Optica Applicata, Vol. XXVI, No. 4, 1996

Instabilities in resonators filled with χ_2-materials

U. PESCHEL, C. ETRICH, F. LEDERER

Institute of Solid-State Theory and Theoretical Optics, Friedrich-Schiller-Universität, 07743 Jena, Germany.

We introduce a general model which describes the action of a planar resonator filled with a quadratically nonlinear material. We investigate the plane wave solutions and their stability behaviour.

1. Introduction

Planar resonators filled with a nonlinear medium belong to basic configurations of nonlinear optics to demonstrate fundamental effects as bistability, switching, self-oscillations and pattern formation. Various types of nonlinearities were studied and common properties could be identified (see, *e.g.*, [1], for and the references therein). In particular, a lot of attention was paid to the investigation of resonant structures filled with a quadratically nonlinear material [2]. Until now the main interest has focused on the efficient frequency conversion, where the distortions of the initial wave were mainly regarded as an unwanted side-effect. Recently, this point of view has changed [3], [4]. The modulations of the pump wave involved in the parametric process were investigated in more detail. A phase shift of the pump wave similar to that caused by a cubic nonlinearity was found to be induced by a subsequent up and down conversion [3]. Hence, in contrast to earlier investigations, we assume the resonator to be pumped by the fundamental harmonic only.

2. Structure under investigation and basic equations

The structure under investigation is shown in Fig. 1. A thin nonlinear crystal (*e.g.*, KTP or $LiNbO_3$) is surrounded by two metal clad mirrors which are assumed to show a high reflectivity for both the fundamental (FH) and second harmonic wave (SH). The resonator is resonant for both harmonics. Under this condition a modal theory can be applied [5]. The structure of the linear transmission functions with respect to spatial β and temporal ω Fourier components can be approximated in each case by a Lorentzian, namely for

– the FH: $$\tau_1(\beta,\omega) \simeq \frac{\hat{\tau}_1}{\beta_1^2|_{\omega=\omega_0} + \left.\frac{\partial\beta_1^2}{\partial\omega}\right|_{\omega=\omega_0}(\omega-\omega_0)-\beta^2},$$

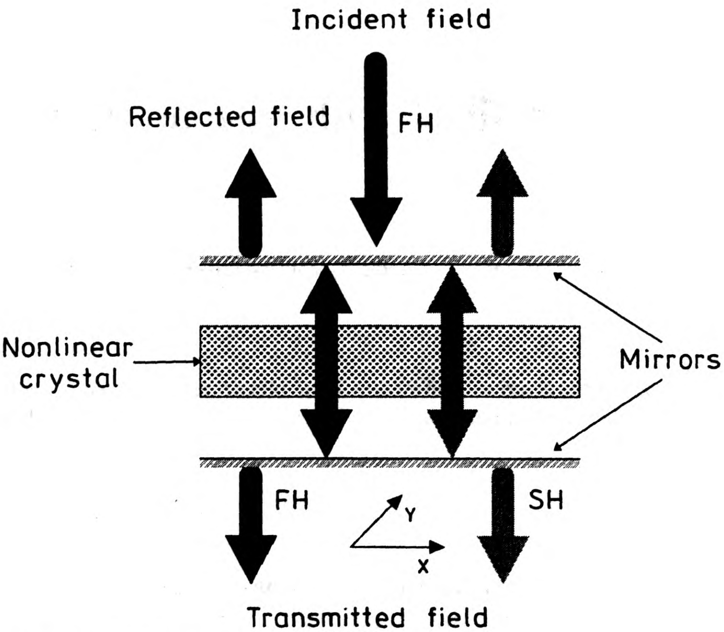

Fig. 1. Structure under consideration

$$- \text{ the SH:} \quad \tau_2(\beta,\omega) \simeq \frac{\hat{\tau}_2}{\beta_2^2|_{\omega=2\omega_0} + \left.\frac{\partial \beta_2^2}{\partial \omega}\right|_{\omega=2\omega_0} (\omega - 2\omega_0) - \beta^2} \tag{1}$$

