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Diffraction measurement methods of sizes of drops in quasi-monodispersive aerosol

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The assumptions of a diffraction method of determining the sizes of drops in the stream of fuel aerosol are presented. The applicability range of the due conventional calculation methods is determined. The systematic errors due to diversified sizes of drops in the sprayed stream of liquid are indicated. The formulae allowing both correction of the measured average diameter $\bar{D}$ and an approximate evaluation of the standard deviation σ of the statistical distribution $\rho(D)$ of drop sizes in the stream examined are derived.

1. Introduction

The quantitative examination of the phenomena taking place during production and next propagation of the aerosol streams depends on the possibility of determining the sizes and velocities of its particles and on the distribution of these magnitudes in space. The stream parameters are determined by the viscosity and surface tension of the sprayed liquid, shape and sizes of the output nozzle and the difference in pressure between the liquid source and the medium to which it is injected.

The measurement of drop diameters is a complex metrological problem since the drops are in motion and their sizes suffer from quick changes in time due to the effect of secondary disintegration as well as evaporation and association.

For the above reasons many measurement techniques applied to estimate the magnitudes of small-sized objects ($10^{-6}-10^{-4}$ m) are of little use in the case of aerosols.

An essential factor limiting the possibility of applying some of the examination methods can also be the time of spraying process, which is very short sometimes. A good example is the injection of fuel into engines with self-ignition being as short as $1-5$ ms. Another difficulty arises from the fact that the degree of pulverization degree differs significantly in particular phases of injection.

The examination of the pulverization degree in continuous operation devices (for example, injectors of airplane turbine engines) is slightly easier, because under settled conditions the geometry parameters of the stream are stable and the time of measurement can be longer.

Recognizing the size distribution of sprayed drops of a liquid and their number per unit volume of the stream renders it possible to explain many phenomena associated with the creation of aerosol and, in prospect, to optimize the parameters of pulverising devices.

The common application of the fluid pulverizing devices to many fields of science and technology creates the necessity of working out and implementing standard methods for their diagnosis. Special significance is attributed to the possibility of estimating the repeatability of the parameters of the fuel pulverization methods in both piston and turbine engines.

An intensive development of measurement techniques including photographic and holographic ones, Doppler anemometry and those exploiting the phenomena of light scattering and diffraction is observed [1], [2].

The basic advantage offered by the diffraction methods of determining the sizes of the aerosol drops is the fact that the parameters of the stream examined do not change in the course of measurement which otherwise happens often when applying other techniques. Therefore, it may be assumed that the light wave passing through the stream has no essential influence on the processes which take place inside the stream.

The sizes of the aerosol drops are easy to estimate in the case of nondispersive medium all the particles of which have the same sizes. The analysis of the diffraction images occurring in a polydispersive medium is a complex problem recognized to an unsufficient degree, so far. In paper [3], a concept of diffractogram analysis for the case of quasi-monodispersive media, *i.e.*, such for which the sizes of drops show only slight spread around their average value is presented.

2. Theoretical principles of the method

2.1. Assumptions of the light scattering model

The aerosol being a dispersive medium, in which fluid is pulverized, is characterized by significant difference in the refractive index between the drops and the surrounding gas. This is the reason for the scattering of the light wave propagating within the pulverized stream. The mathematical interpretation of this process based on complete description of the occurring phenomena constitutes a very complex problem [3] – [13]. In order to obtain relations of practical importance a number of simplifying assumptions is required [14], [15]. In further considerations it has been assumed that:

– dispersive medium (gas) and the pulverized fluid are optically uniform and isotropic,

– incident light wavelength is the same as that of light scattered, which means that quantum phenomena connected with the energy exchange between the light and matter have been neglected,

– drops are spherical and their diameter is greater than the wavelength,

– average distance between the drops is at least several times greater than the wavelength,

– mutual interaction of the waves diffracted at the neighbouring drops is ignored,

– spatial distribution of drops in the stream is totally random and, thus, the drops can be treated as incoherent centra of scattering,

– sizes of the dispersive medium (in the direction of light propagation) are limited to the degree allowing us to assume a single-scattering model.

For the sake of quantitative description of the above assumptions the following dimensionless magnitudes [14] have been introduced:

$x_1 = \frac{D}{\lambda}$ – describes the scattering properties of the drops,

$x_2 = \frac{\bar{l}}{\lambda}$ – describes the spatial structure of the asrosol,

$x_3 = \frac{s}{\lambda}$ – describes the scattering properties of the drop stream,

where: D – diameter of the aerosol, $\bar{l}$ – average distance between the drops in the stream, s – linear size of the volume of dispersive medium in the direction of wave propagation, λ – the wavelength.

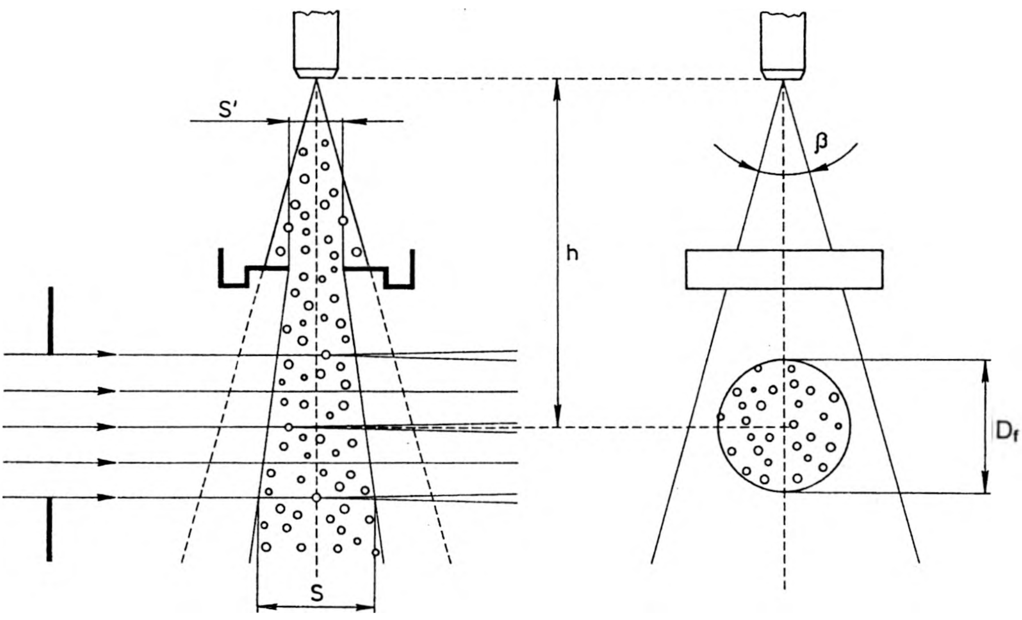

Fig. 1. Reduction of the measurement space by mechanical restriction of pulverization cone.

In the model accepted $x_1 > 1$, $x_2 \gg 1$, and $x_3 \gg 1$, while $x_3 \gg x_2$. The average distance $\bar{l}$ betwen the drops in the aerosol stream increases with the distance h from the nozzle output of the injector (Fig. 1). Knowing the fuel discharge $Q(p)$ and the angle β of pulverization cone the position of measurement space can be chosen in such a way that the condition $x_2 \gg 1$ be fulfilled.

