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Exact algebraic method for design of the model nonastigmatic spherical ophthalmic glasses

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An exact algebraic method for designing the model nonastigmatic spherical ophthalmic glasses is given. The method allows us to determine construction parameters of glasses with an assumed back vertex power, which completely fulfil all conditions of correct performance, and takes into account manufacture recommendations. The method consists in solving the system of nonlinear equations by means of software. Calculation of the parameters of nonastigmatic spherical ophthalmic glasses of 65 mm in diameter for positive and negative back vertex powers were designed of organic material CR39 in the most interesting range of vertex powers from 0 to ±7 with 0.25 D step are presented.

1. Introduction

This work describes a new approach to the design of model nonastigmatic spherical glasses, so called "punktals". Nonastigmatic ophthalmic glasses are used for correction of the eye refraction errors and they should feature corrected astigmatism in the characteristic field of view with low residual astigmatism in intermediate zones of the field of view. The model version of glasses refers to theoretical solutions and does not include any technological simplifications introduced to reduce the optical tooling.

Design of nonastigmatic lenses has although not rich but a rather long history [1]. In 1801, Young arrived at formulae necessary to calculate the astigmatism of an extremely narrow light beam. In the years 1889–1900, Ostwald used the 3rd order aberration method to design nonastigmatic glasses for infinity and obtained for each glass two solutions differing in the convexity. In 1904, Tscherning presented his 3rd order solutions of nonastigmatic glasses in the form of so-called "Tscherning ellipse". It is also worthwhile to mention design works performed between 1903 and 1914 by Gullstrand and Rohr, and works of Ostwald from 1935 to design the nonastigmatic glasses for near vision. Also Wollaston, Schleiermacher, Martin, Percival, Southall and many others contributed to the development of ophthalmic glasses.

Later for precise calculations of astigmatism trigonometric methods were commonly used, and then came the computer-based methods. Today the 3rd order aberration methods are no longer in use. In the case of low-power nonastigmatic glasses the angles of incidence on individual surfaces are moderate and smaller than 10–15°, which explains the past relative usefulness of these methods, especially in situations where tolerances of convexity were rather loose. However the angles of incidence are significant enough to cause deformation compared to the 3rd order aberration calculations.

The new approach presented in this work consists in using exact algebraic equations for computation of nonastigmatic glasses. Besides the correction of astigmatism this method assures that the ophthalmic glasses fulfil exactly all other requirements necessary for proper performance. Hitherto, researchers considered it impossible to give exact algebraic formulae binding the aberrations and astigmatism in particular and its construction and physical parameters. This was due to their extreme complexity. Today we should verify this approach taking into account works of HERZBERGER [2], WALTHER [3], [4], CASTRO-RAMOS *et al.* [5] and the author KRYSZCZYŃSKI's paper [6] was devoted to algebraic computations in correction of aberrations of simple optical systems (minimum of spherical aberration of single lens, a system of two spherical mirrors with zero spherical aberration at the edge of aperture).

2. Meridional pupil ray

The astigmatism of ophthalmic glass is calculated along the meridional pupil ray that determines selected angle of view. During the observation the eye follows the object and rotates. Traditionally it is assumed that the eye's pupil is in its rotation center. In this work, it is assumed that the variables determining the ray tracing in the case of the pupil ray are the consecutive angles of incidence j_1 and j_2 of the ray at the glass surfaces. The Figure shows among others also the parameters describing the pupil ray.

