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Quality criterion for numerical methods in EDFA modelling

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Based on the quality criterion introduced in the paper it is possible to evaluate numerical solutions obtained for different methods used in the analysis of erbium-doped fibre amplifiers (EDFA) and compare them. The 4-th order Runge–Kutta method and average power analysis technique are set against the modified midpoint (MM) method that is a new one in the context of solving EDFA propagation equations. The MM method has been proved a very effective tool in EDFA modelling.

1. Introduction

Optical amplification plays an important role in the C (1530–1560 nm) and L (1570–1610 nm) band optical communication, especially in WDM/DWDM systems. The key component is an erbium-doped fibre amplifier. It is important that one should be able to predict the performance of an amplifier while working conditions vary and choose optimal ones fulfilling the requirements for specific application [1].

To analyse EDFA the model described in detail in [2] was chosen. It is based on a two-level laser system with homogeneous gain effects. It is used to model spectral properties of EDFA. The modelling of the gain and amplified spontaneous emission (ASE) spectrum for a given length of an erbium-doped fibre can be computed under certain assumptions without direct determination of absorption and emission cross sections of a fibre. In this model, a fibre is completely characterized knowing only a few parameters that can be directly obtained from measurements [3]. These are: the small signal gain g_k^*, the small signal attenuation λ_k, and the fibre saturation parameter ζ. Index k is used to keep track of the wavelength λ_k. In this approach [2], two basic equations are used, the transition equation

$$\frac{\bar{n}_2(z)}{\bar{n}_t} = \frac{\sum_k \frac{P_k(z)\alpha_k}{h\nu_k\zeta}}{1 + \sum_k \frac{P_k(z)(\alpha_k + g_k^*)}{h\nu_k\zeta}} \tag{1}$$

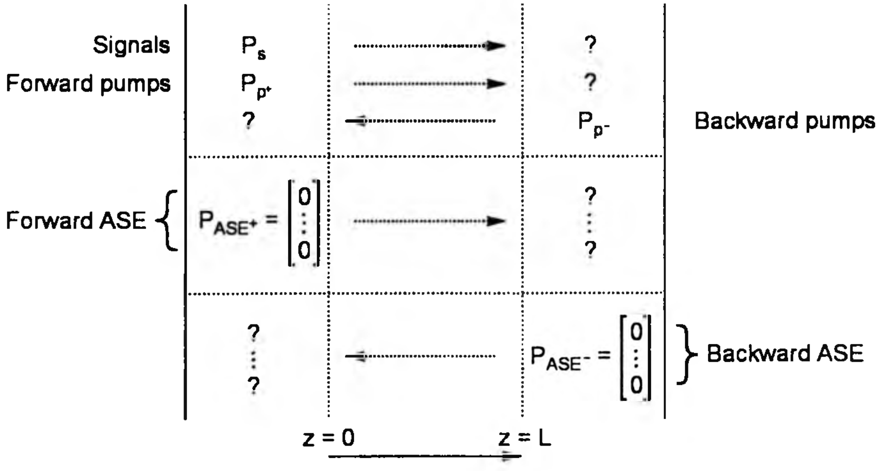

Fig. 1. Boundary conditions in EDFA. Dashed line arrows indicate propagation directions of light beams.

where $\bar{n}_2(z)/\bar{n}_t$ is the ratio between population in the upper state and the density of erbium ions, and the propagation equation

$$\frac{\mathrm{d}P_k}{\mathrm{d}z} = u_k(\alpha_k + g_k^*)\frac{\bar{n}_2(z)}{\bar{n}_t}P_k(z) + u_k g_k^* \frac{\bar{n}_2(z)}{\bar{n}_t} 2h\nu_k\Delta\nu_k - u_k(\alpha_k + \overset{0}{\alpha}_k)P_k. \tag{2}$$

P_k is the power of the k-th light field including full spectra of forward and backward ASE powers, pumps, signals, u_k is equal to +1 for a forward-propagating field and −1 for a backward-propagating field. The elementary bandwidth for ASE noise is $\Delta\nu_k$. The $\overset{0}{\alpha}_k$ accounts for fibre background loss.

