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Self-phase modulation of temporary overlapped chirped pulses

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A self-phase modulation at amplification of temporary overlapped chirped pulses is investigated. The analytical and numerical calculations are used to predict the effects of prepulse intensity increasing and additional satellites arising after recompression as well as the effects of spectrum broadening and shape distortion of the pulses. The experimental results are presented that agree well with the calculations.

1. Introduction

Recently, a great deal of effort has been devoted to generation of intense ultrashort laser pulses with high contrast [1]–[3]. In most of the high-temperature plasma experiments a single ultrashort pulse without any prepulses is required to exclude the pre-plasma influence [3]. Therefore, the decreasing of ultrashort pulse contrast, *e.g.*, due to self-phase modulation during amplification [4], [5] is the factor interfering in realization of plasma experiment.

On the other hand, however, recent investigations have shown that for some applications it is necessary to synthesise prepulses with certain parameters [6], [7]. For example, using a train of pulses with independent flexible control of parameters of the separate pulses, it is possible to achieve very effective electron acceleration in the non-linear plasma waves [6]. It is also known that a prepulse plays an important role in plasma shaping before arrival of the main laser pulse and makes it possible to increase the efficiency of transformation of laser energy to X-ray radiation [7].

It is shown in paper [8] that multipass amplification of a chirped pulse in a regenerative amplifier can result in a decrease of the pulse contrast and, on the other hand, can be a way of producing prepulses necessary for the applications described above. This effect was investigated at low intensity of a pulse, when the interaction of the pulse with amplifying medium is linear [8]. Our paper considers the case of strong pulses propagating in an amplifier, when the interaction is non-linear and, as a result, the self-phase modulation (SPM) of a pulse occurs. We show that SPM of overlapped in time chirped pulses can result particularly in the change of the shape and duration of the pulses after recompression, broadening their spectrum and swapping of energy to satellites.

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2. Self-phase modulation of temporary overlapped chirped pulses

An initial ultrashort pulse is usually bandwidth limited in the chirped-pulse -amplification (CPA) high-power lasers [1], [2], [9]. To achieve high energy level of amplification it is required to inject a long (nanosecond) pulse to a system of amplifiers. The transformation to the long pulse is carried out by a stretcher [1], [2], [10]. As a result of application of the stretcher a chirped pulse arises. This is a square-phase-modulated pulse. Temporal frequency dependence is linear for this pulse. At the expense of cubic non-linearity in amplifiers the additional non-linear phase modulation arises. The value of this modulation can be calculated from B-integral being the measure of a non-linear phase shift [4], [5]. The B-integral is defined as

$$B = \frac{2\pi}{\lambda}\int_0^L n_2|E(z,t)|^2 \mathrm{d}z \tag{1}$$

where: n_2 — non-linear refraction index, λ — wavelength of radiation, $E(z,t)$ — strength of electrical field in a light wave, L — length of propagation path in the amplifiers.

Similarly, the value of bandwidth broadening can be estimated as a speed of change of B-integral

$$\Delta\omega = -\frac{\mathrm{d}}{\mathrm{d}t}B. \tag{2}$$

Thus, the transformation of a single pulse by amplification is connected with distortions of linearity of the pulse chirp. The change of a spectrum is insignificant in comparison with the full spectrum of chirped pulse, because the intensity changes slowly in nanosecond range [5].

In the presence of a train of overlapped chirped pulses the situation is different. In order to analyse this case, we shall consider two identical chirped pulses with time delay Δt. Initial ultrashort pulse can be written down as

$$E = \mathrm{E}_0 \exp\left(\frac{t^2}{\Gamma}\right) \tag{3}$$

where $\sqrt{\Gamma}$ is the temporal width of ultrashort pulse. This pulse is stretched by a grating-based stretcher with a phase function [5]

$$\varphi = \exp\left[-i(\omega-\omega_0)\tau_0+\frac{i}{2\mu}(\omega-\omega_0)^2\right] \tag{4}$$

where ω is the optical frequency, μ is the stretcher parameter and τ_0 is the group delay of the stretcher.

The fields of two nanosecond pulses with square phase modulation after stretching are described by the following expressions [5]:

$$E_0^{(1)}(t) = A_1 \exp(-\beta t^2) \exp(i\alpha t^2),$$
$$E_0^{(2)}(t) = A_2 \exp(-\beta(t-\Delta t)^2) \exp(i\alpha(t-\Delta t)^2) \tag{5}$$

where: A_1, A_2 – amplitudes of light waves, $\beta = \dfrac{\Gamma}{\Gamma^2 + 4/\mu^2}$ – temporal width of the chirped pulse, $\alpha = \dfrac{2}{\mu\Gamma^2 + 4/\mu}$ – speed of frequency change.

