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Scattering by a slit in an infinite conducting screen

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Cylindrical wave diffraction by a slit in an infinite, plane, perfectly conducting barrier in a homogeneous biisotropic medium is investigated. The source point is assumed far from the slit so that the incident cylindrical wave is locally plane. The slit is wide and the barrier thin, both with respect to wavelength. The boundary value problem is reduced to a Wiener–Hopf equation and solved approximately.

1. Introduction

The diffraction of electromagnetic waves by a slit in a screen is an importnat topic in diffraction theory from both theoretical and engineering points of view. As known, certain guiding structures such as microwave passive filters, coupling structures, *etc*., contain thick slits or slots and it is therefore of prime interest to analyse their diffraction characteristics. The main aim of this work is to calculate the scattered wavefield excited by a cylindrical wave incident to a slit in a screen exhibiting conduction in homogeneous biisotropic medium. To the best of our knowledge, the diffraction by a slit in a homogenous biisotropic medium has never been discussed before. This seems to be first and the worthwhile attempt in diffraction theory in a homogeneous biisotropic medium. The source is assumed to be sufficiently far from the slit so that its wavefront is locally plane. Throughout we assume that the field is harmonic in time. The asymptotic analysis of the resulting integrals is only carried far enough to permit the calculation of the diffracted wavefields far from the slit. We anticipate extending the analysis of these integrals, so that expressions for the wavefield in the slit and close to the screen can be obtained, and have therefore given more details of the solution than is necessary to calculate only the farfield results.

Scattering from a slit or strip is a well-studied problem in diffraction theory. Asvestas and Kleinman [1] summarize and review much of the work done on it. Jones [2] and Noble [3] discusses diffraction from a slit or strip using the Wiener–Hopf method. We follow their approach very closely. To calculate the diffracted wavefield

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from the interaction between the edges we assume that the slit is large, with respect to wavelength, and asymptotically approximate several integrals using this assumption. KARP and KELLER [4] calculate this interaction term for diffraction from a slit in a perfectly rigid barrier using the geometrical theory of diffraction (this theory also assumes that the slit is large with respect to wavelength). Lastly, the same overall approach used here has been taken by ASGHAR [5], ASGHAR, HAYAT, and HARRIS [6] and ASGHAR and HAYAT [7].

2. Formulation and solution of the problem

A cylindrical wave is incident at an angle Φ_0 to a slit $x_1 < x < x_2$. We consider Cartesian coordinates (x, y, z) with origin O. The conducting planes $x \le x_1$ and $x \ge x_2$ are assumed to be of infinitesimal thickness. We consider a line source located at (x_0, y_0). Thus, following ASGHAR and HAYAT [8], the problem becomes one of solving the equation

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} + k_{1xz}^2\right) Q_{1y}(x, z) = \delta(x - x_0)\delta(z - z_0). \tag{1}$$

Subject to the boundary conditions:

$$\frac{\partial}{\partial x} Q_{1y}(x, z) \mp \delta \frac{\partial}{\partial z} Q_{1y}(x, z) = 0, \text{ for } z = 0^{\pm}, \quad x \le x_1, \quad x \ge x_2, \tag{2}$$

$$Q_{1y}(x, z^+) = Q_{1y}(x, z^-), \text{ for } z = 0, \quad x_1 < x < x_2,$$

$$\frac{\partial}{\partial z} Q_{1y}(x, z^+) = \frac{\partial}{\partial z} Q_{1y}(x, z^-), \text{ for } z = 0, \quad x_1 < x < x_2 \tag{3}$$

where:

$$\delta = (\gamma_1 k_{2xz}^2 + \gamma_2 k_{1xz}^2)[ik_y(k_{2xz}^2 - k_{1xz}^2)]^{-1},$$

$$k_{1xz}^2 = (\gamma_1^2 - k_y^2),$$

$$k_{2xz}^2 = (\gamma_2^2 - k_y^2).$$

we have also used the implicit dependence $\exp(ik_y y)$ of the field vectors, $\mathbf{Q}_1 = (Q_{1x}, Q_{1y}, Q_{1z})$ is the left-handed Beltrami field and two wave numbers in homogeneous biisotropic medium are:

$$\gamma_1 = \frac{k}{1 - k^2\alpha\beta}\left(\sqrt{1 + \frac{k^2(\alpha - \beta)^2}{4}} + \frac{k(\alpha + \beta)}{2}\right),$$

$$\gamma_2 = \frac{k}{1-k^2\alpha\beta}\left(\sqrt{1+\frac{k^2(\alpha-\beta)^2}{4}}-\frac{k(\alpha+\beta)}{2}\right),$$

$$k = \omega(\varepsilon\mu)^{1/2},$$

ω is frequency, ε and μ are permittivity and permeability scalars, respectively, while α and β are the biisotropy pseudoscalars. The biisotropic medium $\alpha = \beta$ is reciprocal and is called a chiral medium.