In general, the propagation Eq. (2) can be put into the compact vector form

$$\frac{\mathrm{d}\mathbf{P}}{\mathrm{d}z} = \mathbf{f}(z, \mathbf{P}) \tag{3}$$

with boundary conditions at the input and the output of a fibre shown in Fig. 1. This problem is solved by transformation to initial conditions using a relaxation method.

2. Comparison of numerical methods

To solve the propagation equation of form (3) numerical algorithms should be used. The most popular one is the fourth-order Runge–Kutta method [4]. The average power analysis technique is also applied [5]. Here, we present a modified midpoint method that is new in the context of solving propagation equations [6]. In contrast to Runge–Kutta methods it is a multistep one. It is a second-order method, but its order can be easily increased [7]. Those three methods were coded in a special computer

T a b l e 1. Fibre parameters used in modelling.

Parameter	Value
Core radius *a*	1.5 μm
Doped core radius *b*	1.2 μm
Aperture NA	0.25
Metastable lifetime τ	10 ms
Erbium ion density $\bar{n}_t$	6×10^{24} m^{-3}
1530 nm absorption	5.9 dB/m
980 nm absorption	4.2 dB/m
Background loss	3 dB/km
Length	12 m
Overlap at 980 nm	0.67
Overlap at 1530 nm	0.41

programme that was written in Matlab language [8]. The computation involves a dual boundary value problem introduced by the backward ASE and/or backward pumping scheme for the system of differential equations. In all of the iterative forward and backward integrations, the co-propagating and counter-propagating ASE noise is presented in wavelength slots with equal spacing of 0.2 nm in the wavelength range from 1450 nm to 1650 nm. It yields a total of 2002 coupled differential equations if one signal and one pump are considered. It is obvious that this number is increased in the case of DWDM systems and/or more than one pump source.

For simulations a typical erbium-doped fibre was chosen. Its parameters are listed in Tab. 1. Additionally, its absorption and emission coefficients are shown in Fig. 2.

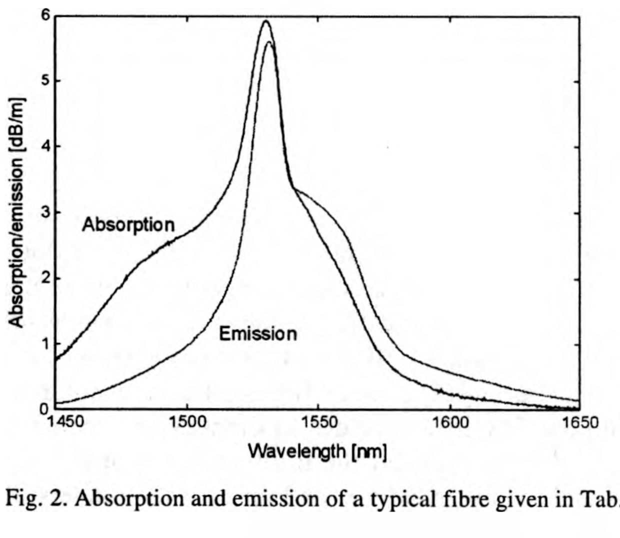

Fig. 2. Absorption and emission of a typical fibre given in Tab. 1.

To compare solutions of numerical methods we propose the following procedure. Discrete values obtained from solving nonlinear set of equations $\mathbf{P}(z_\sigma)$ at a given fibre length division σ are interpolated using spline functions $\tilde{\mathbf{P}}_\sigma(z)$. Then the value

$$\varepsilon = \left\| \frac{\mathrm{d}\tilde{\mathbf{P}}_\sigma(z)}{\mathrm{d}z} - \mathbf{f}(z, \tilde{\mathbf{P}}_\sigma(z)) \right\|, \tag{4}$$

is calculated and it can be recognized as the measure of numerical solution quality.

