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Evaluation of rigid body displacement by differential holographic interferometry*

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The evaluation of a small change in the position of a diffused object by means of differential holographic interferometry is the subject of this paper. The equations of interference fringes will be derived for a quite general displacement of rigid body and for a quite general shape of the object surface. For the plane surface of the object the interference pattern will be analysed in details, and the method for determining the displacement of a rigid body from one interference pattern will be shown.

Introduction

Let us consider an object with optically rough surface. We want to evaluate the displacement of the object caused fly some agent. Assume that the displacement is so small that the differential holographic interferometry can be used. Hereafter, the displacement is understood as a change in the object position without any deformation of the object, i.e. the object is considered as a rigid body. Then the displacement vector Δ of any point on the object surface can be analytically expressed as a function of its coordinates with the help of only six a priori unknown constants in contrast to the case in which a deformation of the object is also admitted and the dependence of the displacement vector Δ on the coordinates of object surface can be an a priori quite unknown function. That is why the displacement of a rigid body is much simpler to study than that of an object which may suffer from simultaneous deformations of its surface. Nevertheless, in papers dealing with estimation of the rigid body displacement, e.g. in [1—7], this problem is discussed in an even simpler form, i.e. the treatment is there reduced to the rigid body displacement considering only basic displacement as the pure translation in a known direction or the pure rotation round a known axis of rotation.

Our chief object in this work is first to find and then to analyse the interference pattern belonging to a general change in the rigid body position. The investigation are restricted to an interferometric arrangement in which the entrance pupil of the optical system is situated at infinity with respect to the examined object, so that the interference fringes are localized on the object surface [8]. For the object illumination a coherent spherical wave is used.

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Theory

The interference fringes are loci of the constant optical path difference δ. The equation for an interference fringe of k-th order lying on the object surface can be written

$$\delta(\boldsymbol{r}_s(u, v)) = k\lambda, \tag{1}$$

where λ is the wavelength, $\boldsymbol{r}_s(x_s, y_s, z_s)$ is the position vector of the object surface and the variables u, v are some curvilinear coordinates of the object surface. In differential holographic interferometry for the path difference δ the following well-known relation holds

$$\delta = (\boldsymbol{s}-\boldsymbol{s}_i)\,\Delta, \tag{2}$$

where $\boldsymbol{s}_i(p_i, q_i, m_i)$ and $\boldsymbol{s}(p, q, m)$ are the unit direction vectors of incident and reflected rays, respectively. When looking for the shape of interference fringe we may take account of the principal reflected rays of the observing optical system, only. By substituting (2) into (1) we can write

$$[\boldsymbol{s}-\boldsymbol{s}_i(\boldsymbol{r}_s(u, v))]\cdot\Delta\,(\boldsymbol{r}_s(u, v)) = k\lambda, \tag{3}$$

where the direction vector $\boldsymbol{s}$ is constant for all the principal rays, the entrance pupil of the observing optical system being situated at infinity. The vector $\boldsymbol{s}_i$ is given by

$$\boldsymbol{s}_i = (\boldsymbol{r}_s-\boldsymbol{r}_i)/R_i, \tag{4}$$

where $\boldsymbol{r}_i(x_i, y_i, z_i)$ is the position vector of the point source illuminating the object, and R_i is the distance along the ray from the point source to a point of the object surface. This distance can be expressed by

$$R_i(\boldsymbol{r}_s) = R_{i0}+\Delta R_i(\boldsymbol{r}_s), \tag{5}$$

where R_{i0} is the constant which equals the distance R_i of the reference ray that goes from the point source to the reference point of the object surface. The reference point of the object surface is chosen somewhere in the middle of the illuminated part of the object surface (see also fig.). For not too large angle subtended by marginal rays the quantity ΔR_i is approximately given by

$$\Delta R_i = (\boldsymbol{r}_s-\boldsymbol{r}_{s0})\boldsymbol{s}_{i0}, \tag{6}$$

where the subscript 0 denotes the quantities which belong to the reference ray. Applying (5) and (6) to (4) we get

$$\boldsymbol{s}_i(\boldsymbol{r}_s) = (\boldsymbol{r}_s-\boldsymbol{r}_i)/[R_{i0}+(\boldsymbol{r}_s-\boldsymbol{r}_{s0})\boldsymbol{s}_{i0}]. \tag{7}$$