Respective refraction angles j_1' and j_2' are calculated from the law of refraction, denoting by n_1 the refractive index of glass. This way we obtain the following formulae:

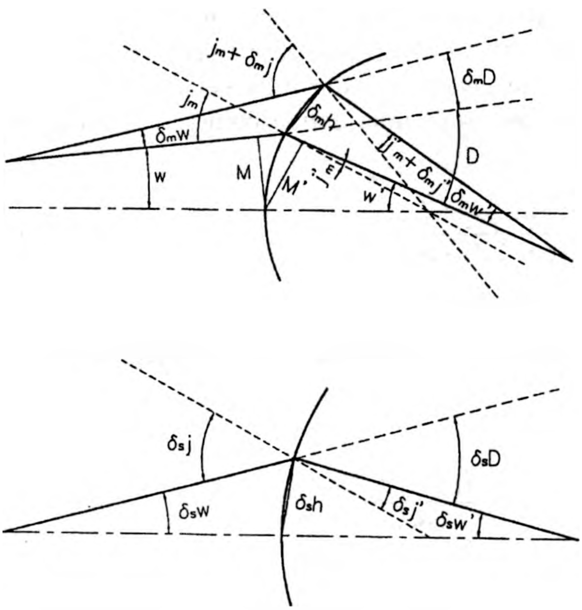
$$j_1' = \arcsin\frac{\sin j_1}{n_1}, \qquad j_2' = \arcsin(n_1 \sin j_2). \tag{1}$$

In the meridional plane the angular deviation D of the ray can be calculated equally as the difference of ray angles with the axis or the difference between the angles of incidence and refraction. In this work, the second possibility is employed. Consecutive angular deviations of the ray D_1 and D_2 at the glass surfaces can be calculated from the following formulae:

$$D_1 = j_1 - j_1', \qquad D_2 = j_2 - j_2'. \tag{2}$$

Assume that the pupil ray in the object plane forms constant angle w_0 with the optical axis. Consecutive angles of refraction w_1 and w_2 can then be calculated from the following formulae:

$$w_1 = w_0 + D_1, \qquad w_2 = w_0 + D_1 + D_2. \tag{3}$$



Parameters of the pupil ray and differential astigmatic meridional and sagittal rays.

From Equations (1)–(3) it is evident that the angular variables j_1 and j_2 describe the angular pupil ray tracing through the ophthalmic glass. To position the ray with respect to the vertex of surfaces we need the coefficients μ_1 and μ_2 described in the book of SMITH [7]. The coefficients are given as follows:

$$\mu_1 = \frac{\cos w_1 + \cos j_1'}{\cos w_0 + \cos j_1}, \qquad \mu_2 = \frac{\cos w_2 + \cos j_2'}{\cos w_1 + \cos j_2}. \tag{4}$$

The shortest distance of the ray to the vertex of the first surface we will denote by M_1 and the center thickness of the glass by d_1. Then the shortest distance of the ray to the vertex of the second surface M_2 and M_2' (before and after refraction) is as follows:

$$M_2 = \mu_1 M_1 - d_1 \sin w_1, \qquad M_2' = \mu_2 M_2 = \mu_1 \mu_2 M_1 - \mu_2 d_1 \sin w_1. \tag{5}$$

Formulae (1)–(3) show that angular variables j_1 and j_2 also describe the location of the pupil ray together with the distance M_1 and thickness d_1. Assuming the angular variables j_1 and j_2 it is possible to determine the pupil ray tracing not knowing the surface curvatures. Curvatures c_1 and c_2 of consecutive glass surfaces depend on the above-mentioned angular and linear variables in accordance with the following formulae:

$$c_1 = \frac{\sin w_0 + \sin j_1}{M_1}, \qquad c_2 = \frac{\sin w_1 + \sin j_2}{M_2}. \tag{6}$$

When calculating astigmatism of ophthalmic lens we also need the oblique thickness along the pupil ray measured between the points of intersection of the ray with surfaces, including the object plane during observation of the near vision ($L \neq 0$). For that purpose we will need sags g_1 and g_2 of intersection points, which can be calculated from the following formulae:

$$g_1 = \frac{\sin(w_0 + j_1)}{\cos w_0 + \cos j_1} M_1, \qquad g_2 = \frac{\sin(w_1 + j_2)}{\cos w_1 + \cos j_2} M_2. \tag{7}$$

The oblique distance d_0^* between the object plane and the first surface, and the oblique thickness d_1^* between the first and the second surfaces can be determined from formulae:

$$d_0^* = \begin{cases} \dfrac{d_0 + g_1}{\cos w_0} & \text{for} \quad L \neq 0 \\ 0 & \text{for} \quad L = 0 \end{cases}, \qquad d_1^* = \frac{d_1 - g_1 + g_2}{\cos w_1}. \tag{8}$$

In Eqs. (8) we assume the conventional zero oblique distance d_0^* in the case of an object being located at infinity ($L = 0$).