The phenomenon of multiple scattering of light can be omitted if the dimensionless parameter

$$S_w = \frac{x_1^2 x_3}{x_2^3} \tag{1}$$

is not greater than 1 [15].

Within the pulverization cone of the injectors of high fuel discharge $Q(p)$ ($Q(p) > 100$ l/h) the fulfilment of the condition $S_w \leqslant 1$ is not possible even for the distance h close to maximum range of drops. Under such circumstances the only way to assure the consistence between the conditions of measurements and the assumptions of the accepted physical model is a mechanical restriction of the stream of the pulverized drop the scheme of which is illustrated in Fig. 1. A slit of width s' located perpendicularly to the direction of the light beam propagation diminishes the dimension s of the measurement space to such degree that $S_w \leqslant 1$.

The introduction of the elements deforming the pulverization cone is an interference into the process of drops disintegration which may be the cause of some systematic measurement errors. However, the advantages following from the simplification of the mathematical description of the occurring phenomena are much more important since they allow us to apply the formulae in which the intensity of the scattered light becomes dependent only on the spectrum of the drop pulverization $\rho(D)$ in the aerosol stream.

2.2. Fraunhofer diffraction by the aerosol drops

The light scattering by the particles of spherical symmetry is decsribed by the Lorentz–Mie theory. Depending on the value of the Mie parameter $x = \pi D/\lambda$ several models can be distinguished, *i.e.*, that of Rayleigh ($x \ll 1$), that of Rayleigh–Gans ($x \approx 1$), that of Fraunhofer ($x > 1$) and that of Van de Hulst ($x \gg 1$). The average value of the drop diameter $\bar{D}$ in the pulverisation spectrum $\rho(D)$ produced by the injectors of the turbine engine happens to be contained within the range of several to tens of micrometers. Thus, the value of Mie parameter is decisive for the applicability of the model of small angle light scattering to the description of the due effects.

If the pulverized stream of liquid is illuminated by a plane monochromatic wave the light scattering phenomenon can be described with Fraunhofer approximation. The intensity distribution of the light scattered by a single spherical particle is given by the relation

$$I(\Theta,D) = \left| \frac{2J_1\left(\frac{\pi D}{\lambda}\sin\Theta\right)}{\frac{\pi D}{\lambda}\sin\theta} \right|^2 [K(\Theta)]^2 \tag{2}$$

where: Θ – diffraction angle of the light wave, J_1 – Bessel function of the first kind and first order, $K(\Theta)$ – Kirchhoff directional coefficient of the form

$$[K(\Theta)]^2 = \frac{1}{2}(1+\cos^2\Theta). \tag{3}$$

The coefficient $K(\Theta)$ defines the light wave amplitude correction needed due to the change of the light propagation direction. This is of significance only for the particles of sizes comparable with the wavelength λ. For the scattering centres of greater dimensions one can assume $K(\Theta) \approx 1$.

Application of the diffraction model to the description of the light scattering by the aerosol drops is also advantageous because of resulting simplicity of expression describing the scattered light intensity $I(\Theta, D)$, which is essential while practically implementing the diffraction method [16], [17].

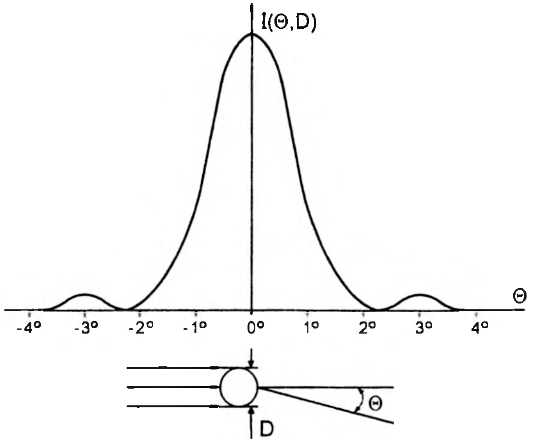

Fig. 2. Intensity distribution $I(\Theta, D)$ of the light scattered by a drop of diameter $D = 20$ μm illuminated by a plane wave of wavelength $\lambda = 0.63$ μm depending on the observation angle Θ.

In Figure 2, the intensity distribution of the light scattered by a drop of 20 μm in diameter illuminated by a plane parallel light beam of $\lambda = 0.63$ μm is presented. This is a characteristic image in the form of Airy rings being composed of bright central maximum surrounded by a series of darkening rings. This image, observed on a screen located at a distance l_e, becomes magnified more and more with the increase of this distance while its intensity decreases.

If in the measurement space there exist N drops, while each of them produces its own diffraction image the analysis of the resultant light intensity distribution becomes more difficult since its shape is influenced by both the diameters D_k of the aerosol drops and their spatial distribution.

Two identical drops diffract a plane wave into a cone of the same solid angle but when the difference of the distances between them (measured in the direction of the light wave) is significant their diffraction images will be Airy rings of different diameters. This disadvantage possibly causing some interpretation ambiguity can be reduced by significant separation of the screen from the measurement space. This exerts, however, some disadvantageous influence on the signal-to-noise ratio in the recorded light intensity distribution since the number of scattering centres, *i.e.*, the particles of the water vapour and all kinds of solid contaminations suspended in the air, increases forming the "diffraction background".

2.3. Fourier transformation of the diffraction image

The difficulties connected with the necessity of taking account of the spatial distribution of the drops in the aerosol stream can be eliminated by Fourier transforming of the diffraction image $I(\Theta, D)$. The following assumptions have been made [2], [16]:

– stream of drops is illuminated by a monochromatic and coherent plane wave,

– wavefront of this wave is of the same phase and amplitude of the due light vector vibrations,

– Fourier transforming objective is free from aberrations.

If within the space where the monochromatic plane propagates there is N_k identical drops of diameter D_k the light wave amplitude in the point (x_n, y_n) of the focal plane will be defined by the relation [6]

$$U_N(x_n, y_n) = U'(x_n, y_n)\sum_{i=1}^{N_k} \exp(ik\Delta_i) \tag{4}$$

in which: $U'(x_n, y_n)$ – amplitude of the wave diffracted by a single drop, k – wave number, and Δ_i – optical path between the drop and the point (x_n, y_n). The light intensity $I_k(x_n, y_n)$ at the given point of the diffraction image is

$$I_k(x_n, y_n) = |U'(x_n, y_n)|^2 \left|\sum_{i=1}^{N_k} \exp(ik\Delta_i)\right|^2, \tag{5}$$

the term $U'(x_n, y_n)$ is an Airy function for the drop of D_k diameter

$$|U'(x_n, y_n)|^2 = \left|\frac{2J_1(z)}{z}\right|^2. \tag{6}$$

The variable z denotes a dimensionless Airy optical unit

$$z = \frac{\pi D_k r_n}{\lambda f} \tag{7}$$

where: f – objective focal length, r_n – radius vector of the point (x_n, y_n).

