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Appearance of wave front dislocations under interference among beams with simple wave fronts

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The appearance of wave front dislocations under interference among beams of simple wave fronts is considered. It is shown that even two beams with smooth wave fronts can cause the formation of screw dislocations. The screw dislocations are formed at the point where the lines of equal intensity of two beams cross the minima of an interference pattern.

1. Introduction

Numerous studies are devoted to the wave front dislocation [1]–[5]. Many of them relate to the statistic fields. However, the origin of dislocations in speckle-field occurs in undeveloped speckle patterns, *i.e.*, by interference of a small number of partial waves [2]. In this case the question arises: how many and what beams are to form screw dislocation of the wave front. This question is under study in the present work.

2. Theory

Let us restrict our considerations to scalar approximation, *i.e.*, when only one field component is analyzed. Consider interference of two coherent unspecified waves U_1, U_2. Phases μ_1, μ_2 and modules of amplitudes A_1, A_2 are of such kind that at any plane of observation, which is perpendicular to z-axis, the analyzed fields obey the wave front approximation [6], *i.e.*, the propagating waves U_1 and U_2 are not subject to any diffraction. At a point (x, y) of the observation plane the modules of amplitudes of interference wave fronts differ slightly and a solution to equation $A_1(x,y) = A_2(x,y)$ is in the form of some line $y = f(x)$. The line of interference fringe minimum and the line of equal amplitude modules always cross at nonzero angle, *i.e.*,

$$\frac{df(x)}{dx} \neq \frac{d\tilde{f}}{dx}$$

where $y = \tilde{f}(x)$ – equation of the line of interference fringe minimum.

The condition of an isolated zero in the point (x_i, y_i) is

$$\begin{cases} \mu_1(x_i, y_i) = \mu_2(x_i, y_i) + \pi = \mu_i, \\ A_1(x_i, y_i) = A_2(x_i, y_i) = A_i. \end{cases} \tag{1}$$

Let us analyze the resulting interference field that is formed by U_1 and U_2 waves. We use a local coordinate system X, Y, Z with the origin coinciding with an amplitude zero point, and the direction of Z-axis coincides with the direction of z-axis of a general coordinate system. Decompose the resulting field U_i (in the vicinity of zero point x_i, y_i) into McLorren series and restrict our considerations to linear terms with respect to X and Y. It is easy to show that the resulting field can be written in the form

$$U_i = \exp[j\mu_i]\{\Delta A_i^x X + \Delta A_i^y Y + jA_i[\Delta\mu_i^x X + \Delta\mu_i^y Y]\} \tag{2}$$

where:

$$\Delta A_i^t = A_{1i}^t - A_{2i}^t, \quad \Delta\mu_i^t = \mu_{1i}^t - \mu_{2i}^t,$$

$$A_{pi}^t = \left.\frac{\partial A_p}{\partial t}\right|_{\substack{x = x_i,\\ y = y_i}} \quad \mu_{pi}^t = \left.\frac{\partial \mu_p}{\partial t}\right|_{\substack{x = x_i,\\ y = y_i}} \quad t = x, y, \quad p = 1,\ 2.$$

Then the phase tangent of the resulting field is described by a relationship

$$\tan\Phi = A_i\frac{\Delta\mu_i^x X + \Delta\mu_i^y Y}{\Delta A_i^x X + \Delta A_i^y Y}. \tag{3}$$

It follows from (3) that the field in the vicinity of zero point is defined as a screw dislocation. The dislocation sign and the phase behaviour while passing zero point depend on the relation between $\Delta\mu_i^t$ and ΔA_i^t. Obviously, an ideal ("classic") helicoid is realized in the case

$$\frac{A_i\Delta\mu_i^x}{\Delta A_i^y} = 1, \quad A_i\Delta\mu_i^y = -\Delta A_i^x. \tag{4}$$

Under any other conditions the helicoid will be deformed.

Let us consider the relation between "phases of screw dislocation". By the phase of screw dislocation we mean some constant phase shift nearby a zero point that can be defined for any specific zero point. It is the phase shift occurring while superimposing the reference field that defines the intensity of interference fringes in the vicinity of zero point (a dark, light or gray fork). Let us introduce the s coordinate along the line of equal amplitude modules (equal intensities). Consider the resulting field when moving along the curve of equal intensities. In this case the relationship for the resulting field can be written as

$$E(s) = 2A(s)e^{j\frac{\mu_1(s)+\mu_2(s)}{2}}\cos\left[\frac{\mu_1(s)-\mu_2(s)}{2}\right]. \tag{5}$$

In the crossing point s_i of the line of equal intensities $y = f(x)$ with i-th zero line (amplitude zero as a function of the spatial coordinates), the exponent index (phase) in (5) is defined by the relationship $\varphi_i = \mu_i + m_i\pi + \frac{\pi}{2}$, while taking condition (1) into consideration.