Apart from the quality of numerical solution of nonlinear set of equations the norm should take into account the desirable convergence of backward ASE to zero at the end of a fibre

$$\tilde{\mathbf{P}}_{\mathrm{ASE}^-}(z = L) = 0. \tag{5}$$

The successive iterations in the relaxation method should lead to a more exact solution. Therefore the next condition should be taken into account

$$\lim_{m \to \infty} |G_m - G_{m-1}| = 0. \tag{6}$$

where G_m and G_{m-1} is the amplifier gain in the m-th iteration and $(m-1)$-th iteration, respectively.

In calculations the least square norm was assumed to evaluate Eq. (4). Considering also Eqs. (5) and (6) the proposed norm took the following form:

$$\varepsilon = \left\{ \left[\frac{1}{2p} \sum_{i=1}^{2p} \frac{1}{L} \int_0^L W(i, z) \left(\frac{\mathrm{d}\tilde{P}_i}{\mathrm{d}z} - f(z_\sigma, \tilde{P}_i) \right)^2 \mathrm{d}z + \frac{1}{ns} \sum_{j=1}^{ns} \frac{1}{L} \int_0^L \left(\frac{\mathrm{d}\tilde{P}_j}{\mathrm{d}z} - f(z_\sigma, \tilde{P}_j) \right)^2 \mathrm{d}z \right. \right.$$
$$\left. \left. + \frac{1}{np} \sum_{l=1}^{np} \frac{1}{L} \int_0^L \left(\frac{\mathrm{d}\tilde{P}_l}{\mathrm{d}z} - f(z_\sigma, \tilde{P}_l) \right)^2 \mathrm{d}z \right] \right\}^{1/2} f(G_m, G_{m-1}) \quad \left[\frac{\mathrm{W}}{\mathrm{m}} \right]. \tag{7}$$

Indexes i, j and l are chosen to represent ASE noise, signals and pumps, respectively. Forward and backward ASE spectra are divided into p elementary wavelength slots. The backward ASE (index i equals from $p + 1$ to $2p$) should converge to zero at the end of the erbium-doped fibre. Therefore, the numerically obtained value of backward ASE power at $z = L$ is replaced with zero in Eq. (4). If the value is close to zero, the difference between the left and right sides of Eq. (3) is small. If it is not, this substitution will give bigger difference. In addition, this difference can be emphasized choosing appropriate weight function $W(i, z)$. The second and the third elements of Eq. (7) account for evaluations of signals and pumps, where ns and np are the numbers of signals and pumps, respectively. L is the length of an erbium-doped fibre. The second condition (6) is represented by the weight function $f(G_m, G_{m-1})$ chosen as

$$f(G_m, G_{m-1}) = 10^{|G_m - G_{m-1}|} \tag{8}$$

If the changes for successive iterations are small the value of the function tends to 1 and it increases dramatically when $G_m \neq G_{m-1}$. Fulfilling the convergence criterion for gain ensures that the analogous criteria for the ASE and the pump power are also fulfilled [9].

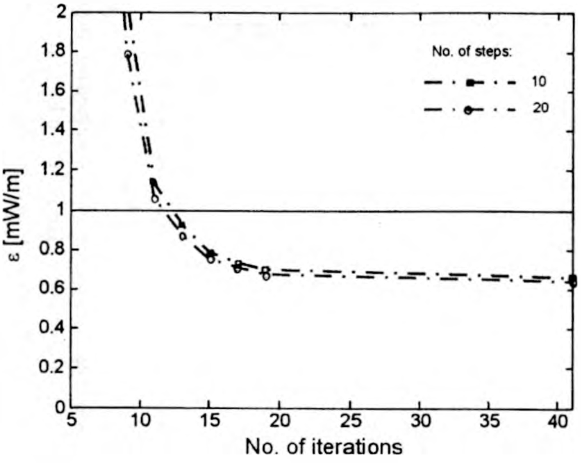

Fig. 3. Values of the norm as a function of the number of iterations.

Table 2. Comparison of numerical methods for *C*-band.