where ω_0 is the carrier frequency of the incident light field at the FH frequency. The position and the width of the resonances are determined by the real and imaginary parts of $\beta_{1/2}^2$. From (1) it is straightforward to obtain the evolution equations for the slowly varying envelopes of the transmitted FH (E_1) and SH (E_2) fields which are driven by an external field (E_{in}) at the FH frequency:

$$\left[i \left.\frac{\partial \beta_1^2}{\partial \omega}\right|_{\omega=\omega_0} \frac{\partial}{\partial t} + \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \beta_1^2|_{\omega=\omega_0} \right] E_1 + \chi_1^{eff} E_1^* E_2 = \hat{\tau} E_{in},$$
$$\left[i \left.\frac{\partial \beta_2^2}{\partial \omega}\right|_{\omega=2\omega_0} \frac{\partial}{\partial t} + \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \beta_2^2|_{\omega=2\omega} \right] E_1 + \chi_2^{eff} E_1^2 = 0. \tag{2}$$

The nonlinearly induced polarization is incorporated in (2), where $\chi_{1/2}^{eff}$ contains the coefficient of the second order nonlinearity as well as an integral over the corresponding field structures weighted by a Green's function [5]. Note that the common phase mismatch enters the equations via those overlap integrals only. The convenient scaling

$$X = \sqrt{\mathscr{I}\beta_1^2|_{\omega=\omega_0}}\, x, \qquad Y = \sqrt{\mathscr{I}\beta_1^2|_{\omega=\omega_0}}\, y, \qquad T = \frac{\mathscr{I}\beta_1^2|_{\omega=\omega_0}}{\left.\frac{\partial \beta_1^2}{\partial \omega}\right|_{\omega=\omega_0}}\, t,$$

$$A_1=\sqrt{\frac{\chi_1^{\text{eff}}\chi_2^{\text{eff}}\left.\dfrac{\partial\beta_1^2}{\partial\omega}\right|_{\omega=\omega_0}}{(\mathscr{I}\beta_1^2|_{\omega=\omega_0})^2\left.\dfrac{\partial\beta_2^2}{\partial\omega}\right|_{\omega=2\omega_0}}}\,E_1,\qquad A_2=\frac{\chi_1^{\text{eff}}}{\mathscr{I}\beta_1^2|_{\omega=\omega_0}}E_2,\qquad A_{\text{in}}=\frac{\hat{\tau}}{\mathscr{I}\beta_1^2|_{\omega=\omega_0}}E_{\text{in}},$$

allows to reduce the number of free parameters considerably. We obtain the following set of equations:

$$\left[i\frac{\partial}{\partial T}+\frac{\partial^2}{\partial X^2}+\frac{\partial^2}{\partial Y^2}+\Delta_1+i\right]A_1+A_1^*A_2=A_{\text{in}},$$

$$\left[i\frac{\partial}{\partial T}+\alpha\left(\frac{\partial^2}{\partial X^2}+\frac{\partial^2}{\partial Y^2}\right)+\Delta_2+i\gamma\right]A_2+A_1^2=0 \tag{3}$$

where $\Delta_{1/2}$ denote the detuning of both waves from the respective resonances and α and γ represent the relative diffraction and damping of the SH wave:

$$\Delta_1=\frac{\mathscr{R}\beta_1^2|_{\omega=\omega_0}}{\mathscr{I}\beta_1^2|_{\omega=\omega_0}},\quad \Delta_2=\frac{\mathscr{R}\beta_2^2|_{\omega=2\omega_0}\left.\dfrac{\partial\beta_1^2}{\partial\omega}\right|_{\omega=\omega_0}}{\mathscr{I}\beta_1^2|_{\omega=\omega_0}\left.\dfrac{\partial\beta_2^2}{\partial\omega}\right|_{\omega=2\omega_0}},\quad \alpha=\frac{\left.\dfrac{\partial\beta_1^2}{\partial\omega}\right|_{\omega=\omega_0}}{\left.\dfrac{\partial\beta_2^2}{\partial\omega}\right|_{\omega=2\omega_0}},$$