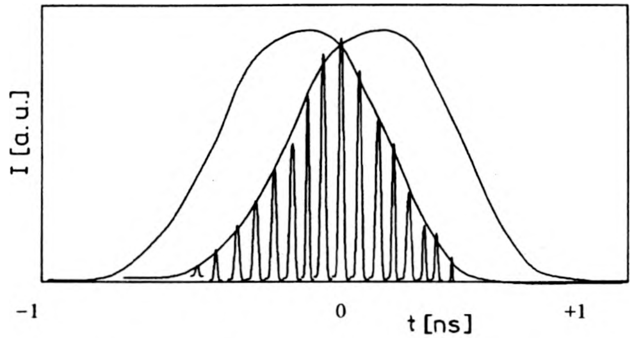

Fig. 1. Modulations of two overlapped chirped pulses.

In the area where pulses overlap we have intensity modulation with beat frequency $f = 2\alpha\Delta t$ (see Fig. 1). Changing a delay between pulses, we can change frequency of temporal modulation of intensity. As a result of these beats the non-linear self-modulation leads to effective bróadening of a spectrum and, as will be shown below, to energy transformation to satellites, which can be seen after recompression.

3. Theoretical considerations

Let us consider the case of two identical overlapped in time chirped pulses passing through the medium (*e.g.*, Nd:glass) with length L and non-linear parameter of refraction n_2. For weak absorption and amplification we shall write the equation for slowly varying amplitude [5]

$$i\frac{\mathrm{d}}{\mathrm{d}z} A = i\frac{\eta}{2} A(z,t) - \frac{n_2 \omega_0}{c} |A(z,t)|^2 A(z,t) \tag{6}$$

where electrical field E is expressed as

$$E(z,t) = A(z,t) \exp(i\omega_0 t) \tag{7}$$

and η is absorption coefficient of non-linear medium.

Boundary conditions on a border of non-linear medium shall be written down as the sum of fields of two identical chirped pulses with relative delay Δt

$$E_1 = E_0 \exp(i\omega_0 t)\left[\exp\left(\frac{t}{\Gamma - 2i/\mu}\right)^2 + \exp\left(\frac{t-\Delta t}{\Gamma - 2i/\mu}\right)^2\right]. \tag{8}$$

Solving Equations (6), we obtain a field at the output of the medium

$$E_2 = E_1 \exp\left(\eta \frac{L}{2}\right)\exp(i\beta). \tag{9}$$

For our case we have

$$B = K I_2$$

where

$$I_2 = \exp\left[-\left(\frac{1}{\Gamma+2i/\mu}+\frac{1}{\Gamma+2i/\mu}\right)(t-\Delta t)^2\right] + \exp\left[\frac{1}{\Gamma+2i/\mu}+\frac{1}{\Gamma-2i/\mu}\right)t^2\right]$$

$$+\exp\left[-\frac{t^2}{\Gamma+2i/\mu}-\frac{(t-\Delta t)^2}{\Gamma-2i/\mu}\right]+\exp\left[-\frac{(t-\Delta t)^2}{\Gamma-2i/\mu}-\frac{t^2}{\Gamma+2i/\mu}\right],$$

$$K = 2\frac{\pi n_2}{\lambda}\frac{\exp(\eta l)-1}{\eta}E_0^2.$$

To calculate the field after passage of non-linear medium, we can expand the exponent in formula (9) to the power series of argument

$$\exp(iKI_2) = \sum_{n=0}^{\infty}\frac{1}{n!}(KI_2)^n.$$

Formally, this can be done for any argument, as the radius of convergence of power series is infinite. For B-values less than 1 the series decreases from the first term, and it is possible to reject the upper terms of series. Decomposing I_2 to the form of a binominal formula, inserting it to formula (9), and then using recompression transformation $F^{-1}\varphi_{\mathrm{compr}}F$, we can have the output pulse. In the last expression F is the Fourier-transformation, and

$$\Phi_{\mathrm{compr}} = \exp\left[i(\omega-\omega_0)\tau_0 - \frac{i}{2\mu}(\omega-\omega_0)^2\right]$$

is the phase function of the compressor [5]. Thus, we could obtain general expression for compressed pulse in the form

$$E_{\mathrm{out}} = E_0 \exp\left(\eta\frac{L}{2}\right)\frac{\exp(i\omega_0 t)}{2\sqrt{\pi}}\sum_{n=0}^{N}\sum_{a=0}^{n}\sum_{b=0}^{n}\sum_{c=0}^{n}\sum_{d=0}^{n}\sum_{j=1}^{2}\frac{(iK)^n}{a!\cdot b!\cdot c!\cdot d!}$$

$$\times\frac{1}{\sqrt{1+\dfrac{2i\alpha_n}{\mu}}}\exp\left(\frac{-\beta_{1,j}\beta_{2,j}}{\alpha_n}\Delta t^2\right)\exp\left[\frac{-(t-t_j)^2}{4/\alpha_n+2i/\mu}\right] \tag{10}$$

where:

$$t_j = t_1 + \frac{\beta_{2,j}}{\alpha_n}\Delta t, \quad \alpha_n = \frac{(2n+1)\Gamma + 2i/\mu}{\Gamma^2 + 4/\mu^2},$$

and β_{ij} are the constants, depending only on μ, Γ and a, b, c, d index.