3. Astigmatic rays

The formulae given by Young concern the extremely thin pencil of rays in two perpendicular planes: meridional and sagittal. From his formulae it is evident that the pencil of rays performs differently in both planes producing astigmatism as the result. Astigmatism control is a difficult task for the designers of optical systems. In the case of ophthalmic glasses the situation is easier because in principle it is the only aberration that requires correction. Other aberrations, such as distortion or transversal chromatism in the medium power range are rather small and can remain uncorrected.

Less known is the angular version of Young's formulae. In this version, the auxiliary angles $\delta_m w$ and $\delta_s w$ and differential heights $\delta_m h$ and $\delta_s h$ are introduced in two perpendicular planes. Thus the formulae take the following form:

$$n' \cos j' \delta_m w' - n \cos j \delta_m w = \delta_m h (n' \cos j' - n \cos j) c,$$

$$\delta_m h_{+1} \cos j_{+1} = \delta_m h \cos j' - d^* \delta_m w,$$

$$n' \delta_s w' - n \delta_s w = \delta_s h (n' \cos j' - n \cos j) c,$$

$$\delta_s h_{+1} = \delta_s h - d^* \delta_s w. \tag{9}$$

In Equation (9) n and n' denote the refractive indices, and subscript $+1$ denotes the next surface of the system. This version simplifies the notation reducing by one exponent of the cosine present in the original Young's formulae. Further we will take advantage of formulae (9) because they are very convenient to use.

3.1. Differential meridional ray

We assume conventionally the entrance differential angle $d_m w_0$ between the meridional and pupil rays as equal to

$$\delta_m w_0 = \begin{cases} -0.01 & \text{for} \quad L \neq 0 \\ 0 & \text{for} \quad L = 0 \end{cases}. \tag{10}$$

For further consideration it would be favourable to increase the number of angular parameters describing the meridional ray. We will introduce as variables the differential angles $\delta_m j_1$ and $\delta_m j_2$, respectively, for the consecutive surfaces of the glass. After differentiation of the refraction law we obtain the following relation between the angles of incidence and refraction

$$\delta_m j' = \frac{n \cos j}{n' \cos j'} \delta_m j. \tag{11}$$

By analogy with formula (2) we will introduce the meridional deviations $\delta_m D$ of the differential ray as the differences between the differential angles of incidence and refraction. For consecutive surfaces we obtain the following meridional differential deviations:

$$\delta_m D_1 = \delta_m j_1 - \delta_m j_1',$$

$$\delta_m D_2 = \delta_m j_2 - \delta_m j_2'. \tag{12}$$

Replacing formula (11) for each surface into formulae (12) we obtain the relation between the deviation and variable differential angles:

$$\delta_m D_1 = \left(1 - \frac{\cos j_1}{n_1 \cos j_1'}\right) \delta_m j_1,$$

$$\delta_m D_2 = \left(1 - \frac{n_1 \cos j_2}{\cos j_2'}\right) \delta_m j_2. \tag{13}$$

The consecutive differential angle of refraction $\delta_m w_1$ and $\delta_m w_2$ with the pupil ray can be calculated from the following formulae:

$$\delta_m w_1 = \delta_m w_0 + \delta_m D_1,$$

$$\delta_m w_2 = \delta_m w_0 + \delta_m D_1 + \delta_m D_2. \tag{14}$$

We will conventionally assume the entrance differential height $\delta_m h_0$ of the meridional ray at the point of intersection with the object plane as equal to

$$\delta_m h_0 = \begin{cases} 0 & \text{for} \quad L \neq 0 \\ 1 & \text{for} \quad L = 0 \end{cases}. \tag{15}$$

