded from our considerations because in this case the method fails.

Some comment concerning various calculational algorithms supporting the methods of the filter designing, mentioned in the introduction of this paper, should be done. In the set of two filters, with the first filter transforming the amplitude, and the second one transforming the phase, which transforms collimated Gaussian beam into that of uniform intensity distribution, the procedure is repeated from the one to the next plane and back, until the result is self-consistent [2].

It can be seen from our considerations that the real propagation process is irreversible. This is the consequence of using the theory which goes beyond the frame of geometrical optics.

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