At $t_1 = t_2$ we come to the formula describing transformation of a single pulse in non-linear medium

$$E_{out} = 2E_0 \exp\left(\eta \frac{L}{2}\right) \frac{\exp(i\omega_0 t)^n}{\sqrt{\pi}} \sum_{n=0}^{N} \frac{(4iK)}{n!} \left[\frac{1}{\sqrt{1+2i\alpha_n/\mu}} \exp\left(\frac{-(t-t_0)^2}{1/4\alpha_n + 2i/\mu}\right)\right]. \quad (11)$$

The result of calculations using this formula is in good agreement with the numerical calculations presented in paper [5].

As this stage we can see that the system of discrete satellites with the general period Δt arises (see the index in the exponential term in formula (10)). We shall consider the properties of these satellites, taking into account that parameter $\Gamma\mu$ is much less than 1. In fact, this parameter is equal to the ratio of duration of a short pulse and a stretched one. In our experiments, for example, it is equal to 0.002. We can modify the expressions which are included in formula (10) with the accuracy of small paremeter $\mu\Gamma$. The expression for the satellite with number ν will be as follows:

$$E_\nu = \sqrt{E_0 \exp\left(\alpha \frac{L}{2}\right)} \frac{\exp(i\omega_0 t)}{\pi\sqrt{\pi}} \sum_{n=0}^{N} \frac{(iK)^n}{n!} \frac{1}{\sqrt{1+(2i\alpha_n/\mu)} \exp\left(\dfrac{(t-\nu\Delta t)^2}{1/4\alpha_n + 2i/\mu}\right)}$$

$$\times \left[\exp\left(\frac{i\mu\Gamma(t-\nu\Delta t)(\nu(2n+1)-n)\Delta t}{1/4\alpha_n + 2i/\mu}\right) A_{n,\nu}\right.$$

$$\left. + \exp\left(\frac{i\mu\Gamma(t-\nu\Delta t)(\nu(2n+1)-n-1)\Delta t}{1/4\alpha_n + 2i/\mu}\right) B_{n,\nu}\right], \quad (12)$$

$$A_{n,\nu} = \exp\left(-\frac{i\mu}{2}(\nu^2 - \nu)\Delta t^2\right) \int_{-\pi}^{\pi} (1+\cos(x))^n \cos(\nu x)\,dx,$$

$$B_{n,\nu} = \exp\left(-\frac{i\mu}{2}((\nu-1)^2 - \nu + 1)\Delta t^2\right) \int_{-\pi}^{\pi} (1+\cos(x))^n \cos((\nu-1)x)\,dx.$$

This is a finite expression to investigate the satellites and energy transformation.

4. Discussion

Let us consider in detail formula (12) for the satellites. First, we take into account the case of small B. This assumption permits us to reject the high order digit of the series. The first factor before the sum shows constant coefficient and carrier frequency. The dependence on K indicates that the result depends on B-integral. The first temporal factors in the sum have the Gaussian structure. The factor $(t-\nu\Delta t)^2$

shows that the temporal centre of the satellite with number v is $v\Delta t$. The second factor demonstrates the frequency shift. The additional frequency is

$$\frac{\partial\varphi}{\partial t}=\frac{\mu\Gamma\Delta t}{(1/4\alpha_n)+(2i/\mu)}.$$

The term in the denominator of an index of Gaussian function becomes real with infinitely small parameter $\mu\Gamma$

$$\frac{1}{4\alpha_n}+\frac{2i}{\mu}=(2n+1)\Gamma+O(\mu\Gamma)$$

where $O(\mu\Gamma)$ is the infinitely small value. It means that the field represents the sum of bandwidth limited Gaussian pulses with equal relative delays and durations $\tau=(2n+1)\Gamma$. The frequency shift must be $\frac{\mu\Delta t}{2n+1}$ and is equal to $\mu\Delta t$ at $n=0$. In comparison, the value of beat frequency $f=2\Delta t\alpha=\frac{4\Delta t\mu}{(\mu\Gamma)^2+4}$ and $f=\mu\Delta t$ at infinitely small $\mu\Gamma$.

The above consideration shows that at Gaussian distribution of intensity the bandwidth of satellites decreases with growing number of satellites arising. This is connected with the shape of overlapped area. The B-integral has various values at the bottom and at the top of chirped pulse, and the swapping of energy is more intensive at the top of Gaussial pulse. Another property of the satellites is the spectral shift. All pulses undergo non-linear phase perturbation with a beat frequency of two initial pulses: $\Delta\omega=\mu\cdot\Delta t$. There are terms with multiple shifts, but their intensity values are relatively small. The broadening of full spectrum, which was observed in the experiment (see further), is explained by this shift. Really, all terms of a series (12) are spectral limited pulses with enlarged duration, and they cannot give new spectral components, though the shape of the pulse varies. These conclusions are proved to be true on the basis of experiment presented in paper [11]. In this experiment, two temporary overlapped chirped pulses were introduced in regenerative amplifier. After recompression pulses were directed onto spectral-resolved grating and then to the streak-camera. Even at small values of B-integral there were satellites with a central frequency shift. These satellites had narrower spectrum in relation to a basic one, and their distance from basic pulse was equal to the time interval between two initial pulses.