	RK4	Modified midpoint method $n = 4$	Modified midpoint method $n = 8$	Modified midpoint method $n = 16$	Average power analysis
$L = 10$ m					
ε [mW/m]	0.885	0.892	0.892	0.891	0.883
G [dB]	27.64	27.73	27.72	27.71	27.72
NF [dB]	3.79	3.77	3.77	3.77	3.77
$L = 18$ m					
ε [mW/m]	0.547	0.644	0.618	0.615	0.735
G [dB]	28.86	29.14	29.02	28.99	28.64
NF [dB]	4.02	3.98	4.00	4.00	3.95
$L = 32$ m					
ε [mW/m]	0.252	0.263	0.371	0.362	0.267
G [dB]	18.57	19.63	19.83	19.71	19.22
NF [dB]	3.52	3.50	3.50	3.50	3.49

$P_s = 1$ μW, $\lambda_s = 1.55$ μm, $P_p = 20$ mW, $\lambda_p = 0.98$ μm, $W(p+1 \leq i \leq 2p,\ L - \Delta L \leq z \leq L) = 10$, $W(p+1 \leq i \leq 2p,\ 0 \leq z \leq L - \Delta L) = 1$, $W(1 \leq i \leq p,\ 0 \leq z \leq L) = 1$, $\Delta L = 0.1$ m.
RK4 – 4-th order Runge–Kutta method.

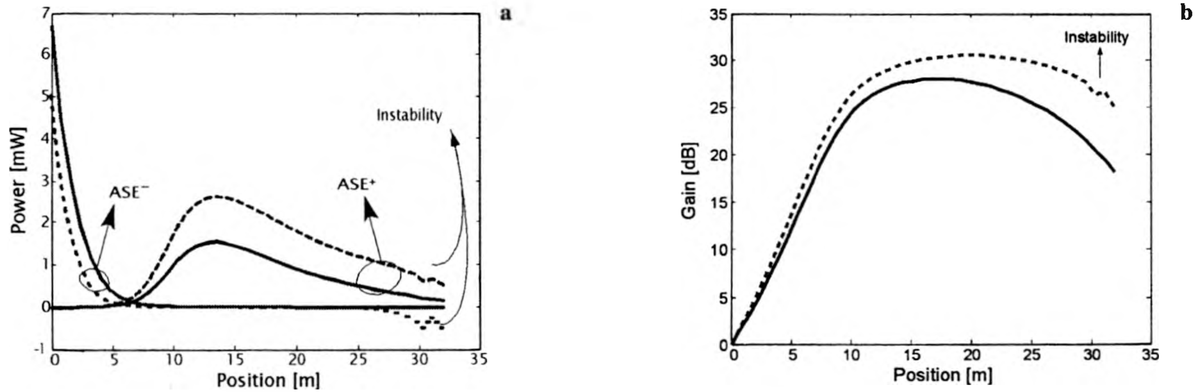

Fig. 4. Influence of the stabilization procedure on solution. The bold lines indicate solution after stabilization. Forward and backward ASE as a function of position in the fibre (**a**) and the gain as a function in the fibre (**b**).