The exponential term in expression (4) describes the interference between the light wave following from the optical path difference between the scattering centres and the point (x_n, y_n). If the aerosol drops are distributed in the measurement space in a way totally random and their position changes in time this sum is proportional to the number N_k [6]

$$I_k(x_n, y_n) \simeq N_k |U'(x_n, y_n)|^2. \tag{8}$$

This approximation can be applied ouside the central region of the zero order diffraction fringe since for $r \to 0$ the light intensity $I_k(r)$ is proportional to N_k^2 [10], [11].

When there exist drops of different diameters D_k $(k = 1, 2, \dots, M)$ within the measurement space each group of drops produces a separate diffraction image. Due to superposition of $I_k(x_n, y_n)$ distributions the resultant diffraction image of the intensity $I(x_n, y_n)$ takes the form

$$I(x_n, y_n) \simeq \sum_{k=1}^{M} I_k(x_n, y_n) = \sum_{k=1}^{M} N_k \left| \frac{2J_1(z)}{z} \right|^2. \tag{9}$$

Passing from the discrete distribution of the drop diameters D_k to a continuous one described by the pulverisation spectrum $\rho(D)$ relation (9) can be expressed in the form

$$I(r) = \int_0^\infty \rho(D) \left| \frac{2J_1(z)}{z} \right|^2 \mathrm{d}D. \tag{10}$$

The Fourier transform of the plane wave scattered by the aerosol stream allows us to eliminate the influence of the spatial distribution of the drops on the light distribution $I(r)$ in the focal plane. The motion of the drops in the paraxial region does not change the diffraction image.

The distance between the stream and the objective is defined by the phase modulation of the Fourier spectrum at particular points of the image plane. In the case of photographic recording or visual observation of the diffraction pattern the phase of the light wave has no significance since neither the photographic emulsion nor the eye are phase-sensitive [18]. The phase modulation is essential when holographically recording the Fourier spectrum.

Optical diffractometers are applied to the measurement of the sizes of the microobjects of transversal dimensions ranging between 5 μm and 500 μm. They are based on the Fraunhofer diffraction and according to the way they are realized they can be divided into two groups: lensless and of lens type. In the first case, there are no optical elements between the measurement space and the screen while in the second case there are some lens systems performing the transformations of the diffractive image (Fig. 3).

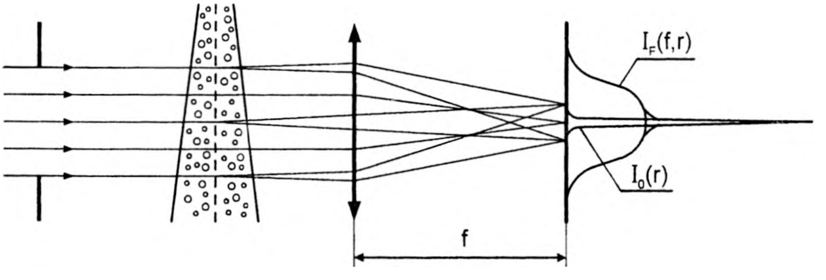

Fig. 3. Intensity distribution of the light in the focal plane after a plane wave passed through the aerosol stream.

In both variants of the method the far field diffraction is employed since the distance of the diffraction pattern is much greater than the transversal dimensions of the object scattering the light. This occurs when the distance from the screen l_e or the focal length f of the objective fulfil the condition [6], [8], [18]

$$l_e \gg \frac{D^2}{2\lambda}, \quad f \gg \frac{D^2}{2\lambda}. \tag{11}$$

For practical realisation of the measurements the distances l_e and f are usually some tens times greater. There is a tendency to make the distance $r_d = r_{i+1} - r_i$ between the two neighbouring rings hardly visible and almost unmeasurable. Usually, it is assumed that the value of the diffraction interval r_d should not be less than 2 mm [18].

3. Calculation of the average diameter $\bar{D}$ of the aerosol drops

3.1. Monodispersive medium

In Figure 4, a dimensionless light intensity distribution $\Lambda_m(z)$ was shown for a monodispersive medium

$$\Lambda_m(z) = \frac{I_F(z)}{I_0} = \left|\frac{2J_1(z)}{z}\right|^2 = \left|\sum_{k=0}^{\infty} \frac{(-1)^k z^{2k}}{2^{2k+1}(k+1)!k!}\right|^2. \tag{12}$$

The consecutive dark and bright rings occur for dimensionless coordinates z_i taking the values:

$$z_{0.5} = 3.832, \quad z_{1.0} = 5.136, \quad z_{1.5} = 7.016,$$
$$z_{2.0} = 8.417, \quad z_{2.5} = 10.17, \quad z_{3.0} = 11.62. \tag{13}$$

The half-values of the index i denote local minima of light intensity $\Lambda_m(z)$ while the entire values – the local maxima.

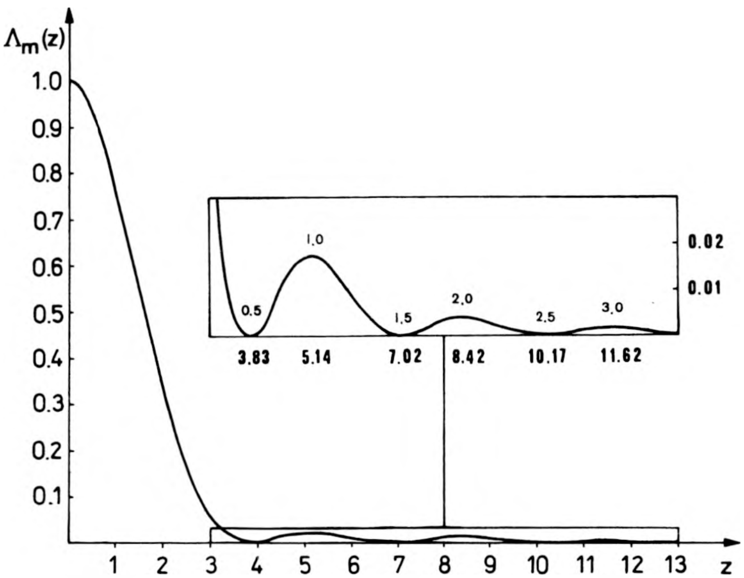

Fig. 4. Dimensionless intensity distribution of the light diffracted on a single drop.

Determining the positions of consecutive extrema r_i from the diffractograms $I_F(f,r)$ the diameters of the drops can be calculated [13], [18]

$$D_m = \frac{z_i \lambda \sqrt{r_i^2+f^2}}{r_i}. \tag{14}$$

Substituting the values z_i to Eq. (14) the well known formulae are obtained [6], [8], [16]

$$D_m \simeq 1.22\frac{\lambda f}{r_{0.5}} \simeq 1.64\frac{\lambda f}{r_{1.0}} \simeq 2.2\frac{\lambda f}{r_{1.5}} \simeq \dots . \tag{15}$$

The intensities of transmission maxima are very small but, in spite of this, the consecutive bright and dark rings are well visible and can be recorded by scanning the diffraction image or indirectly taking advantage of the photographic techniques.

The possibilities of observing this rings follows from high contrast K between the neighbouring extrema of the light intensity $I_F(f,r)$.