For laser systems generating high-peak-power pulses such a high contrast of radiation is necessary, so that prepulse cannot result in plasma production. However, in powerful laser systems the chirped pulse accumulates significant B-integral upon amplification. Under these conditions it is possible to expect significant deterioration of the contrast, which can affect plasma experiment.

To demonstrate the practical contrast deterioration, we executed calculations using formula (10), showing arising number of satellites in the presence of weak postpulse under conditions of self-modulations (see Fig. 2). The calculations were

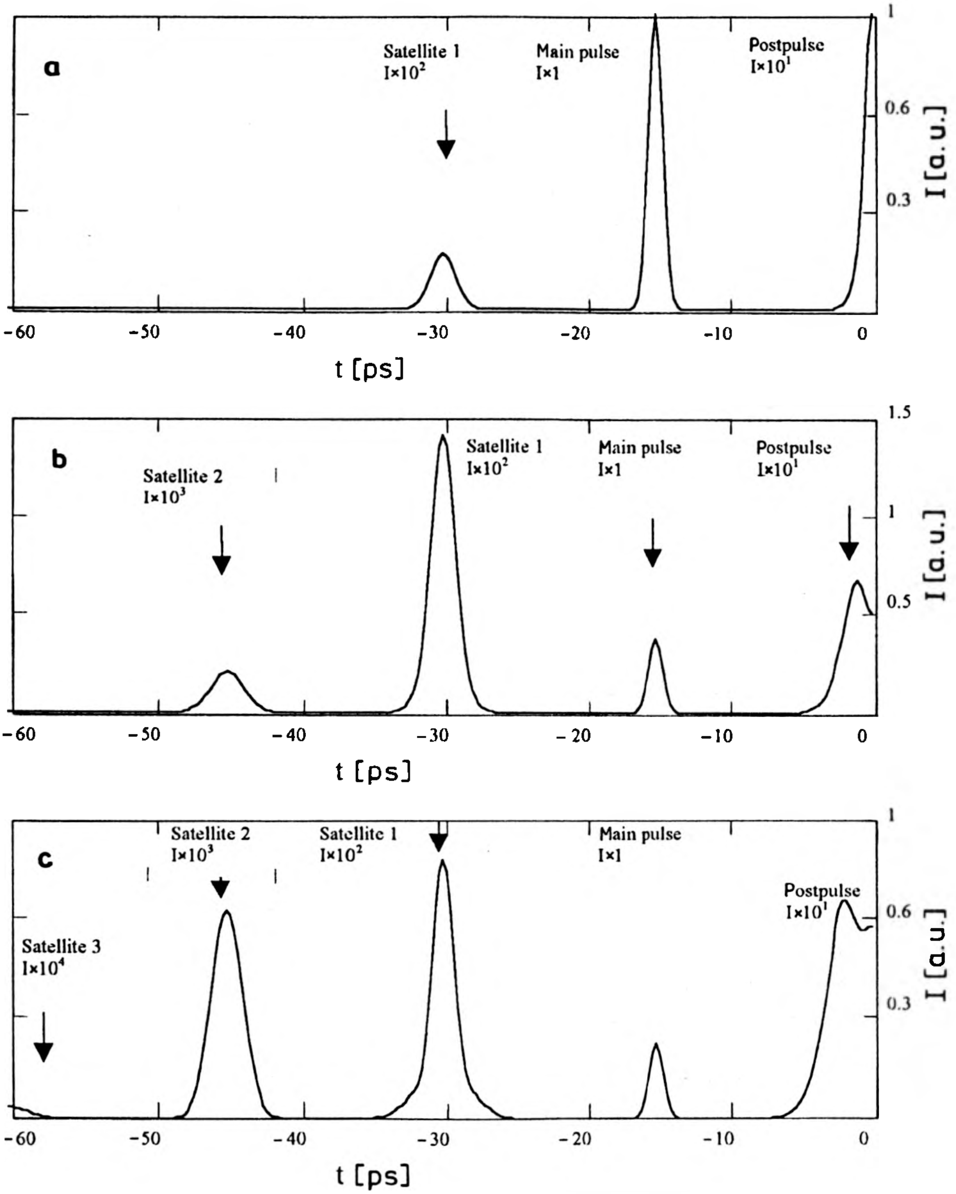

Fig. 2. Arise and increase of prepulses under following initial conditions: duration of chirped pulses — 1 ns, compressed pulses — 1 ps, delay between main pulse and postpulse — 15 ps, main pulse to postpulse intensity ratio — 10 times, value of B-integral: 0.5 (a), 3 (b), 5 (c).