The differential heights $\delta_m h_1$ and $\delta_m h_2$ at consecutive glass surfaces obtained from formulae (9) are as follows:

$$\delta_m h_1 = \frac{\delta_m h_0 - d_0^* \delta_m w_0}{\cos j_1},$$

$$\delta_m h_2 = \frac{\cos j_1' \delta_m h_1 - d_1^* \delta_m w_1}{\cos j_2}. \tag{16}$$

The location of the meridional image t_2' along the pupil ray does not depend on the entrance angle (Eq. (10)) and height (Eq. (15)) but mainly on the variable angles of incidence j_1 and j_2, and differential angles $\delta_m j_1$ and $\delta_m j_2$. This location can be calculated from the following formula:

$$t_2' = \cos j_2' \frac{\delta_m h_2}{\delta_m w_2}. \tag{17}$$

Curvatures c_1 and c_2 of glass surfaces calculated from Young's formulae (9) depend on the above-mentioned angular pupil and differential meridional variables in accordance with the following formulae:

$$c_1 = \frac{n_1 \cos j_1' \delta_m w_1 - \cos j_1 \delta_m w_0}{(n_1 \cos j_1' - \cos j_1) \delta_m h_1},$$

$$c_2 = \frac{\cos j_2' \delta_m w_2 - n_1 \cos j_2 \delta_m w_1}{(\cos j_2' - n_1 \cos j_2) \delta_m h_2}. \tag{18}$$

Curvatures calculated from the differential meridional ray (Eqs. (18)) must conform to the respective curvatures calculated from the pupil ray.

3.2. Differential sagittal ray

By analogy to formula (10) we assume conventionally the entrance differential angle $\delta_s w_0$ of the sagittal ray with the pupil ray to be equal to

$$\delta_s w_0 = \begin{cases} -0.01 & \text{for} \quad L \neq 0 \\ 0 & \text{for} \quad L = 0 \end{cases}. \tag{19}$$

We introduce as variables the differential angles $\delta_s j_1$ and $\delta_s j_2$ for consecutive glass surfaces. The refraction of sagittal ray is similar to that of paraxial one. The relation between the angles of incidence and refraction is thus given by the following formula:

$$\delta_s j' = \frac{n}{n'}\delta_s j. \tag{20}$$

By analogy to Eq. (12) we introduce the sagittal deviations $\delta_s D$ of the differential sagittal ray as the differences between its differential angles of incidence and refraction. For consecutive surfaces we obtain the following sagittal differential deviations:

$$\delta_s D_1 = \delta_s j_1 - \delta_s j_1',$$

$$\delta_s D_2 = \delta_s j_2 - \delta_s j_2'. \tag{21}$$

Replacing formula (20) taken for each surface into formulae (21) we obtain the relation between these deviations and variable differential angles:

$$\delta_s D_1 = \left(1 - \frac{1}{n_1}\right) n_1 \delta_s j_1,$$

$$\delta_s D_2 = (1 - n_1)\delta_s j_2. \tag{22}$$

Consecutive differential angles of refraction $\delta_s w_1$ and $\delta_s w_2$ related to the pupil ray can be calculated from the following formulae:

$$\delta_s w_1 = \delta_s w_0 + \delta_s D_1,$$

$$\delta_s w_2 = \delta_s w_0 + \delta_s D_2. \tag{23}$$

The entrance differential height $\delta_s h_0$ of the sagittal ray at the point of intersection of the pupil ray with the object plane we conventionally assume as equal to

$$\delta_s h_0 = \begin{cases} 0 & \text{for} \quad L \neq 0 \\ 1 & \text{for} \quad L = 0 \end{cases}. \tag{24}$$