3.2. Quasi-monodispersive media

The light intensity $I_F(f,r)$ after the plane wave have passed through the aerosol stream of diversified sizes of drops is described by Eq. (10).

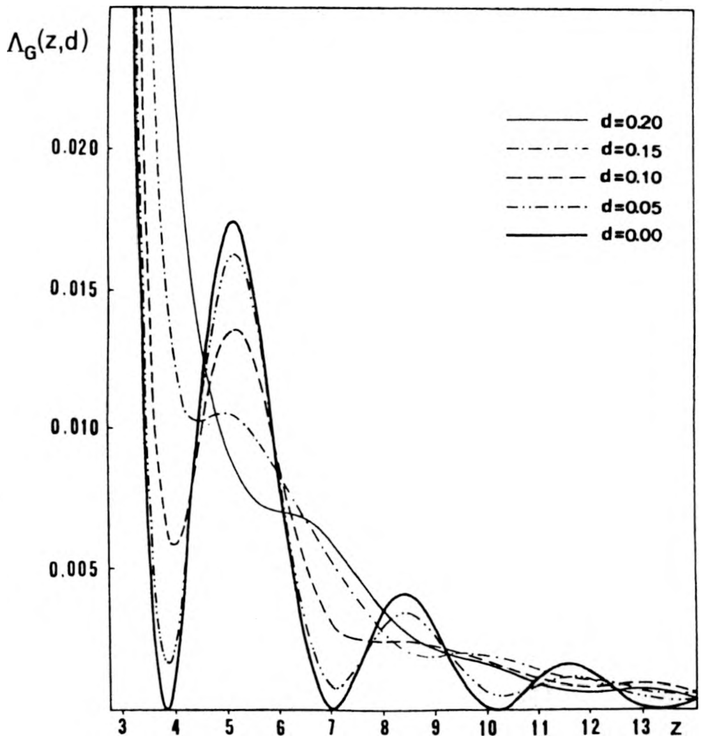

Fig. 5. Decay of the diffraction rings for the quasi-monodispersive medium of Gauss distribution $\rho_G(D)$ with the increase of the relative standard deviation d.

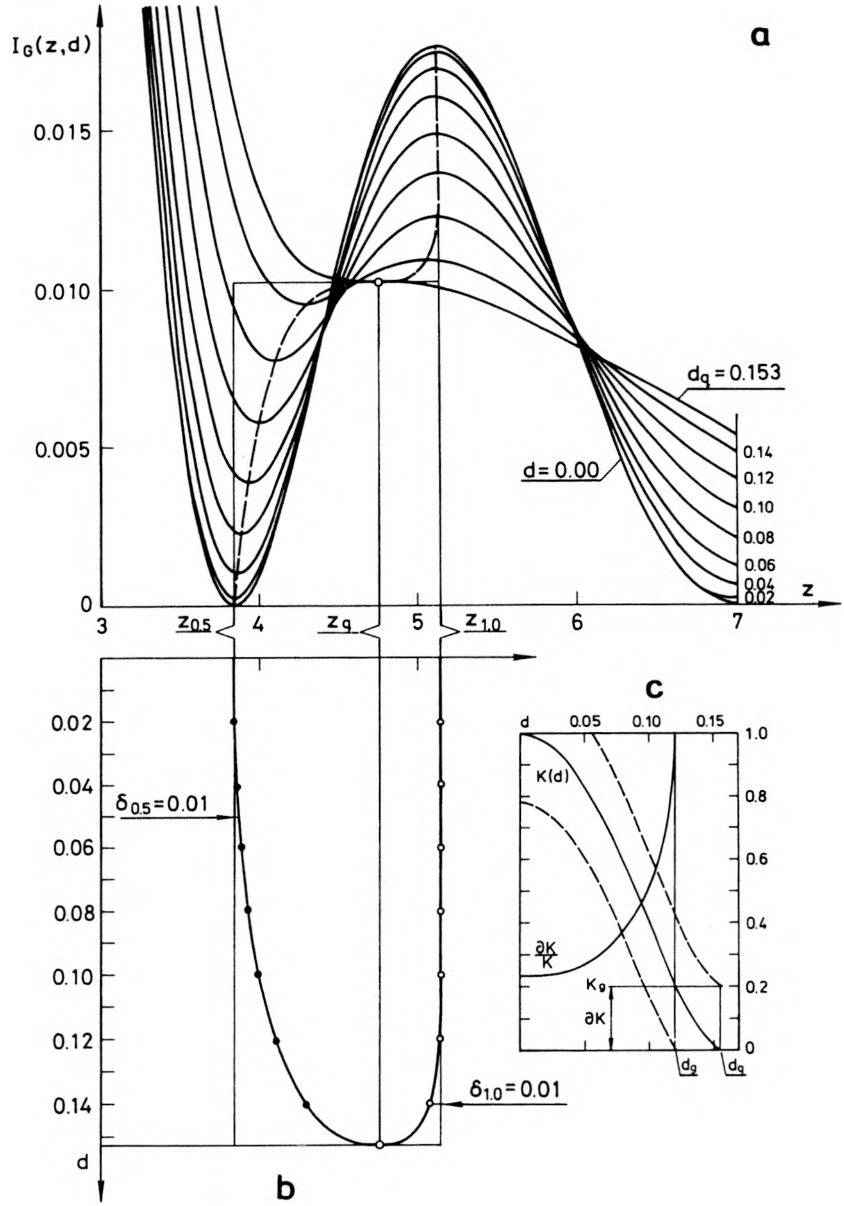

Fig. 6. Decay of the first dark and bright diffraction rings for a medium of normal distribution $\rho_G(D)$ (a). The change of position of the light intensity extrema $z_{0.5}$ and $z_{1.0}$ versus the relative standard deviation d (b),. Contrast $K(d)$ between bright and dark diffractive rings (c).

As a measure of size dispersion in the pulverized liquid the ratio of the standard deviation σ and the average value $\bar{D}$ of the function $\rho(D)$ is assumed

$$d = \frac{\sigma}{\bar{D}}. \tag{16}$$

The quantitative estimation of the influence of the relative standard deviation d on the dimensionless light intensity distribution $\Lambda_\rho(z)$ was carried out by comparing it with the distribution $\Lambda_m(z)$ for the monodispersive medium. It has been assumed that

$$\bar{D}_\rho = D_m \text{ and } z = \frac{\pi \bar{D}_\rho r}{\lambda f} = \frac{\pi D_m r}{\lambda f}. \quad (17)$$

In Figure 5, several curves $\lambda_G(z)$ calculated for the medium the drop sizes of which are described by normal Gauss distribution $\rho_G(D)$ are presented. Together with the increase of value d consecutive diffraction rings disappear and for $d = 0.2$ the function $\Lambda_G(z)$ is monotonic in the whole range.

Figure 6 shows the way the first dark and bright rings disappear. The corresponding extrema of light intensity $z_{0.5}(d)$ and $z_{1.0}(d)$ approach each other and for $d_q = 0.153$ and $z_q = 4.76$ a point of inflexion of the curve $\Lambda_G(z)$ appears. The first dark ring increases gradually its diameter while the bright ring remains unchanged. In a quantitative way this is described by the parameter

$$\delta(d) = \frac{z_i(d) - z_i(0)}{z_i(0)} = \frac{z_i(d) - z_m}{z_m}. \quad (18)$$

Simultaneously, this parameter is a measure of a systematic error which apepars when relation (14) is used to calculate the average diameter $\bar{D}$ of the drops in the quasi-monodispersive medium; formula (14) being derived for monodispersive aerosol (Fig. 7).