For a unique solution to the problem we require that Q_{ly} must satisfy the radiation condition in the limit $\sqrt{x^2+z^2} \to \infty$ and the edge condition $Q_{1y}(x, 0) = O(1)$ and $\frac{\partial}{\partial z}Q_{1y}(x, 0) = O(x^{-1/2})$ as $x \to x_1^+, x_2^-$.

We decompose the total field as

$$Q_{1y} = \begin{cases} Q_{1y}^{\mathrm{inc}} - Q_{1y}^{\mathrm{ref}} + \Phi & \text{for} \quad z \geq 0 \\ \Phi & \text{for} \quad z \leq 0 \end{cases}, \tag{4}$$

where Q_{1y}^{inc} is the solution of inhomogeneous wave equation corresponding to the incident wave, Q_{1y}^{ref} is the reflected wave, and Φ is the solution of the homogeneous wave equation which gives the diffracted field, *i.e.*,

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}+k_{1xz}^2\right)Q_{1y}^{\mathrm{inc,ref}} = \delta(x-x_0)\delta(z\mp z_0), \tag{5}$$

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}+k_{1xz}^2\right)\Phi = 0. \tag{6}$$

We define the Fourier transform pair with respect to x as

$$\Psi(\nu, z) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}\Phi(x, z)\exp(i\nu x)\mathrm{d}x, \tag{7}$$

$$\Phi(x, z) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}\Psi(\nu, z)\exp(i\nu x)\mathrm{d}\nu, \tag{8}$$

with identical definitions for the other fields (incident and reflected). The solutions of Eq. (5) when $r_0 \to \infty$ can be written as:

$$Q_{1y}^{\mathrm{inc}}(x, z) = b\exp[i(k_{1x}\,x + k_{1z}\,z)],$$

$$Q_{1y}^{\text{ref}}(x,z) = b\exp[i(k_{1x}x - k_{1z}z)] \tag{9}$$

where:

$$x_0^2 + z_0^2 = r_0^2,$$

$$b = -\frac{1}{2i}\frac{1}{\sqrt{2\pi k_{1xz}r_0}}\exp[i(k_{1xz}r_0 - \pi/4)],$$

$$k_{1x} = -k_{1xz}\cos\Phi,$$

$$k_{1z} = -k_{1xz}\sin\Phi,$$

$$\pi \le \Phi \le 3\pi/2.$$

With the help of Eqs. (4) and (9) the boundary conditions (2) and (3) in terms of diffracted field Φ can be written as

$$\frac{\partial\Phi}{\partial x}(x,0^+) - \delta\frac{\partial\Phi}{\partial z}(x,0^+) = 2i\delta k_{1z}\, b\exp(ik_{1x}x),$$

$$\frac{\partial\Phi}{\partial x}(x,0^-) - \delta\frac{\partial\Phi}{\partial z}(x,0^-) = 0, \tag{10}$$

for $x \le x_1$ and $x \ge x_2$, and

$$\Phi(x,0^+) = \Phi(x,0^-),$$

$$\frac{\partial\Phi}{\partial z}(x,0^+) - \frac{\partial\Phi}{\partial z}(x,0^-) = -2ibk_{1z}\exp(ik_{1x}x), \tag{11}$$

for $x_1 < x < x_2$.