Differential heights $\delta_s h_1$ and $\delta_s h_2$ on the consecutive glass surfaces obtained from Eq. (9) are the following:

$$\delta_s h_1 = \delta_s h_0 - d_0^* \delta_s w_0,$$

$$\delta_s h_2 = \delta_s h_1 - d_1^* \delta_s w_1. \tag{25}$$

The location of sagittal image s_2' along the pupil ray does not depend on the entrance angle (Eq. (19)) and height (Eq. (24)) but mainly on the variable incident angles j_1 and j_2 and differential angles $\delta_s j_1$ and $\delta_s j_2$. This location can be calculated from the following formula:

$$s_2' = \frac{\delta_s h_2}{\delta_s w_2}. \tag{26}$$

The curvatures c_1 and c_2 of consecutive glass surfaces depend on the above -mentioned pupil and sagittal angular variables in accordance with the following formulae:

$$c_1 = \frac{N\delta_s w_1 - \delta_s w_0}{(N\cos j_1' - \cos j_1)\delta_s h_1}, \quad c_2 = \frac{\delta_s w_2 - N\delta_s w_1}{(\cos j_2' - N\cos j_1)\delta_s h_2}. \tag{27}$$

Curvatures calculated from the differential sagittal ray (Eqs. (27)) must conform to the respective curvatures calculated from pupil (Eq. (6)) and differential meridional (Eq. (18)) rays.

4. Conditions of correct performance

Performance of the nonastigmatic ophthalmic glass is characterized by the back vertex power. This power denoted as BVP is a function of construction parameters such as: surface curvatures c_1 and c_2, thickness of glass d_1 and the refractive index n_1 of glass in accordance with following formula:

$$\mathrm{BVP} = P_2 + \frac{P_1}{1 - 0.001 P_1 \frac{d_1}{n_1}}. \tag{28}$$

Surface powers P_1 and P_2 expressed in diopters, found in formula (28), can be calculated from the following formulae:

$$P_1 = 1000(n_1 - 1)c_1, \quad P_2 = 1000(1 - n_1)c_2. \tag{29}$$

Perfectly designed positive ophthalmic glass should feature minimum edge thickness d_e at the outer diameter Φ, that depends on geometric construction parameters according to the formula

$$d_e = d_1 - x_1 + x_2. \tag{30}$$

Sags denoted by x_1 and x_2 in Eq. (30) at the height $h = \Phi/2$ are determined from the formulae:

$$x_1 = \frac{h^2 c_1}{1 + \sqrt{1 - h^2 c_1^2}}, \quad x_2 = \frac{h^2 c_2}{1 + \sqrt{1 - h^2 c_2^2}}. \tag{31}$$

The enterence positive glass edge thickness concerns the initial situation before we start to process the glass to obtain different outer shapes, *e.g.*, oval, pilot or square. Negative ophthalmic glasses have fixed minimum center thickness along the optical axis.

The condition for correct performance of ophthalmic glass is the correct location of the exit pupil p_2'. This location is calculated from the pupil ray with the use of formulae (3) and (5) as follows:

$$p_2' = \frac{M'}{\sin w_2'}. \tag{32}$$

The most important parameter characterizing the performance of ophthalmic glasses is astigmatism (Ast) for the characteristic angle of view w_{ch}. Astigmatism expressed in diopters (D) is calculated based on the location of images determined in formulae (17) and (26)

$$\mathrm{Ast}(w_{ch}) = \left(\frac{1}{t_2'} - \frac{1}{s_2'}\right)1000. \tag{33}$$

The condition for correct performance of the model ophthalmic glasses is zero astigmatism $\mathrm{Ast}(w_{ch}) = 0$ D in the characteristic angle of view. According to formulae given earlier all conditions for correct performance of glasses can be presented in the form of functions of linear and angular variables.

5. Algebraic method for the design of ophthalmic glasses

All dependences given in this work were defined as mutually nested functions of angular and linear variables. Owing to that we can describe very complex dependences in a simple and clear manner and solve them with the use of advanced professional software. In this work, the Mathcad software was used. The exact algebraic method of design of nonastigmatic spherical glasses consists in solving the system of nonlinear equations.