carried out in the first power term of the small ratio of weak postpulse to main pulse. It can be seen (Fig. 2) that a weak prepulse arises and it grows with an increase of B-integral (see Fig. 3a). At an initial intensity ratio of the postpulse to the main pulse $\alpha = 0.1$, duration of initial pulses 1 ps, duration of chirped pulses 1000 ps and delay between initial pulses 15 ps the first prepulse intensity grows approximately

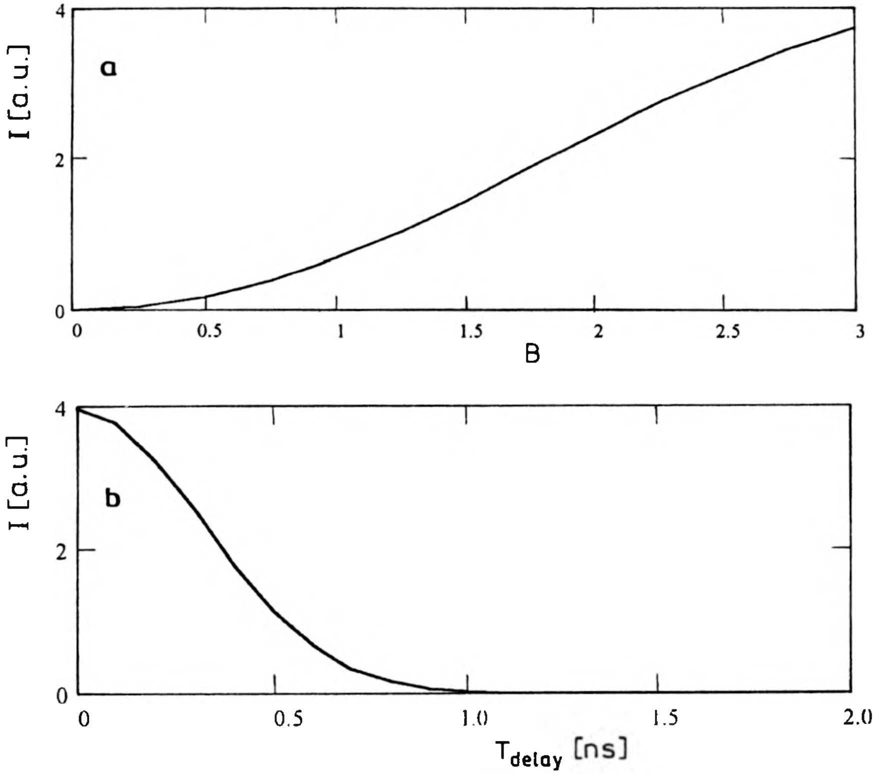

Fig. 3. Increase of first satellite intensity with B-integral under conditions of Fig. 2 (a). Decrease of first satellite intensity with time delay between main pulse and postpulse under conditions of Fig. 2 (b).

proportionally to $\alpha^2 B$. The arising of prepulse in the presence of postpulse is proportional to a square of the intensity ratio of postpulse to main pulse. For example, if we have the contrast on postpulse 10^3 and $B = 3$, we can have the contrast about 3×10^6 on prepulse after self-modulation process.