The solution of boundary value problem consisting of Eq. (6) and boundary conditions (10) and (11) has been obtained by employing the procedure used in ASGHAR, HAYAT and AYUB [9]. Omitting the details the diffracted field is directly given by

$$\Phi = \Phi_1(x,z) + \Phi_2(x,z) \tag{12}$$

where:

$$\Phi_1 = \frac{i\sin\Phi\sin\theta}{4\pi\sqrt{rr_0}}\mathcal{F}_1(-k_{1xz}\cos\theta)\exp[ik_{1xz}(r+r_0)],$$

$$\Phi_2 = \frac{i\sin\Phi\sin\theta}{4\pi\sqrt{rr_0}}\mathcal{F}_2(-k_{1xz}\cos\theta)\exp[ik_{1xz}(r+r_0)],$$

$$\mathcal{F}_1(-k_{1xz}\cos\theta) = AB - CD,$$

with:

$$A = \frac{S_+(-k_{1x})}{S_+(-k_{1xz}\cos\theta)\sin\theta} + \frac{\delta\,\mathrm{sgn}(z)L_+(-k_{1x})}{\cos\theta L_+(-k_{1xz}\cos\theta)},$$

$$B = \frac{\exp[i(k_{1x} - k_{1xz}\cos\theta)x_2]}{(k_{1x} - k_{1xz}\cos\theta)},$$

$$C = \frac{S_-(-k_{1x})}{S_-(-k_{1xz}\cos\theta)\sin\theta} + \frac{\delta\,\mathrm{sgn}(z)L_-(-k_{1x})}{\cos\theta L_-(-k_{1xz}\cos\theta)},$$

$$D = \frac{\exp[i(k_{1x} - k_{1xz}\cos\theta)x_1]}{(k_{1x} - k_{1xz}\cos\theta)},$$

$$\cos\Phi + \cos\theta \neq 0,$$

$$\mathcal{F}_2(-k_{1xz}\cos\theta) = (E+F)G + (H+I)J$$

with:

$$E = [R_1(-k_{1xz}\cos\theta)\exp(ik_{1x}x_1) - C_3T(-k_{1xz}\cos\theta)]\frac{\delta\,\mathrm{sgn}(z)}{\cos\theta L_+(-k_{1xz}\cos\theta)},$$

$$F = [R_1(-k_{1xz}\cos\theta)\exp(ik_{1x}x_1) - C_1T(-k_{1xz}\cos\theta)]\frac{1}{\sin\theta S_+(-k_{1xz}\cos\theta)},$$

$$G = \exp[(-ik_{1xz}\cos\theta)x_2],$$

$$H = [R_2(k_{1xz}\cos\theta)\exp(ik_{1x}x_2) - C_4T(k_{1xz}\cos\theta)]\frac{\delta\,\mathrm{sgn}(z)}{\cos\theta L_-(-k_{1xz}\cos\theta)},$$

$$I = [R_2(k_{1xz}\cos\theta)\exp(ik_{1x}x_2) - C_2T(k_{1xz}\cos\theta)]\frac{1}{\sin\theta S_-(-k_{1xz}\cos\theta)},$$

$$J = \exp[(-ik_{1xz}\cos\theta)x_1],$$

$$S(\nu) = S_+(\nu)S_-(\nu) = 1 + \frac{\nu}{\delta\sqrt{k_{1xz}^2 - \nu^2}},$$

$$L(\nu) = L_+(\nu)L_-(\nu) = \gamma S(\nu) = (K_+S_+)(K_-S_-),$$

$$K_+ = \sqrt{\nu + k_{1xz}}, \quad K_- = \sqrt{\nu - k_{1xz}},$$

$$S_{\pm}(\nu) = S_{\pm}(0)\exp[\int\Psi_{\pm}(\nu)\mathrm{d}\nu],$$

$$S_{+}(0) = S_{-}(0) = 1,$$

$$\Psi_{+}(\nu) = -\Psi_{-}(-\nu) = -\frac{1}{2(\nu + k_{1xz})} - \frac{\delta_1 k_{1xz} F(\nu, \nu_1)}{\pi(1+\delta_1^2)(\nu_1 - \nu_2)} + \frac{\delta_1 k_{1xz} F(\nu, \nu_2)}{\pi(1+\delta_1^2)(\nu_1 - \nu_2)},$$

$$F(\nu, \nu_0) = \frac{f(\nu) - f(\nu_0)}{\nu - k_{1xz}\nu_0}, \qquad \delta_1 = 1/\delta,$$

$$f(p) = \frac{\cos^{-1}(p/k_{1xz})}{\sqrt{k_{1xz}^2 - p^2}}, \qquad \nu_1 = \frac{1}{\sqrt{1+\delta_1^2}}, \qquad \nu_2 = -\nu_1,$$