In the case of positive glasses it is necessary to solve the system of 8 nonlinear equations with 8 unknowns. The unknowns include:

– the angles of incidence at the glass surface of: the pupil rays j_1 and j_2, the differential meridional rays j_{m1} and j_{m2}, and the sagittal rays j_{s1} and j_{s2};

– two linear parameters: center thickness d_1 of the glass along the optical axis, and the shortest distance M_1 of the incident ray from the vertex of the first surface.

Nonlinear equations concern: required back vertex power, location of the exit pupil, the minimum edge thickness of glass, correction of dioptric astigmatism to zero, conformity of curvatures of the first surface calculated for the meridional and pupil rays, conformity of curvatures of the second surface calculated for the meridional and pupil rays, conformity of curvatures of the first surface calculated for the sagittal and pupil rays, and conformity of curvatures of the second surface calculated for the sagittal and pupil rays.

To start the calculation it is necessary to fix the values of global constants and initial values of variables. Global constants are: refractive index n_1 of glass, object vergence in diopters L and the outer glass diameter Φ. Initial values of variables are determined with the use of the trial-and-error method. Once set the values are useful for a large group of glasses of various powers because the solution only slightly depends on initial values.

Equations of conformity of the curvatures of surfaces calculated with the use of different rays should be multiplied by weight coefficients to reduce the errors to minimum. Such an operation guarantees that the parameters of all three rays concern the same and common optical system.

After determination of unknowns the calculations of curvatures or radii of curvatures can be made with the use of an arbitrary ray. For verification purposes usually they are calculated by means of three methods (rays) in accordance with formulae (6), (18) and (27).

In the case of negative glasses the algebraic method of design becomes slightly simpler. The number of nonlinear equations and unknowns is reduced to 7. The thickness of glass is not a variable any more and remains in the group of global variables.

6. Model nonastigmatic spherical glasses

The present method of design of nonastigmatic spherical ophthalmic glasses was used for exemplary calculations of the construction parameters of model ophthalmic glasses of a given range of back vertex power, which completely fulfil all conditions of correct performance. Nonastigmatic positive and negative ophthalmic glasses were designed. The following assumptions were made: range of back vertex power from 0 to ±7 in steps of 0.25 D, outer diameter $\Phi = 65$ mm, material: Columbian resin CR39 with $n_e = 1.500$, location of the exit pupil $p_2' = 25$ mm, minimum edge thickness for the positive glasses $d_e = 0.8$ mm, characteristic one-side angle of view $w_{ch} = 15°$, dioptric astigmatism equal to zero for characteristic angle of view, and calculation for three object vergences $L = 0, -2, -4$ D (distance from the object ∞ 500 and 250 mm, respectively).

Assumed angle $w_{ch} = 15°$ reflects approximately the situation where the text line on the portrait A4 page is read from the distance of 250 mm ($L = -4$ D) or the text line on the landscape A4 page is read from the distance of 500 mm ($L = -2$ D). Calculated construction parameters (radii of surfaces R_1, R_2 and thickness d_1) of nonastigmatic positive glasses can be found in Tab. 1.

Table 1 presents the solutions with the longest radii (Ostwald type). It is a bit difficult to obtain this kind of glasses with zero astigmatism in the end of BVP range. The solution of Wollaston type of glasses can be avoided when we assume certain value of residual astigmatism lower than the eye's tolerance. As we see from Tab. 1, thicknesses determined in BVP range to 1 D are too small from technological

Table 1. Model positive spherical ophthalmic glasses, diameter Φ = 65 mm, material CR39.