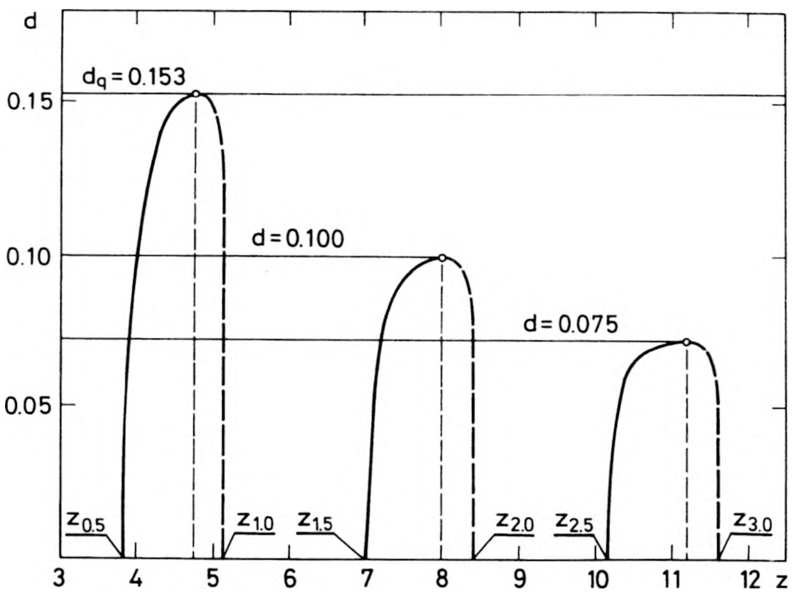

Fig. 7. Change of position of six consecutive extrema of light intensity $\Lambda_G(z, d)$ for a quasi-monodispersive medium of Gauss distribution $\rho_G(D)$.

Assuming $\delta(d) < 0.01$, a conventional criterion for applicability of Eq. (14), we get $d < 0.05$ for the first dark fringe and $d < 0.14$ for the bright one. Herefrom it follows that the first maximum of the light intensity is a more stable information carrier about the drop sizes in the aerosol stream than the first minimum of the $\Lambda_G(z)$ distribution.

The diffraction rings of higher orders disappear in an analogous way. This is illustrated in Fig. 8, in which the positions $z(d)$ of six consecutive extrema for dimensionless light intensity distribution $\Lambda_G(z)$ are presented.

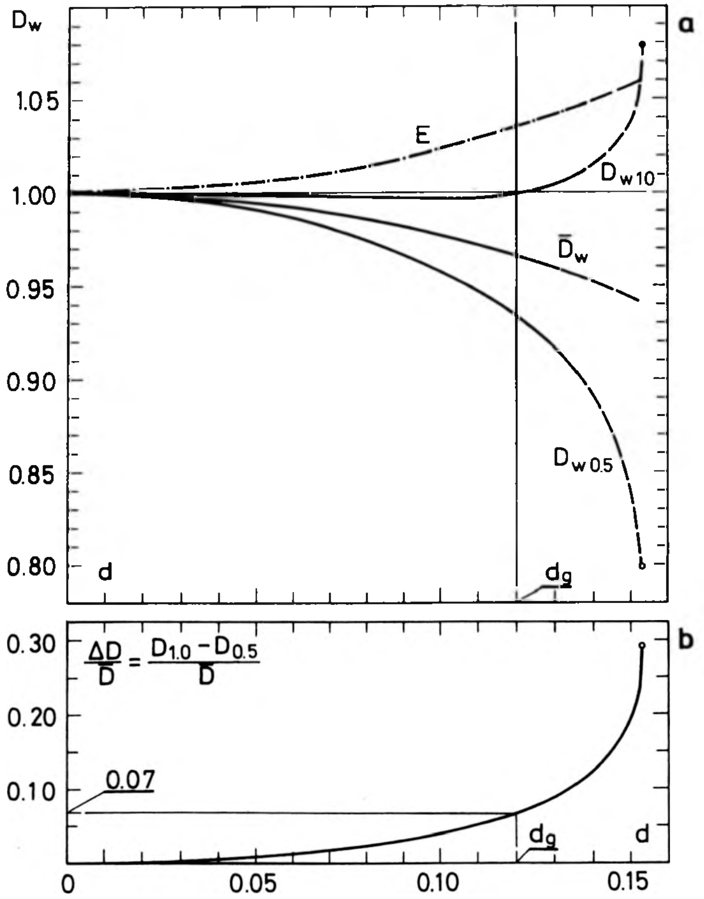

Fig. 8. Dependence of dimensionless diameters $D_{w0.5}$, $D_{w1.0}$, $\bar{D}_w$ of relative difference $\Delta D/\bar{D}$ and the correcting coefficient E on the parameter $d = \sigma/D$.

The calculations carried out for other functions $\rho(D)$ gave similar results. It should be emphasized that for the symmetric distributions, *i.e.*, such that fulfil the condition

$$\rho(D) = \rho(2\bar{D} - D) \tag{19}$$

the dependences $z_i(d)$ are almost identical with those presented in Fig. 8. Only for the distribution function $\rho(D)$ of significant asymmetry coefficient γ small differences in the shape of dependences $z_i(d)$ appear in the vicinity of the points of inflexion [19].

The presented manner of diffraction ring decay can be considered as being representative and the limiting value of the relative standard deviation $d_q = 0.153$ as characteristic of the majority of typical statistical distributions $\rho(D)$.

The value $d_q = 0.153$ allows us to understand the concept of "quasi-monodispersive" medium in a more precise way. This means an aerosol in which the drop sizes show only slight spread around the average value $\bar{D}$. An experimental criterion, allowing us to define the stream of pulverized liquid as quasi-monodispersive, is a possibility of applying relation (14) to calculate the average diameter $\bar{D}$ of the drops. This is directly connected with the appearance of the dark and bright rings in the diffractive image. All the rings disappear if the relative standard deviation $d = d_q$ (Figs. 6 and 7) and thus for $d \geqslant d_q$ the medium should be treated as polydispersive one.

If at least two dark rings appear in the diffractogram of the examined stream the average diameter $\bar{D}$ of the stream of drops is defined by averaging the values D_i calculated for four consecutive extrema of the light intensity.