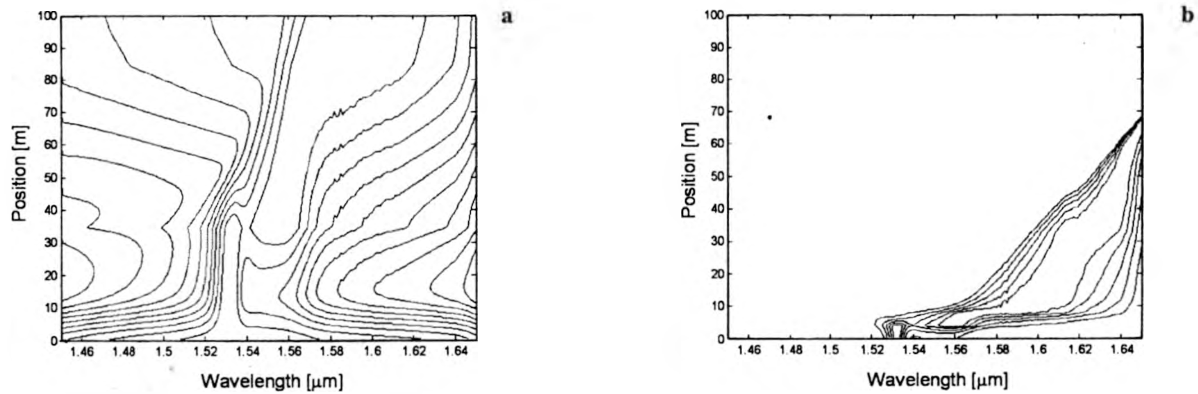

Fig. 5. Propagation of ASE noise in the forward (**a**) and backward (**b**) direction. The ASE noise power at a given position in the fibre was normalised to the maximum power of an ASE wavelength at this point. The z-axis is logarithmic. The EDF is 100 m long, the signal wavelength and pump wavelength is 1.60 mm and 0.98 mm, respectively. The signal power is 1 mW and the pump power is 100 mW.

Table 3. Comparison of numerical methods for L-band.

	$\lambda_s = 1.57$ µm		$\lambda_s = 1.59$ µm		$\lambda_s = 1.61$ µm	
	$\overline{\text{RK4}}$	MMM	$\overline{\text{RK4}}$	MMM	$\overline{\text{RK4}}$	MMM
$P_s = 1$ mW						
ε [mW/m]	3.76	**0.78**	2.43	**0.78**	2.19	**0.72**
G [dB]	14.66	16.78	14.10	16.85	13.52	16.81
NF [dB]	4.08	3.64	3.74	3.46	3.64	3.41
Calculation time [s]	10.1	**0.7**	8.6	**0.7**	7.2	**0.7**
$P_s = 32$ µW						
ε [mW/m]	2.62	**0.73**	2.26	**0.82**	2.15	1.26
G [dB]	28.36	31.38	27.14	31.01	23.68	27.55
NF [dB]	3.49	3.26	3.64	3.51	3.69	3.55
Calculation time [s]	8.6	**0.8**	7.2	**0.7**	7.2	**0.7**

$P_s = 1$ µW, $\lambda_s = 1.55$ µm, $P_p = 20$ mW, $\lambda_p = 0.98$ µm, $W(p+1 \leq i \leq 2p, L - \Delta L \leq z \leq L) = 10$, $W(p+1 \leq i \leq 2p, 0 \leq z \leq L - \Delta L) = 1$, $W(1 \leq i \leq p, 0 \leq z \leq L) = 1$, $\Delta L = 0.1$ m.
$\overline{\text{RK4}}$ – 4-th Runge–Kutta method with inversion level averaging, MMM – modified midpoint method with parameter $n = 4$.

The norm (7) for typical amplifier work conditions has been checked. The results comply with those expected. They are collected in Fig. 3. If the number of iterations is increased the value of the norm ε decreases. There can be seen a point where both curves begin to saturate. Thus, it can be said that if the value of the norm is less than 1 mW/m the solution is acceptable.

Next, the following methods were taken into account: the 4-th order Runge–Kutta, modified midpoint method and average power analysis. Three different lengths of erbium-doped fibre were chosen for which the norms were checked at the same configuration. The results including also the values of gain and noise figure are collected in Tab. 2. Generally, it follows that if the length of erbium-doped fibre increases the differences in the values gain between the three methods considered also increase. Noise figure is almost the same for all cases. The value of the norm ε is less than 0.8 mW/m and the difference between three methods is less than 0.2 mW/m. Although the norm proposed is a measure of solution quality it is rather impossible to determine which method is the best and whether its solution is close to exact one or not (as it is unknown) at that level of the norm accuracy. The curves for gain and noise figure at the same length of the EDF (erbium-doped fibre) for all the methods practically overlap and therefore they are not distinguished.