BVP	$L = 0$ [D]			$L = -2$ [D]			$L = -4$ [D]		
[D]	R_1	R_2	d_1	R_1	R_2	d_1	R_1	R_2	d_1
0.25	79.983	82.929	1.07	102.989	108.186	1.07	140.279	150.450	1.06
0.50	72.037	77.102	1.36	89.978	98.333	1.35	117.736	132.875	1.34
0.75	68.325	75.438	1.66	84.158	95.603	1.63	108.083	128.231	1.62
1.00	65.804	74.908	1.96	80.309	94.768	1.92	101.855	126.918	1.89
1.25	63.801	74.838	2.27	77.312	94.705	2.21	97.096	126.961	2.17
1.50	62.077	74.989	2.58	74.777	95.022	2.50	93.138	127.687	2.45
1.75	60.526	75.253	2.89	72.532	95.545	2.79	89.687	128.798	2.73
2.00	59.095	75.572	3.19	70.976	97.130	3.08	86.590	130.141	3.01
2.25	57.750	75.911	3.50	68.593	96.882	3.37	83.758	131.625	3.29
2.50	56.473	76.246	3.82	66.815	97.603	3.67	81.137	133.190	3.57
2.75	55.249	76.557	4.13	65.133	98.321	3.96	78.689	134.791	3.85
3.00	54.070	76.830	4.45	63.532	99.014	4.26	76.386	136.395	4.13
3.25	52.926	77.050	4.76	61.999	99.662	4.55	74.209	135.971	4.41
3.50	51.814	77.203	5.09	60.527	100.248	4.85	72.142	139.491	4.70
3.75	50.726	77.275	5.41	59.106	100.756	5.15	70.173	140.928	4.98
4.00	49.758	75.494	5.53	57.732	101.167	5.45	68.291	142.256	5.27
4.25	48.606	77.108	6.08	56.399	101.465	5.76	66.488	143.447	5.55
4.50	47.563	76.830	6.42	55.102	101.628	6.07	64.755	144.471	5.84
4.75	46.524	76.389	6.78	53.836	101.637	6.38	63.086	145.300	6.13
5.00	45.481	75.752	7.14	52.597	101.468	6.70	61.476	145.900	6.43
5.25	44.423	74.871	7.52	51.379	101.095	7.02	59.917	146.239	6.72
5.50	43.333	73.675	7.92	50.178	100.486	7.35	58.406	146.279	7.02
5.75	42.182	72.044	8.34	48.986	99.604	7.68	56.937	145.982	7.32
6.00	40.902	69.716	8.83	47.797	98.402	8.03	55.504	145.307	7.63
6.25	39.244	65.759	9.46	46.600	96.817	8.39	54.102	144.208	7.94
6.50	37.637	61.694	10.20	45.378	94.758	8.76	52.725	142.637	8.26
6.75	37.612	63.242	10.49	44.104	92.077	9.16	51.366	140.535	8.58
7.00	37.584	64.852	10.78	42.725	88.483	9.61	50.018	137.833	8.91

point of view but they do follow earlier assumption (d_e = 0.8 mm). Results of calculation confirm earlier observation that the radii of glass curvatures elongate when the object is getting closer to the eye. However, the assumption of common solution for $L = -4$ D independent of object location leads to the impairment of visual comfort for $L = 0$ D. It is a good idea to assume common solution for $L = -2$ D because the present astigmatism in the characteristic angle for $L = 0$ D and $L = -4$ D is then lower than the astigmatism tolerances of the eye equal 0.12–0.15 D.

Table 2 presents the calculated construction parameters of nonastigmatic negative ophthalmic glasses, which also fulfil all assumptions. Zero glasses (BVP = 0 D) were also added to this group. As it is evident from Tab. 2 thicknesses of negative glasses

Table 2. Model negative spherical ophthalmic glasses, diameter Φ = 65 mm, material CR39.