However, if only one dark ring can be observed in the diffraction image there appears an uncertainty in the results of calculation due to the relation

$$D(r_{0.5}) < D(r_{1.0}). \tag{20}$$

A question arises of which of the two values is closer to the average diameter $\bar{D}_p$ of the pulverization spectrum $\rho(D)$. In Figure 8, the dimensionless values of the aerosol drop diameter D_{wi} are presented as depending on the relative standard deviation d defined by the relations:

$$D_{w0.5} = \frac{D(r_{0.5})}{D_m}, \quad D_{w1.0} = \frac{D(r_{1.0})}{D_m}, \quad \bar{D}_w = \frac{D_{w0.5}+D_{w1.0}}{2}. \tag{21}$$

The calculations carried out on the basis of the first light intensity extremum position produce a result underrated by few to several percent, *i.e.*, $D_{w0.5} \ll 1$. Within the range $d = 0-0.12$ the drop diameter $D_{1.0}(d)$ is almost the same as the diameter D_m of the comparable monodispersive medium and thus $D_{w1.0} \simeq 1$ (minimum value $D_{w1.0} = 0.997$). Significant increase of $D_{w1.0}$ for the relative standard deviation $d = 0.14-0.153$ (Fig. 8a) has no greater significance in the measurement method applied since it is a range of disappearance of two last diffraction fringes.

From the above analysis it follows that the diameter $D_{0.5}(d)$ of the aerosol drops determined on the basis of the first minimum of the light intensity is always less than the average value $\bar{D}_p$ while $D_{1.0}(d) \simeq \bar{D}_p$. Can we, thus, give up the calculation of the diameter $D_{0.5}(d)$, limiting ourselves to calculating $D_{1.0}(d)$ only?

In order to answer this question both the systematic error (which occurs after applying relations strictly valid for media of dimension uniformity to the quasi-monodispersive aerosols) and the random error (connected with the accuracy of experimental data obtained from diffractogram) should be taken into account.

A characteristic feature of the appearing image is significant diversification of the light intensity $A_m(z)$ between the central point ($z = 0$) and the consecutive bright diffraction rings ($A_m(0) = 1$, $A_m(z_{1.0}) = 0.0175$, $A_m(z_{2.0}) = 0.0042$, $A_m(z_{3.0}) = 0.0016$). For this reason the first dark ring is the best "visible" one.

After carrying out a detailed analysis it has been stated that the accuracy of position determination for this ring is greater almost by an order of magnitude than that of other extrema and this is true irrespective of the way the diffraction image is recorded. Therefore, it appears aimful to calculate the average $\bar{D} = (D_{0.5} + D_{1.0})$ and to take advantage of systematic character of changes $\bar{D}(d)$ and make a suitable correction. This is suggested by the fact that the position $z_i(d)$ of the extrema of transmission is, in principle, independent of statistical distribution $\rho(D)$ of the drop sizes in the aerosol stream. Only in the vicinity of points of inflexion $z(d_q)$, *i.e.*, for $d \geqslant 0.9 d_q$ a slight influence of the distribution type on the character of dependence $z_i(d)$ is observed.

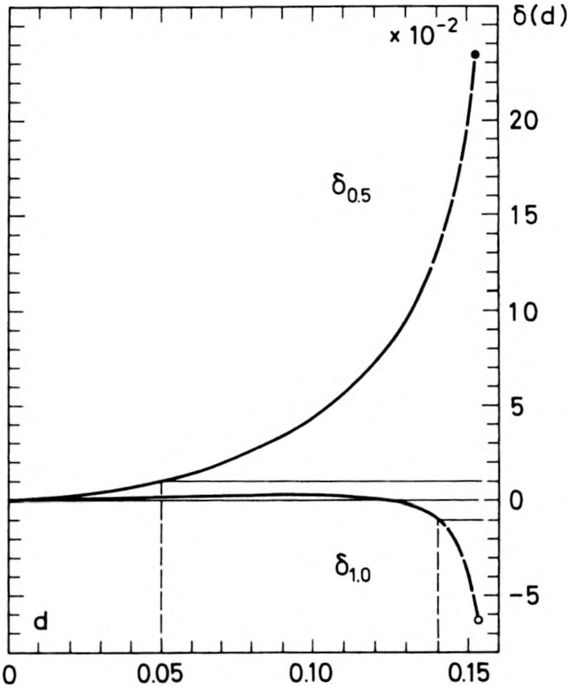

Fig. 9. Relative error $\delta(d)$ of the position $z_i(d)$ of the light intensity extrema.

In Figures 8 and 9, the broken line is used to mark those fragments of dependences $\delta(d)$ and $D_w(d)$ which are characteristic of the Gauss distribution.

The corrections of the average value $\bar{D}$ can be made by introducing the coefficient $E(d)$ fulfilling the condition

$$E(d)\bar{D}_w = 1. \tag{22}$$

Figure 8a illustrates the dependence of this coefficient on the relative standard deviation d. The value of d, however, is not known a priori and therefore it is reasonable to refer to a magnitude possible to determine directly from the experimental data. This role is best played by the relative difference of the drop diameters $\Delta D/\bar{D}$ calculated on the basis of position of the first two extrema of light intensity (Fig. 8b)

$$\frac{\Delta D}{\bar{D}} = \frac{D_{1.0}-D_{0.5}}{D}. \tag{23}$$

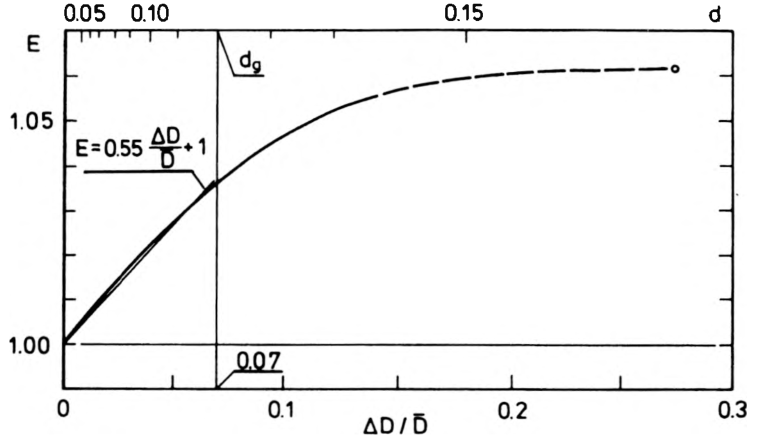

Fig. 10. Dependence of the correcting coefficient E on the relative difference $\Delta D/\bar{D}$.

Figure 10 shows the correction coefficient $E = E(\Delta D/\bar{D})$ as dependent on the value of the above difference.

Before approximation of the function $E(\Delta D/\bar{D})$ the range of relative standard deviation d, for which it is possible to determine the position of diffraction extrema on the basis of experimental data, must be determined.

A measure of the light intensity differentiation $I_i(r_i)$ between the maximum and minimum is the contrast coefficient [18]

$$K(d) = \frac{I_{1.0}-I_{0.5}}{I_{1.0}+I_{0.5}}. \tag{24}$$

Its value diminishes from a unit (monodispersive medium) to zero at the point of inflexion z_q, in which $I_{0.5}(d_q) = I_{1.0}(d_q)$ (polydispersive medium).