$$\gamma=\frac{\mathscr{I}\beta_2^2|_{\omega=2\omega_0}\left.\dfrac{\partial\beta_1^2}{\partial\omega}\right|_{\omega=\omega_0}}{\mathscr{I}\beta_1^2|_{\omega=\omega_0}\left.\dfrac{\partial\beta_2^2}{\partial\omega}\right|_{\omega=2\omega_0}}.$$

3. Plane wave solutions

First we study the stationary plane wave solutions of (3). Neglecting the derivatives the SH wave can be expressed by the FH explicitly

$$A_2^0=-\frac{(A_1^0)^2}{\Delta_2+i\gamma}. \tag{4}$$

This is inserted into the equation for the FH resulting in

$$\left[\Delta_1+i-\frac{\Delta_2-i\gamma}{\Delta_2^2+\gamma^2}|A_1^0|^2\right]A_1^0=A_{\text{in}}. \tag{5}$$

This equation is similar to the one obtained in the case of a cubic nonlinearity with a certain amount of two photon absorption [3]. By changing the sign of the detuning of the SH wave one can switch between effectively focusing ($\Delta_2<0$) and defocusing ($\Delta_2>0$) behaviour. The nonlinearity induced losses are proportional to γ and are caused by SH light leaving the cavity.

Due to the additional nonlinearly induced losses the bistable behaviour is not as pronounced as in the case of a real valued cubic nonlinearity. Nevertheless, plane

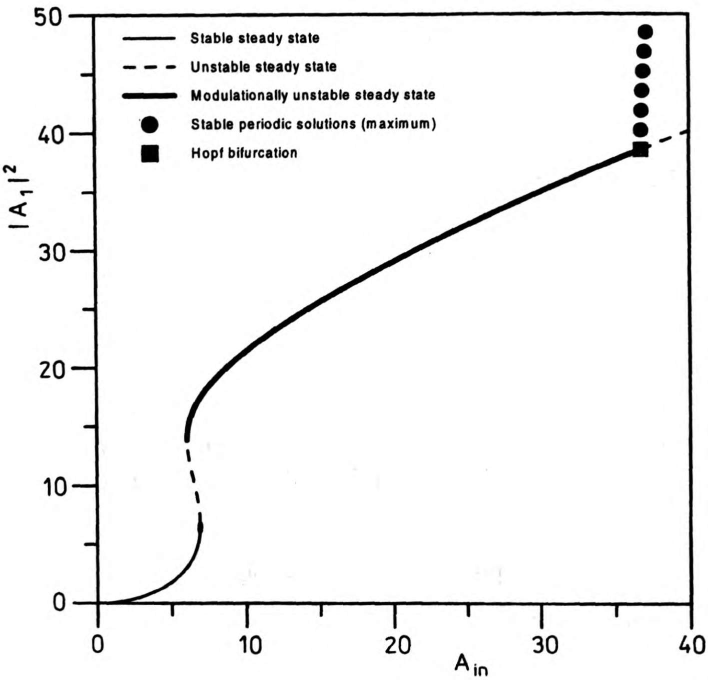

Fig. 2. Bistable hysteresis loop of plane-wave solutions and various instabilities ($\Delta_1 = 4$, $\Delta_2 = 4$, $\alpha = 0.5$, $\gamma = 0.6$)

wave bistability can be obtained (see Fig. 2), if the following conditions are met:

$$\Delta_1\Delta_2 > 0, \quad \frac{|\Delta_2|(|\Delta_1| - \sqrt{3})}{\sqrt{3}|\Delta_1| + 1} > \gamma. \tag{6}$$

4. Stability analysis

An important issue is the stability of the plane wave solutions. In what follows we study the action of a small perturbation added to the plane-wave background

$$A_n(X, Y, T) = A_n^0 + \delta A_n \exp(\lambda T)\exp(K_X X + K_Y Y). \tag{7}$$

If the perturbation grows in time ($\mathscr{R}\lambda > 0$), the solution is unstable.