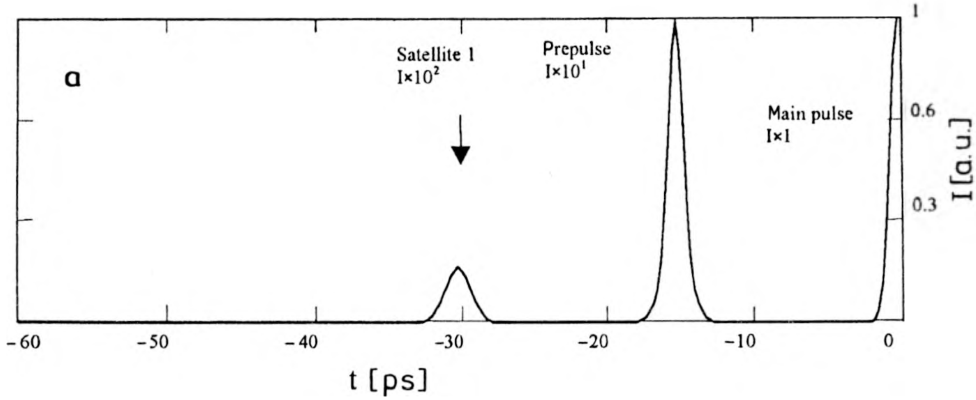

Fig. 4a

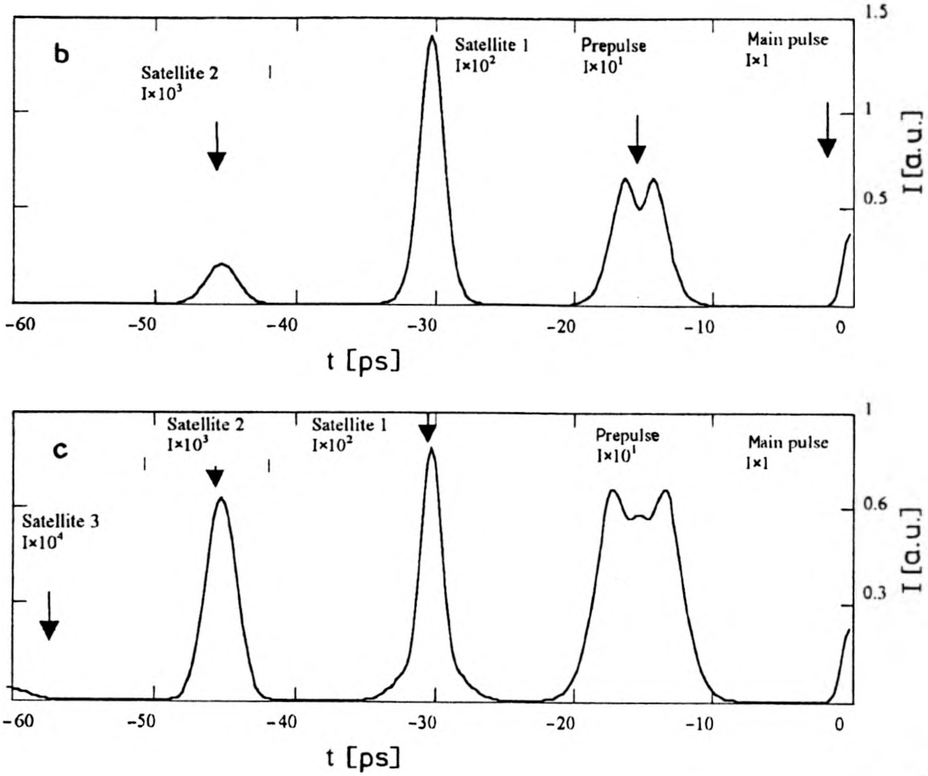

Fig. 4b,c

Fig. 4. Increase of prepulses under following initial conditions: duration of chirped pulses – 1 ns, compressed pulses – 1 ps, delay between main pulse and prepulse – 15 ps, main pulse to prepulse intensity ratio – 10 times, value of B-integral: 0.5 (a), 3 (b), 5 (c)

It is also shown that behaviour of prepulse intensity does not change strongly with changing delay between pulses within the interval of overlapping. The factor of transformation begins to fall only in case of incomplete overlapping of pulses (Fig. 3b).

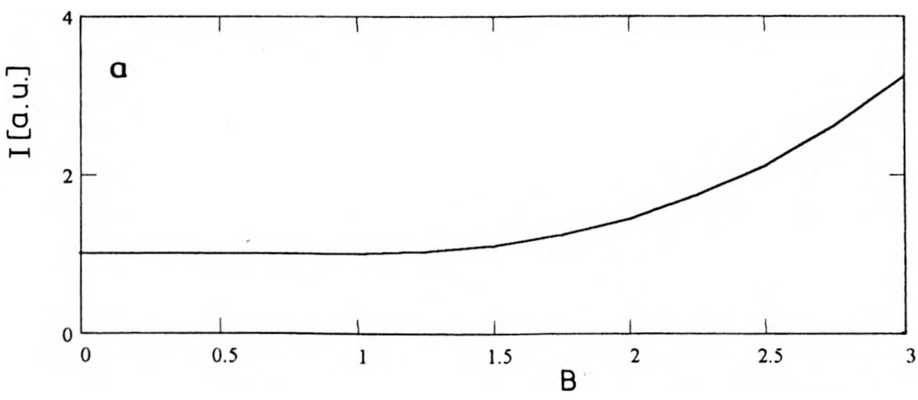

Fig. 5a

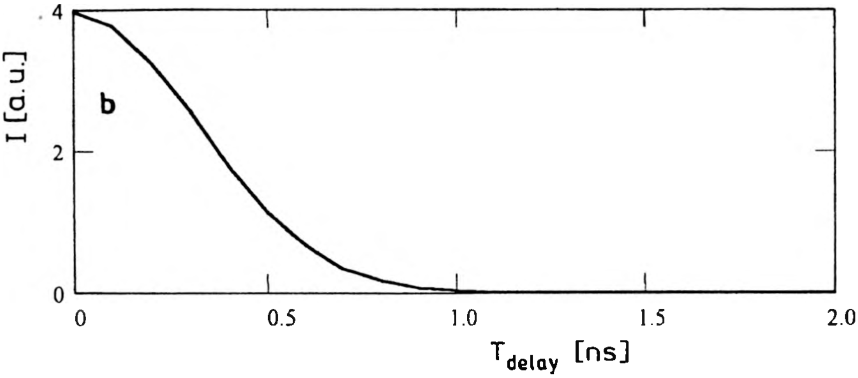

Fig. 5b

Fig. 5. Increase of prepulse intensity with B-integral under conditions of Fig. 4 (a). Decrease of first satellite intensity with time delay between main pulse and prepulse under conditions of Fig. 4 (b).