The possibility of extrema identification in the recorded diffraction image diminishes significantly when the value of the difference $I_{1.0}-I_{0.5}$ is comparable with the accuracy of its determination. Then, a real decay of contrast occurs since it can achieve the zero value (within the random error). As a limiting coefficient of contrast $K_g(d_g)$ there is assumed the one for which the equivalent conditions are fulfilled:

$$K_g(D_g)-|\delta K| = 0 \quad \text{or} \quad \frac{|\delta K|}{K_g(d_g)} = 1. \tag{25}$$

Assuming that the measurement accuracy for the light intensity $I(r)$ is $dI/I(0) = 0.002$, the value $K_g = 0.2$ was determined which corresponds to the limiting standard deviation $d_g = 0.12$ (Fig. 6c).

The assumed accuracy $dI/I(0) = 0.002$ is of asymptotic character since it constitutes a threshold for the detection possibilities for the majority of standard methods of image recording.

For $d_g = 0.12$, the relative diameter of drops is $\Delta D/\bar{D} = 0.07$ (Fig. 8**b**) and thus the correction coefficient $E = E(\Delta D/\bar{D})$ presented in Fig. 10 can be approximated by a straight line within this range

$$E(\Delta D/\bar{D}) = 1+0.55\frac{\Delta D}{\bar{D}} \quad \text{for} \quad \frac{\Delta D}{\bar{D}} \leqslant 0.07. \tag{26}$$

Taking advantage of relations (21) and (22) the dependence for the corrected drop diameter is obtained

$$\bar{D}_k = \bar{D}\left(1+0.55\frac{\Delta D}{\bar{D}}\right) \quad \text{for} \quad \frac{\Delta D}{\bar{D}} \leqslant 0.07. \tag{27}$$

The diameter $\bar{D}_k$ calculated in this way differs slightly from $D_{1.0}$, in other words

$$\bar{D}_k \simeq D_{1.0} \quad \text{for} \quad \frac{\Delta D}{\bar{D}} \leqslant 0.07. \tag{28}$$

Thus, there appears a necessity to evaluate the purposefulness of calculating the values $D_{0.5}$, $D_{1.0}$, $\bar{D}$ and $\Delta D/\bar{D}$ in order to make correction of the results obtained approximately equal to $D_{1.0}$. A detailed analysis of the measurement accuracy, in which the influence of the diffraction background and systematic errors connected with the technical implementation of the method discussed were taken into account, indicates some supremacy of this slightly "circuitous" way of calculating the average diameter $\bar{D}_k$ of the drops. The most essential reason for its application is the possibility of exploiting the value of $\Delta D/\bar{D}$ to determine the relative standard deviation $d = \sigma/D$ of the statistical distribution $\rho(D)$ of the drop sizes in the aerosol stream examined.

4. Determination of the standard deviation σ of the statistical distribution $\rho(D)$ of the aerosol drop sizes

The observation of the diffraction image occurring while the light wave passes through the quasi-monodispersive aerosol stream allows us to evaluate the standard deviation σ of the function describing the statistical distribution $\rho(D)$ of the drop sizes.

In Figure 5, the decay of consecutive diffraction rings with the increase of dimensionless parameters $d = \sigma/D$ is shown. The number of rings constitutes the basis for qualitative estimation of this parameter characterizing the spread of the drop sizes in the pulverization spectrum $\rho(D)$.

The values of consecutive maxima of light intensity $I(z_i)$, $i = 1, 2, 3, \ldots$, are very small as compared to the intensity in the image centre $I(0)$, (Fig. 4), but, in spite of this, they are well visible due to high contrast to the neighbouring minima $I(z_i)$, $i = 0.5,\ 1.5,\ 2.5,\ \ldots$.

The determination of the number p of dark rings for the case of the photographic recording of the diffraction image is facilitated by the fact that the exposure curve for negative materials is of logarithmic character.

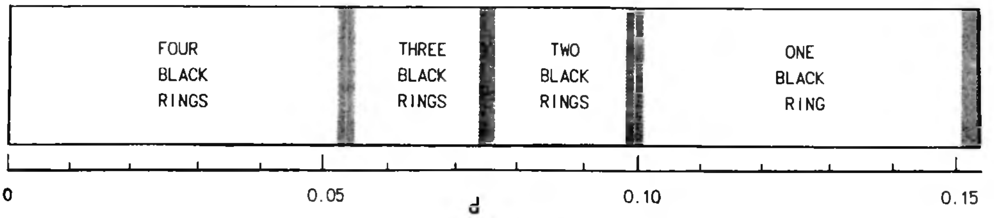

Fig. 11. Number of dark rings p visible in the diffraction image as dependent on relative standard deviation d.

Knowing the value p it is possible, taking advantage of Fig. 11, to estimate approximately the relative standard deviation d. The error ΔD of such estimation is equal to the width of the interval corresponding to the number of observed dark diffraction fringes.

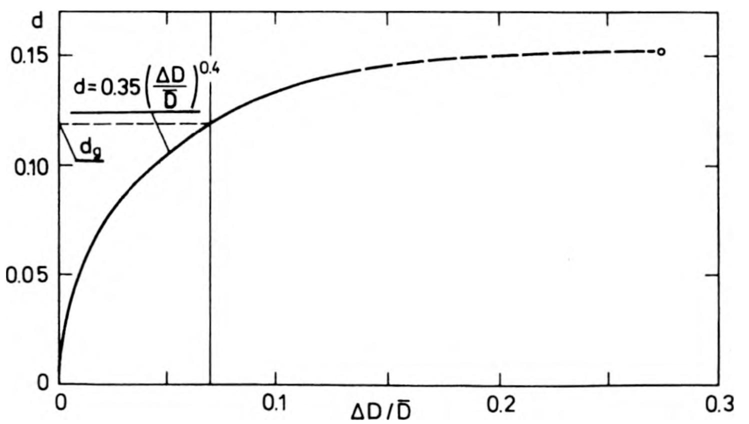

Fig. 12. Mutual dependence between the relative standard deviation d and the relative difference of diameters $\Delta D/\bar{D}$.

Systematic character of the changes in first minimum position of light intensity $I(z_{0.5})$ allows us to determine the parameter d with a much better accuracy. Figure 12 illustrates the dependence between the value d and the relative difference $\Delta D/\bar{D}$ of the drop diameters. Within the interval $d = 0 - d_g$ which corresponds to the range of $\Delta D/\bar{D} = 0-0.07$ we can take advantage of the approximation

$$d \simeq 0.35\left(\frac{\Delta D}{\bar{D}}\right)^{0.4}, \quad \text{for } \frac{\Delta D}{\bar{D}} \leqslant 0.07. \tag{29}$$

Multiplying the value d calculated according to Eq. (29) by the corrected diameter $\bar{D}_k$ the standard deviation σ of the drop pulverization spectrum $\rho(D)$ is obtained. Relation (29) is independent of the kind of distribution function $\rho(D)$. This follows from the fact that different equations applied to the statistical description of the drop diameters in the stream referred to the dimensionless diameter $D/\bar{D}$ for $d = \sigma/\bar{D} =$ idem, give the distribution functions $\rho_i(D/\bar{D})$ for $d \leqslant d_g$ ($d = 0-0.12$), which show no essential difference within this range. This conclusion is formulated taking advantage of the equations used most frequently to approximation of the drop diameter distribution $h(D_i)$ determined experimentally, *i.e.*, those of Rosin–Rammler, Nukijama–Tanasawa and Gauss.