Before we come to spatially modulated perturbations ($K_{X,Y} \neq 0$), we study the action of homogeneous perturbations first. In the case of bistability the plane wave solutions destabilize at the first limit point (at limit point: $\lambda = 0$) and stabilize at the second one. Additionally, we found Hopf bifurcations which are characterized by a nonvanishing imaginary part of the eigenvalue ($\lambda = \pm i\omega$) at the critical point. Self-oscillations start if the intensities of both fields cross a line defined by

$$|A_1^0|^2 = \frac{\gamma}{2(1+\gamma)^2}\Big[|A_2^0|^2 - 2(1+\gamma)^2 - (\Delta_1+\Delta_2)^2 + 4\Delta_2(\Delta_1+\Delta_2) + 4\Delta_2^2\,\frac{(1+\gamma)^2+(\Delta_1+\Delta_2)^2}{|A_2^0|^2-2(1+\gamma)^2-(\Delta_1+\Delta_2)^2}\Big], \quad (8)$$

where both intensities are given by Eqs. (4) and (5). Self-oscillations we found to be very important in the system under investigation. They are driven by a competition between FH and SH fields. The development of a Hopf bifurcation is demonstrated in Figs. 2 and 3. Above a certain threshold intensity the self-oscillations them-

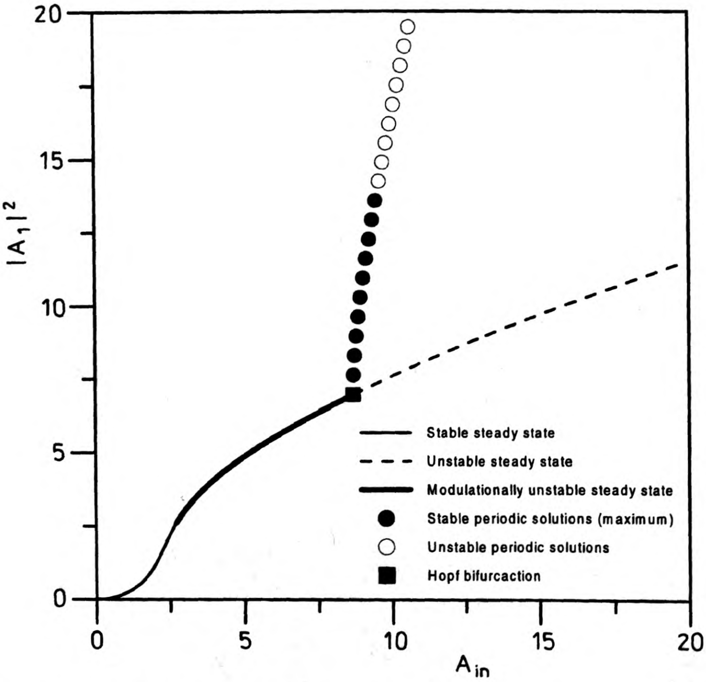

Fig. 3. Monostable hysteresis loop of plane-wave solutions and various instabilities ($\Delta_1 = 4$, $\Delta_2 = 4$, $\alpha = 0.5$, $\gamma = 0.6$)

selves destabilize again (see Fig. 3). Period doubling occurs followed by chaotic bahaviour.

Let us now investigate the plane-wave solution with respect to spatially modulated perturbations ($K_{X,Y} \neq 0$). Again we obtain both real and complex valued bifurcations. If we evaluate the hysteresis loops in more detail, we find modulational instabilities to be always present before plane wave instability sets in (see Fig. 4). Hence all the plane wave effects mentioned above as bistability and self oscillations require the exclusion of transverse effects by, *e.g.*, spatial confinement.