In this case s can be connected with a phase surface of either the first or the second beam. It is obvious that the exponent index does not change while passing through the point s_i in the xy-plane at a small distance ds (that tends to zero) from the point s_i. Then the exponent index defines dislocation phase in the point s_i. The difference between the phases in adjacent minima $(i, i+1)$ of interference pattern is defined by the relationship

$$\Delta\varphi_i = \mu_{i\pm 1} - \mu_i \pm \pi. \tag{6}$$

Thus, when scrutinizing (6) along the line of equal intensities, that is, on one of the phase surfaces, it appears to differ by π for two adjacent points s_i. Specifically, for two quasi-plane waves it is reflected in the fact that in adjacent minima of an interference pattern a phase differs by π, if an observation plane is orthogonal to one of the wave vectors of interference beams or to bisectrix of an angle between them. On superposition of a reference plane wave, it will be observed in the form of division of a light and dark strip in two adjacent minima. If the angle between interfering beams is small, the patterns can be seen in any position of the observation plane.

Note that owing to the restrictions adopted by us to the changing of the wave front phases (a smooth phase change), the changing of a wave front that is equal to the distance between adjacent minima can be treated as a linear one. In other words, the result of interference can be understood as interference of two plane fronts. Hence, for any wave fronts a phase of helicoids in adjacent minima of an interference pattern differs practically by π.

3. Computer simulation

The case under consideration was checked by computer simulation. The result of interference between two quasi equal intensity plane waves and reference beam is presented in Fig. 1. The vertical fringes correspond to the field with dislocations. Horizontal fringes result from the superposition of these fields and reference beam. As an illustration (Fig. 1**a**) two dislocations have been chosen that are in adjacent interference minima. The corresponding zones are singled out by white rectangles. Interference bifurcations are marked in the figure by letters A and B. As one can see from the figure, the dislocation phases in adjacent minima differ by π. It is manifested by a dark fork A, which appears as a kind of inclusion in a light fork B. The behaviour of intensity waves is illustrated in Fig. 1**e**. As can be seen from Eq. (3), the fork direction is defined by the intensity gradient. This fact is illustrated in Fig. 1**b**, **c**. We used for calculations an intensity change that is shown in Fig. 1**f**. The forks of different directions (dislocations of different signs) are marked by letters A and B. The lines of equal intensity are designated by arrows. Note that the features of interference patterns fade and become undeterminable, as the distance between the equal intensity lines is decreased. In the limiting case of its confluence (Fig. 1**g**), the screw dislocations anihilate. As a result, a saddle point occurs. The phase behaviour in the vicinity of this point may be interpreted as an edge dislocation with zero length. The interferogram for this case is presented in Fig. 1**d**.