5. Results of measurements

The detailed analysis of the light intensity distribution $I(r)$ in the diffraction images of the quasi-monodispersive aerosol have been carried out because of significant divergence between the drop diameters $D_{0.5}$ and $D_{1.0}$ calculated from the relations valid in a rigorous way for the monodispersive medium. The difference between these diameters amounts to a few percent and is of systematic character, since always $D_{1.0} \geqslant D_{0.5}$.

Relation (27) allowing us to correct the results of measurements of the diameter $\bar{D}$ of the drops of the pulverized stream of the liquid and relation (29) rendering the determination of the relative standard deviation $d = \sigma/\bar{D}$ possible have been confirmed experimentally.

The results of measurements obtained by the diffraction method and the photographic one were compared. In Figure 13, a fragment of the light intensity distribution $I(r)$ is presented which was recorded in the direct way by scanning the diffraction image created after the light wave passed through the pulverization cone of the starting injector of K 108-767 type. After determining the radii of the diffraction rings $r_{0.5}$ and $r_{1.0}$ the magnitudes characterizing the drop sizes were calculated: $D_{0.5} \simeq 30.4 \pm 0.3$ μm, $D_{1.0} = 31.8 \pm 0.4$ μm and $\Delta D/\bar{D} = 0.045$.

The corrected diameter of the drops was $\bar{D}_k = 3.19 \pm 0.4$ μm, and the relative standard deviation $d = 0.1$, therefore, $\sigma = 3.2$ μm. The geometrical parameters of the measurement line (Fig. 1) took the values: $h = 250$ μm, $D_f = 50$ mm, $s' = 2$ mm, and $f = 0.5$ m [20], [21].

The assumed slit width is several times less than the value s_{max} assuring the consistency of the measurement conditions with the accepted model of diffraction.

Mechanical restriction of the pulverization cone to such a thin layer was necessary due to the requirements of the photographic techniques as well as to the possibility of unique analysis of the images. Figure 14 presents the histogram of the pulverisation spectrum of the starting injector of K 108-767 type under the fuel

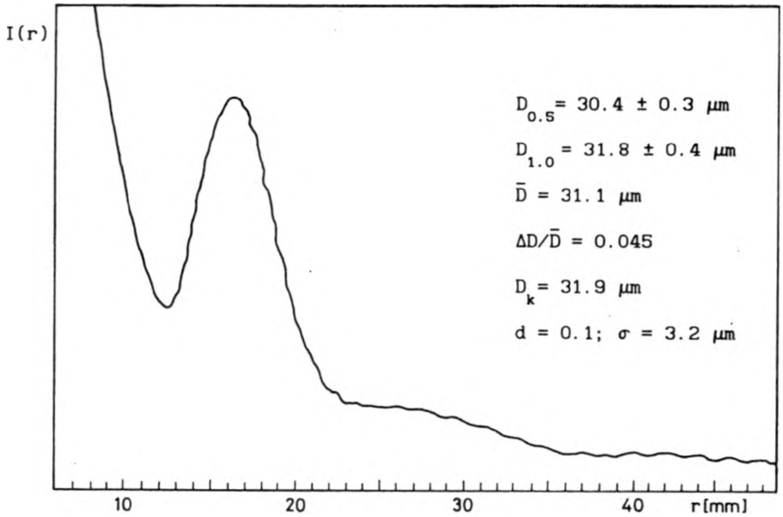

Fig. 13. Light intensity distribution $I(r)/I(0)$ after the light wave passed through the pulverisation cone of the starting injector of K 108-767 type of the Lis-5 engine (pressure of the fuel forcing $p = 0.5$ MPa, the fuel delivery $Q(p) = 8$ l/h, angle of the pulverization cone $\beta = 70°$, $h = 250$ mm, $D_f = 50$ mm, $s = 2$ mm, $f = 0.5$ m).

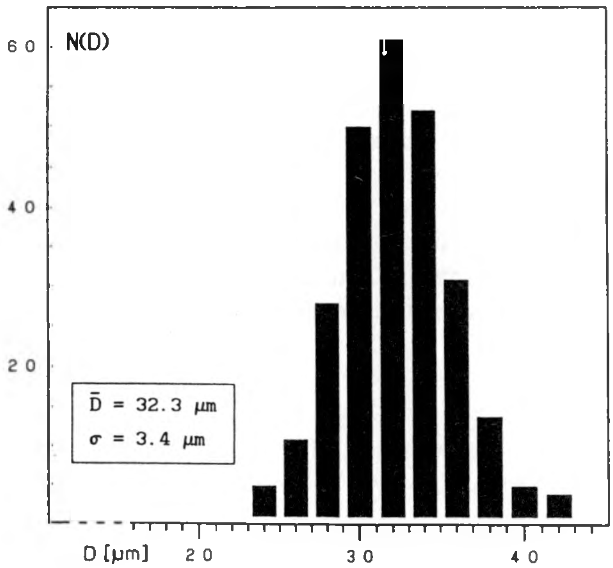

Fig. 14. Histogram of the drop pulverization spectrum of the starting injector of K 108-767 type of the Lis-5 engine produced by the photographic method.

forcing pressure $p = 0.5$ MPa. The average diameter of the drop $\bar{D} = 32.3 \pm 0.5$ μm is (within the measurement error) consistent with the corrected value od $\bar{D}_k$ obtained by the diffraction method.

The correctness of formulae (27) and (29) is confirmed by examination of the light diffraction on different solid structures (such as suitably prepared photographic film, diffraction gratings of thin glass fibres, glass spheres, biologicals and the like) of properties similar to those of quasi-monodispersive structures.

6. Summarising remarks

The magnitude of the drops $\bar{D}$ is estimated on the basis of position of the light intensity extrema $I(r)$ in the diffraction image. The diversification of the drops sizes in the pulverized liquid stream causes diminishing of the contrast between dark and bright rings. It has been stated that for the relative standard deviation $d \geqslant 0.153$ there appears a total decay of all the diffraction fringes ($dI(r)/dr < 0$) independently of the character of the statistical distribution function $\rho(D)$ describing the spectrum of the drop sizes in the aerosol stream.

It has been shown that the maxima of the light intensity $I(r_i)$, $i = 1, 2, \ldots$, are more stable carriers of information about the drop sizes in the quasi-monodispersive medium than minima of $I(r_i)$, $i = 0.5, 1.5, \ldots$. The position of the dark rings changes in a systematic way with the increase of the relative standard deviation $d = \sigma/D$ and has been used to perform its approximate estimate.

The exploitation of the laser light eliminates the effect of chromatic aberration which increases the possibility of proper correction of the remaining aberrations of optical imaging by the objective applied in diffractometer. This problem has been discussed in detail in paper [22].

The qualitative estimation of the relative standard deviation of the pulverization spectrum $\rho(D)$ for the quasi-monodispersive aerosol on the basis of the number of dark fringes visible in the diffraction image is very convenient because it allows us to estimate quickly the value of d (with the error within the range 20–40%) from the table presented in Fig. 11. The simplest way of determining the position extrema of the light intensity $I(r)$ is an indirect measurement consisting in a photographic recording of the diffraction image and its microphotometric analysis.

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