But even if the plane wave solution is unstable the system may approach a new stable but spatially modulated state. In the case of a real valued bifurcation we

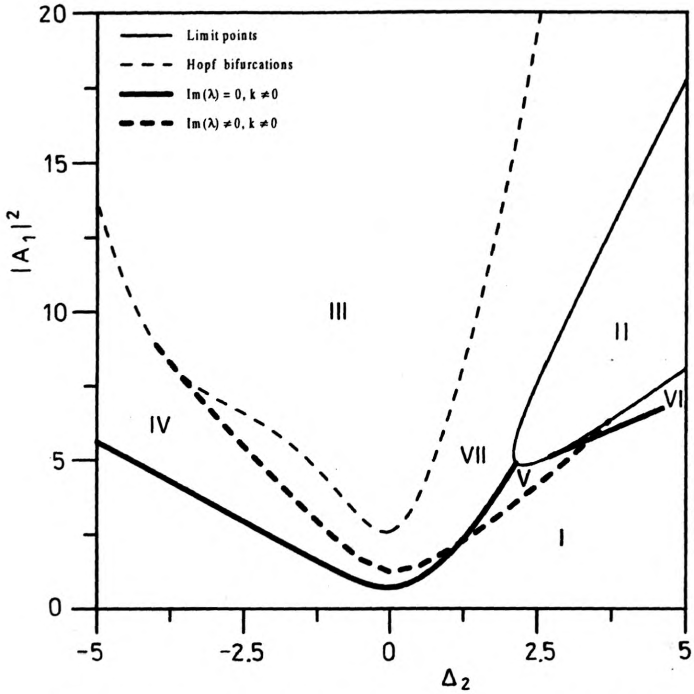

Fig. 4. Domains of stability and instability of a stationary plane wave solution ($\Delta_1 = 4$, $\alpha = 0.5$, $\gamma = 0.6$). I — stable, II — domain of bistability, III — instability due to a Hopf bifurcation, IV–VII — modulationally unstable