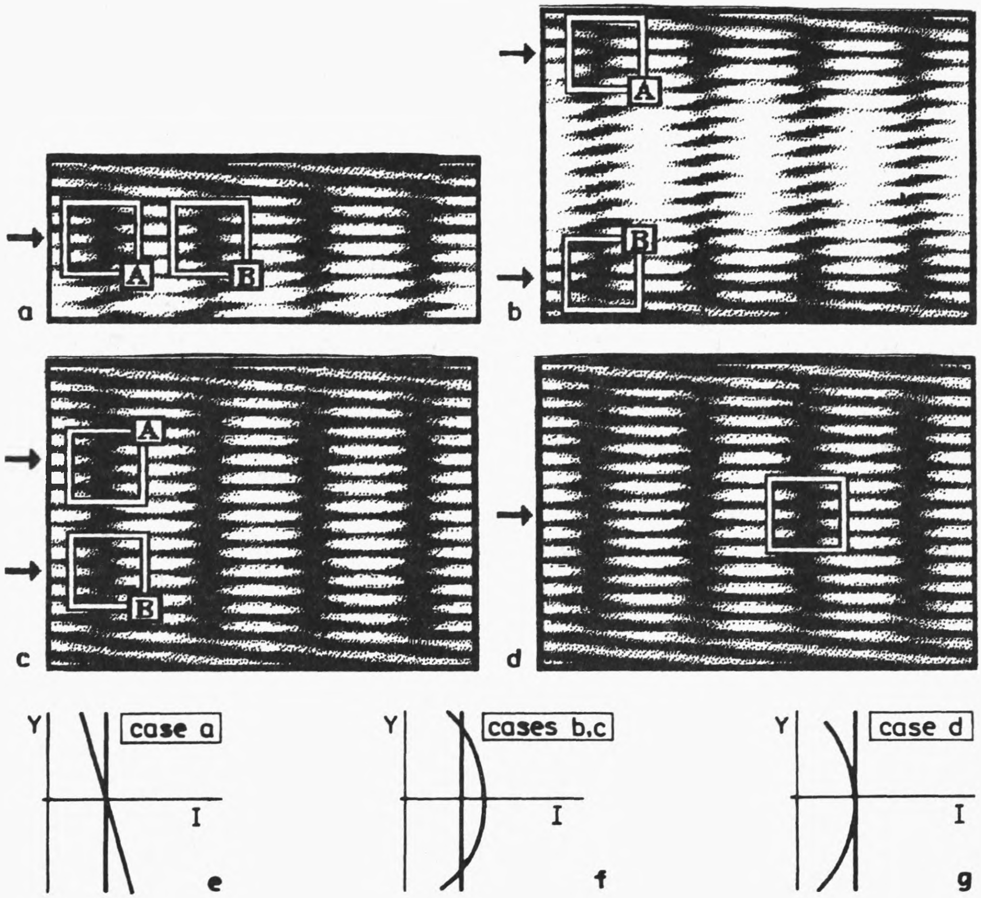
A
B
a
b
c
d
Y
I
case a
cases b,c
case d
e
f
g

Fig. 1.

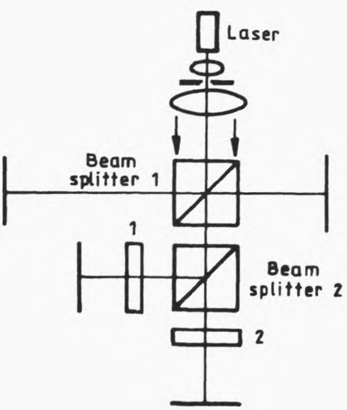
Laser
Beam
splitter 1
1
Beam
splitter 2
2

Fig. 2.

Note that the lingering edge dislocation is a nongeneric case. The interference pattern often observed in practice, as a pattern with shifted fringes, characterizes the fields of complex phase structure. These fields are the ones with smoothly, though quickly changing phase, or a set of screw dislocations, which are united by parts of smooth phase surface. The adjacent dislocations are of different signs. The origin of new dislocations conforms to the rule which is established in [7].

4. Experimental

Experimental checking was realized according to the simple optical scheme that is shown in Fig. 2. Experimental arrangement consists of two Michelson interferometers. Two wedges with logarithmic transmittance (1.2) are deposited in the legs of the second interferometer. This interferometer forms the field with screw dislocations. The wedges were installed in such a way that modulation of amplitude was realized in the direction of minima of interference pattern. The first interferometer enables formation of interference patterns with reference beam.

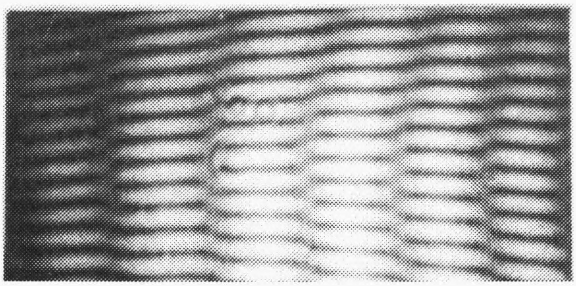

Fig. 3.

The results of the experiment are shown in Fig. 3. As illustrated in this figure, the screw dislocations result from interference of two quasi-plane fronts at the crossing of lines of the same intensity and minima of an interference pattern.

5. Concluding remarks

The above investigation indicates that formation of screw dislocations is possible under interference even between two beams with smooth wave fronts. The screw dislocations are formed in a cross point of the lines of equal intensities of beams and minima of an interference pattern. In the general case, the phase surface in the vicinity of the field points of such kind is a deformed helicoid. It seems likely to assume that the origin of screw dislocation in statistic fields occurs to be in the same way under interference of elementary wave fronts, being formed by a scattering surface. This statement can be experimentally checked for model scattering objects.

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