Other results are shown by calculations with initial prepulse presence. Prepulse intensity grows with αB, where α is the initial intensity ratio of prepulse to main pulse. As in the case of initial postpulse presence, satellites arise (see Fig. 4) and this effect decreases with incomplete overlapping of pulses (Fig. 5b).

5. Experimental results

To check the theoretical considerations we have used the Nd:glass CPA laser system [12] including a mode-locked master oscillator generating 1-ps pulse, a stretcher elongating the pulse duration up to 450 ps and a regenerative amplifier (RA) amplifying the pulse up to several mJ. The multipass amplifier with a Q-switch and cavity dumping represented the RA. Between the master oscillator and the RA an electrooptical temporal selector comprising a Pockels cell and a quadropolar deflector was situated. The own contrast of this installation was of $\geqslant 10^3$.

In the laser system used, it was possible to inject into the RA either a single 450-ps pulse or a train of 450-ps pulses. The time interval between pulses in the train

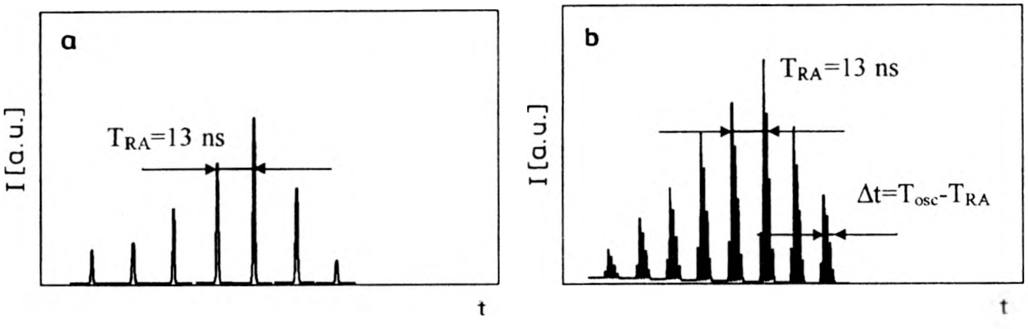

Fig. 6. Train of pulses in the output of the RA upon injection of a single pulse into the RA (a). Train of pulses in the output of the RA upon injection of a pulse train into the RA (b).

was equal to the cavity round trip time of the master oscillator T_{osc}. The cavity round trip time of the RA, T_{RA} was chosen close to T_{osc} ($T_{RA} \approx T_{osc}$). In the case when the single pulse was injected into the RA, a train of single pulses developed over the period equal to T_{RA} (Fig. 6a). However, when the train of pulses was injected, a train of pulses overlapped in time raised (Fig. 6b). The time intervals between overlapped pulses corresponded to the difference $T_{osc} - T_{RA}$. These effects were observed with the use of oscilloscope and fast streak camera. For the case of weak injected pulses (when SPM is negligible), the effects were described in paper [8]. Our considerations relate to the case of strong pulses, when the interaction of pulses with amplifying medium is non-linear and SPM is important. In our experiment we observed the infleunce of SPM on the spectrum of pulses in the output of regenerative amplifier.