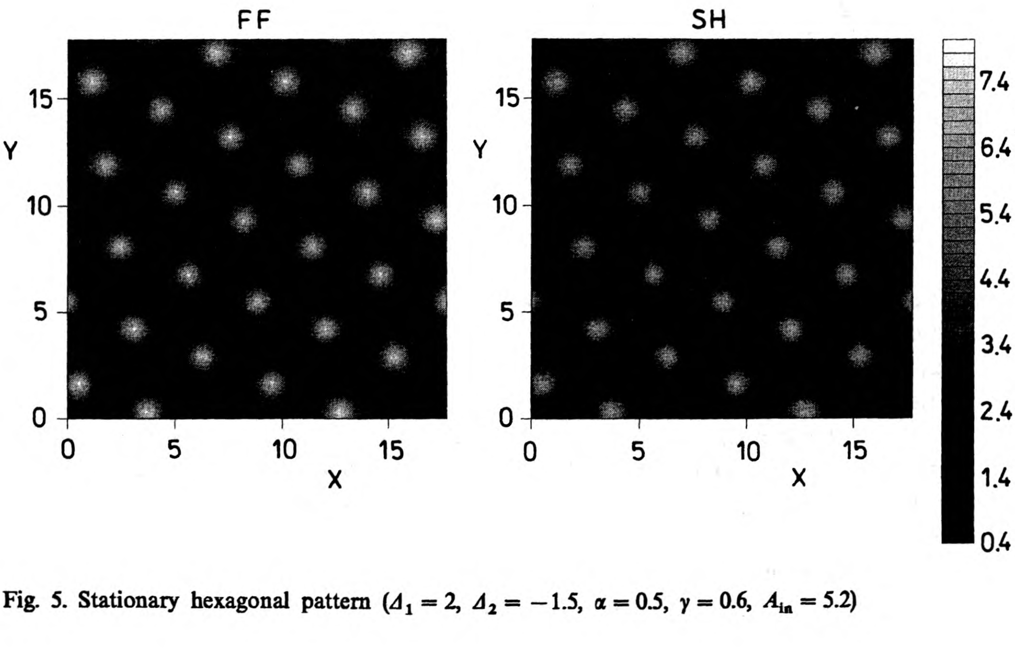

Fig. 5. Stationary hexagonal pattern ($\Delta_1 = 2$, $\Delta_2 = -1.5$, $\alpha = 0.5$, $\gamma = 0.6$, $A_{in} = 5.2$)

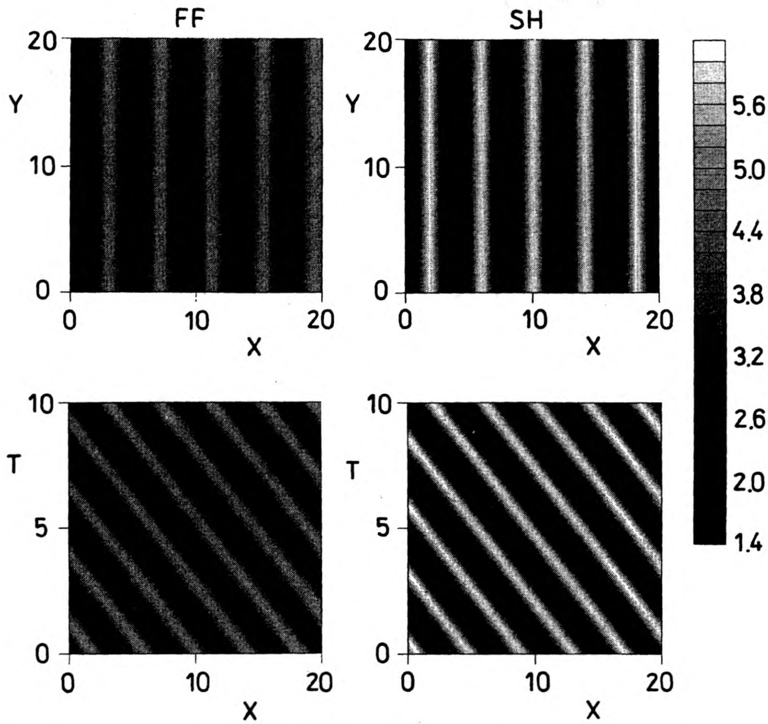

Fig. 6. Running waves ($\Delta_1 = 2$, $\Delta_2 = 1.5$, $\alpha = 0.5$, $\gamma = 0.6$, $A_{in} = 3.2$)

obtained a resting hexagonal pattern (see Fig. 5). We found stable travelling waves at the complex valued bifurcations (see Fig. 6).

5. Conclusions

We developed a consistent model to describe the evolution of FH and SH fields in a double resonant Fabry–Perot resonator filled with a quadratically nonlinear material. The plane wave solutions are similar to those obtained for a cubic nonlinearity. We found plane wave bistability and very dominant self-oscillations. If transverse effects play any role all the plane wave effects are covered by the onset of modulational instabilities. Both stationary patterns and running waves are found in the system

References

[1] Abraham N. B., Firth W. J., J. Opt. Soc. Am. B **7** (1990), 951.
[2] See, for example, the special issue on parametric oscillation and amplification, J. Opt. Soc. Am. B **10** (1993), 1655.
[3] Reinisch R., Popov E., Neviere M., Opt. Lett. **20** (1995), 854.

[4] Sibilia C., Re A., Fazio E., Bertolotti M., J. Opt. Soc. Am. B 13 (1996), 1151.
[5] Peschel T., Lederer F., Phys. Rev. B 46 (1992), 7632.

Received November 27, 1996