BVP	$L = 0$ [D]			$L = -2$ [D]			$L = -4$ [D]		
[D]	R_1	R_2	d_1	R_1	R_2	d_1	R_1	R_2	d_1
0.00	76.208	75.608	1.8	106.252	105.652	1.8	153.878	153.278	1.8
–0.25	63.753	59.402	1.8	83.130	79.260	1.8	96.422	91.441	1.8
–0.50	70.662	65.475	1.8	77.455	71.370	1.8	89.023	81.239	1.8
–0.75	58.156	52.981	1.8	65.840	59.424	1.8	78.248	69.548	1.8
–1.00	49.321	44.450	1.6	57.662	51.271	1.6	70.100	61.070	1.6
–1.25	55.078	47.999	1.6	65.342	55.773	1.6	80.948	66.955	1.6
–1.50	58.984	49.730	1.6	70.800	58.033	1.6	89.112	69.982	1.6
–1.75	62.120	50.665	1.6	75.319	59.271	1.6	96.1556	71.645	1.6
–2.00	65.810	51.804	1.4	80.772	60.781	1.4	104.932	73.678	1.4
–2.25	68.222	51.923	1.4	84.437	60.942	1.4	111.119	73.870	1.4
–2.50	70.547	51.896	1.4	88.043	60.906	1.4	117.387	73.785	1.4
–2.75	72.834	51.764	1.4	91.656	60.730	1.4	123.863	73.508	1.4
–3.00	75.702	51.868	1.2	96.268	60.860	1.2	132.327	73.638	1.2
–3.25	77.954	51.562	1.2	99.979	60.451	1.2	139.456	73.039	1.2
–3.50	80.247	51.219	1.2	103.830	59.994	1.2	147.096	72.376	1.2
–3.75	82.593	50.848	1.2	107.847	59.499	1.2	155.344	71.664	1.2
–4.00	85.440	50.633	1.0	112.820	59.207	1.0	165.859	71.218	1.0
–4.25	87.900	50.201	1.0	117.226	58.634	1.0	175.663	70.405	1.0
–4.50	90.445	49.758	1.0	121.882	58.048	1.0	186.475	69.579	1.0
–4.75	93.083	49.305	1.0	126.826	57.453	1.0	198.463	68.742	1.0
–5.00	95.822	48.846	1.0	132.065	56.847	1.0	211.849	67.899	1.0
–5.25	98.971	48.381	1.0	137.688	56.242	1.0	226.907	67.053	1.0
–5.50	101.638	47.913	1.0	143.685	55.630	1.0	243.983	66.206	1.0
–5.75	104.733	47.441	1.0	150.119	55.017	1.0	263.523	65.361	1.0
–6.00	107.965	46.968	1.0	157.043	54.403	1.0	286.114	64.519	1.0
–6.25	111.344	46.494	1.0	164.515	53.790	1.0	312.540	63.682	1.0
–6.50	114.882	46.020	1.0	172.610	53.178	1.0	343.880	62.850	1.0
–6.75	118.591	45.545	1.0	181.411	52.569	1.0	381.652	62.025	1.0
–7.00	122.483	45.072	1.0	191.017	51.964	1.0	428.073	61.207	1.0

were adopted with the use of step method starting from 1.8 mm, which simulates existing constructions. Solution of negative glasses of Ostwald type for the entire range of BVP does not bring too much trouble. Table 2 confirms the tendency of radii to elongate when the object is getting closer to the eye.

7. Conclusions

It is evident from this work that designing the nonastigmatic spherical ophthalmic lenses with the use of exact algebraic method instead of the simplified 3rd order methods is possible. The method was tested for correctness in exemplary calculations of low-diopter nonastigmatic spherical glass of both positive and negative BVP.

The results concern the theoretical solutions of model glasses because they do not include any technological simplifications introduced to reduce the tooling.

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References

[1] Melanowski W.H., *Ophthalmic Optics*, (in Polish), [Ed.] PZWL, Warszawa 1950.
[2] Herzberger M., J. Opt. Soc. Am. **53** (1963), 661.
[3] Walther A., J. Opt. Soc. Am. A **13** (1996), 523.
[4] *Ibidem*, p. 1763.
[5] Castro-Ramos J., Cordero-Davila A., Vazquez-Montiel S., Gale D., Appl. Opt. **37** (1998), 5193.
[6] Kryszczyński T., Proc. SPIE **3820** (1998), 336.
[7] Smith W.J., *Modern Optical Engineering, The Design of Optical Systems*, McGraw-Hill, New York 1966.

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