$$C_1 = \frac{1}{S_+^2(k_{1xz}) - T^2(k_{1xz})}[G_2(k_{1xz})S_+(k_{1xz}) + G_1(k_{1xz})T(k_{1xz})],$$

$$C_2 = \frac{1}{S_+^2(k_{1xz}) - T^2(k_{1xz})}[G_1(k_{1xz})S_+(k_{1xz}) + G_2(k_{1xz})T(k_{1xz})],$$

$$C_3 = \frac{1}{L_+^2(k_{1xz}) - T^2(k_{1xz})}[G_4(k_{1xz})L_+(k_{1xz}) + G_3(k_{1xz})T(k_{1xz})],$$

$$C_4 = \frac{1}{L_+^2(k_{1xz}) - T^2(k_{1xz})}[G_3(k_{1xz})L_+(k_{1xz}) + G_4(k_{1xz})T(k_{1xz})],$$

$$G_1(\nu) = P_1(\nu)\exp(ik_{1x}x_2) - R_1(\nu)\exp(ik_{1x}x_1),$$

$$G_2(\nu) = P_2(\nu)\exp(ik_{1x}x_1) - R_2(\nu)\exp(ik_{1x}x_2),$$

$$G_3(\nu) = P_3(\nu)\exp(ik_{1x}x_2) - R_1(\nu)\exp(ik_{1x}x_1),$$

$$G_4(\nu) = P_4(\nu)\exp(ik_{1x}x_1) - R_2(\nu)\exp(ik_{1x}x_2),$$

$$P_{1,2}(\nu) = \frac{S_+(\nu) - S_{\pm}(-k_{1x})}{\nu \pm k_{1x}},$$

$$P_{3,4}(\nu) = \frac{L_+(\nu) - L_{\pm}(-k_{1x})}{\nu \pm k_{1x}},$$

$$R_{1,2}(\nu) = \frac{E_0[W_0\{-i(k_{1xz} \mp k_{1x})(x_2 - x_1)\} - W_0\{-i(k_{1xz} + \nu)(x_2 - x_1)\}]}{2\pi i(\nu \pm k_{1x})},$$

$$T(\nu) = \frac{1}{2\pi i} E_0 W_0[-i(\nu + k_{1xz})(x_2 - x_1)],$$

$$E_0 = 2\exp(i\pi/2)\frac{\exp[ik_{1xz}(x_2 - x_1)]}{(x_2 - x_1)^{1/2}},$$

$$W_0(z) = \Gamma(3/2)\exp(z_1/2)(z_1)^{-1/4} W_{-3/4,\,1/4}(z_1),$$

$W_{i,j}$ is a Whittaken function, $z_1 = -i(\nu + k_{1xz}(x_2 - x_1))$, and $S_+(\nu)$, $K_+(\nu)$ and $L_+(\nu)$ are regular for $\mathrm{Im}\nu > -\mathrm{Im}k_{1xz}$ and $S_-(\nu)$, $K_-(\nu)$ and $L_-(\nu)$ are regular for $\mathrm{Im}\nu < \mathrm{Im}k_{1xz}$.

3. Concluding remarks

In this work, the scattering of cylindrical wave by a slit in an infinite perfectly conducting screen in homogeneous biisotropic medium has been analyzed through the Wiener–Hopf technique. The final approximate solution is uniformly valid for all angle of incidence and observation except for the grazing incidence. By using Q_{1y} we can write the other components of $\mathbf{Q}_1$ through the following equation:

$$\nabla \times \mathbf{Q}_1 = -\gamma_1 \mathbf{Q}_1. \tag{13}$$

Similarly the analysis for the right-handed Beltrami field $\mathbf{Q}_2$ can be discussed. Furthermore, for a chiral medium ($\alpha = \beta$) we have:

$$\gamma_1 = k(1 - k\beta)^{-1},$$

$$\gamma_2 = k(1 + k\beta)^{-1}. \tag{14}$$

The diffracted field for the chiral media can easily be found using Eq. (14) in Eq. (12). In nature these media occur as the stero-isomers of organic chemistry which betray circular birefringence or optical activity at optical frequencies. With advances in crafting macromoleculear polymers [10], it is conceivable [11], [12] that media displaying their chirality at microwave frequencies may become commonly available.

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