The problem which arises when the EDF length is longer than the optimal one for gain is random convergence. Instead of averaging level inversion for successive iterations to stabilize the algorithms like it was done in [10] we used another approach. Namely, we replaced minus values of power obtained in the solution with zero. The

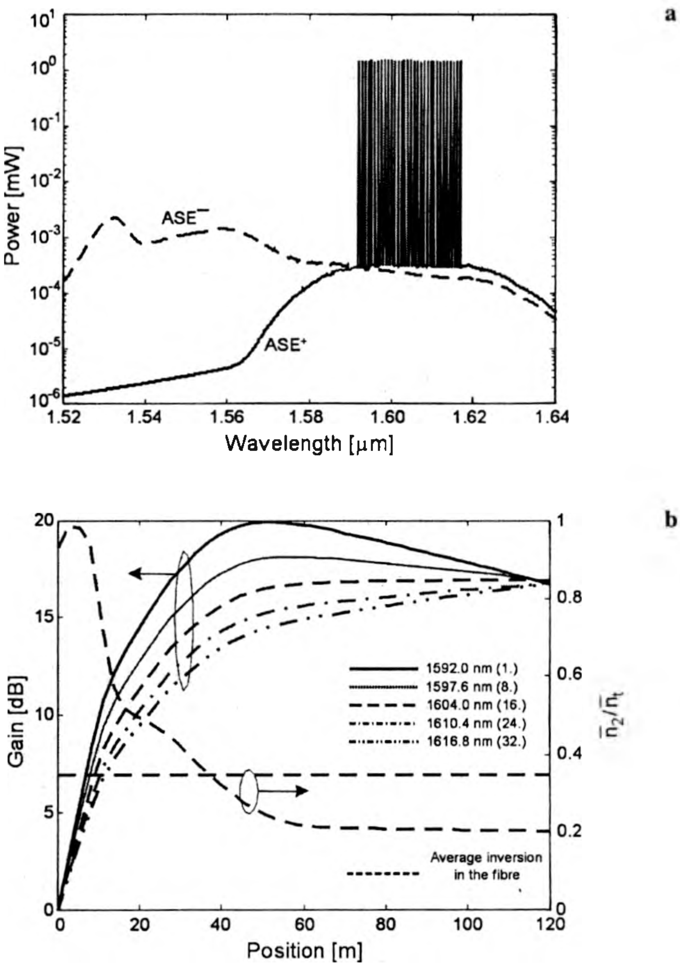

Fig. 6. The spectrum of 32 WDM channels (of –15 dBm power each) and ASE noise at the end of the fibre (**a**). Gain of 32 WDM channels as a function of position in the fibre. Each the 8th channel is shown. Additionaly, the inversion level as a function of position is shown. The number in brackets stands for the number of the channel (**b**). For both figures the EDF is 120 m long, the pump power and its wavelength are 100 mW and 0.98 μm, respectively. The wavelength of the first channel is 1592 nm, channel spacing is 0.8 nm.

effect of such a procedure is shown in Fig. 4 and it was used for the length of erbium-doped fibre equal to 32 m in Tab. 2.

Taking into account the stabilization procedure proposed further investigation was focused on *L*-band amplifiers. We compared modified midpoint method with the Runge–Kutta method with additional inversion level averaging. The results are presented in Tab. 3. It turned out that the value of the norm ε for the MM method was approximately three times smaller and the calculation time was less than ten times.

As an example of modelling using the MM method the ASE noise power propagation in 100 m long erbium-doped fibre is shown in Fig. 5. The ASE noise power at a given position in the fibre was normalised to the maximum power of an ASE wavelength at that point to emphasize the ASE noise power pumping effect [11]. Next example concerns propagation and spectrum of 32 WDM channels. It is shown in Fig. 6.

3. Conclusions

The modified midpoint method presented in the paper, new in the context of solving propagation equations, is very useful. This method has properties comparable to those of existing methods in the case of modelling the *C* band EDFA, and much better features in the case of *L*-band amplifiers. The quality criterion introduced in the paper allows evaluation of numerical solutions obtained with different methods. The criterion is calculated as an error between the left and right sides of propagation equation. Additionally, the weight functions are employed to emphasise convergence of the backward ASE to zero at the end of the fibre, and the convergence of the gain factor of the EDFA.

Concluding, the presented MM method of solving propagation equations is a very good tool for EDFA modelling, and the quantitative measure of quality of numerical solutions allows us to evaluate different methods in an objective way.

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