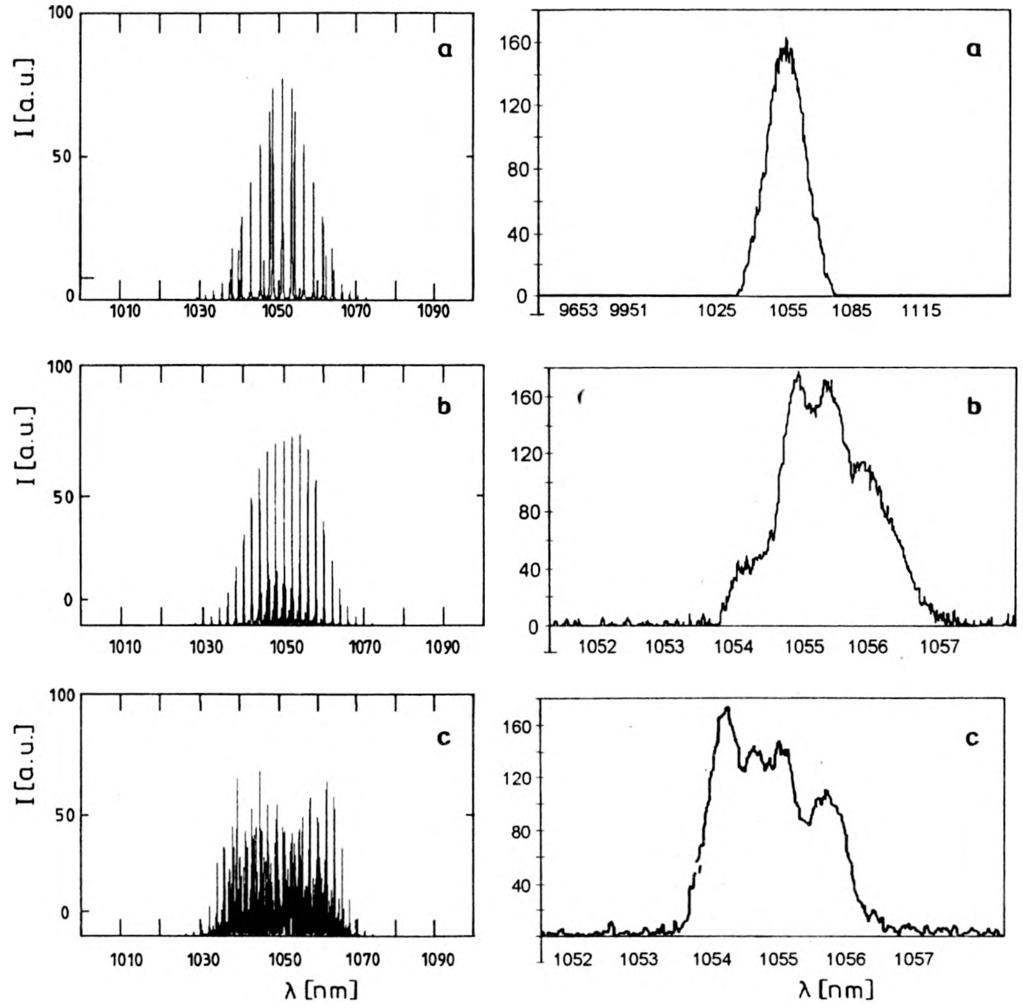

Fig. 7. On the left — spectra obtained from calculations for various values of *B*-integral: 0.5 (a), 3 (b), 5 (c). On the right — spectra measured in the experiment.

We used the Nd:glass rod of the RA as a non-linear medium in our experiment. For estimation of B-integral we registered amplification of the RA at one pass with the use of an oscilloscope. Measuring output energy of the RA and summarising B-integral we estimated its value. Considering transverse intensity distribution of light as a Gaussian one, we calculated effective value of B. Pulses injected to the RA were registered with the use of the streak camera.

The spectrum of the pulse in the output of the RA was measured by usual spectrometer. The output spectra were investigated at various levels of energy (hence, at different B-integral accumulated inside the RA cavity). We observed effective broadening of the spectrum upon amplification of the train of chirped pulses in the RA. In the case of single pulse amplification the effect of broadening of a spectrum was not observed. On the left-hand side of Fig. 7, spectra of amplified pulses obtained from numerical simulations for various values of B-integral are presented. Quite a good conformity with the results of measurements can be seen.

6. Conclusions

Our investigations show that SPM of temporary overlapped chirped pulses can result in:

– swapping of energy from a main pulse to satellites,
– changing the shape and duration of recompressed ultrashort pulse,
– increasing prepulse intensity,
– arising of a prepulse in the presence of postpulse,
– significant broadening of the spectrum of overlapped pulses.

Therefore, for obtaining high-contrast ultrashort pulses from a CPA laser system, the accumulated B-integral for the system should be as low as possible and overlapping of chirped pulses in amplifying medium should be avoided.

References

[1] PERRY M.D., MOUROU G., Science **264** (1994), 917.
[2] MOUROU G., Appl. Phys. B **65** (1997), 205.
[3] NANTEL M., ITATANI J., TIEN A.CH., *et al.*, IEEE J. Select. Topics Quantum Electron. **4** (1998), 449.
[4] KONOPLEV O.A., MAYEHOFER D.D., *ibidem*, 459.
[5] PERRY M.D., DITMIRE T., STUART B.C., Opt. Lett. **19** (1994), 2149.
[6] UMSTADTER D., ESAREY E., KIM J., Phys. Rev. Lett. **72** (1994), 1224.
[7] NAKANO H., NISHIKAWA T., AHN H., UESUGI N., Appl. Phys. B **63** (1996), 107.
[8] VANKOV A.B., KOZLOV A.A., CHIZHOV S.A., YASHIN V.E., Opt. Spektr. **84** (1998), 94 (in Russian).
[9] STRICKLAND D., MOUROU G., Opt. Commun. **56** (1985), 219.
[10] BADZIAK J., CHIZOV S.A., KOZLOV A.A., *et al.*, Opt. Commun. **134** (1997), 495.
[11] LIU A.X., WAGNER R., MAKSIMCHUK A., *et al.*, Opt. Lett. **20** (1995), 1163.
[12] VANKOV A.B., KOZLOV A.A., CHIZHOV S.A., YASHIN V.E., Kvant. Elektron. **22** (1995), 583.

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