The quantities R_{i0}, $\boldsymbol{r}_i$, $\boldsymbol{r}_{s0}$, $\boldsymbol{s}_{i0}$ and $\boldsymbol{s}$ are the constants of the given interferometric arrangement. Inserting the expression (7) into the equation (3) we have

$$\{\boldsymbol{s}[R_{i0}+(\boldsymbol{r}_s-\boldsymbol{r}_{s0})\boldsymbol{s}_{i0}]-(\boldsymbol{r}_s-\boldsymbol{r}_i)\}\,\Delta(\boldsymbol{r}_s) = \\ = k\lambda[R_{i0}+(\boldsymbol{r}_s-\boldsymbol{r}_{s0})\boldsymbol{s}_{i0}], \tag{8}$$

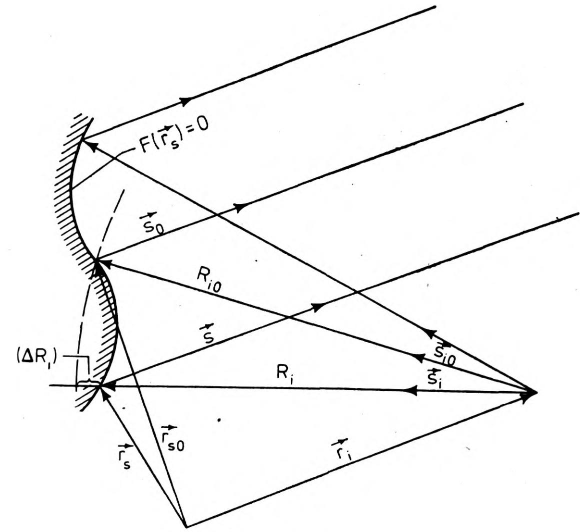

Fig. 1

where the vector $\boldsymbol{r}_s$ is still the function of the coordinates u, v.

Now we want to investigate more closely the vector function $\Delta(\boldsymbol{r}_s)$. The general change of the position of a rigid body can always be broken down in a pure translation and a pure rotation. The axis of rotation can pass through any point of the space. For every axis of rotation having the right direction given by the unit vector $\boldsymbol{o}(\alpha, \beta, \gamma)$ such angle of rotation ψ and such vector of translation $\boldsymbol{t}(a, b, c)$ can be found that both these motions give together the investigated change of the object position. If the axis of object rotation goes through the origin of the coordinate system, the displacement vector Δ at an object point given by the position vector $\boldsymbol{r}_s$ can be expressed by

$$\Delta = (\boldsymbol{o} \times \boldsymbol{r}_s)\psi + \boldsymbol{t} \tag{9}$$

or introducing the rotation vector $\boldsymbol{O}(A, B, C)$, where

$$A = \alpha\psi, \quad B = \beta\psi, \quad C = \gamma\psi, \tag{10}$$

it may be written as

$$\Delta = \boldsymbol{O} \times \boldsymbol{r}_s + \boldsymbol{t}. \tag{11}$$

The angle ψ is positive or negative, depending upon the counter clockwise or clockwise rotation, respectively, when looking against the vector $\boldsymbol{o}$ and assuming the right-hand coordinate system.

Substituting (11) into (8) we easily find

$$([\boldsymbol{O}\boldsymbol{r}_s\boldsymbol{s}] + \boldsymbol{t}\boldsymbol{s} - k\lambda)[R_{i0} + (\boldsymbol{r}_s - \boldsymbol{r}_{s0})\boldsymbol{s}_{i0}] + [\boldsymbol{O}\boldsymbol{r}_s\boldsymbol{r}_i] + \boldsymbol{t}(\boldsymbol{r}_i - \boldsymbol{r}_s) = 0. \tag{12}$$

Now let us introduce the position vector $\boldsymbol{r}_I(x_I, y_I, z_I)$ to determine the plane of the interference pattern in the object space of the observing optical system, and let the position vector $\boldsymbol{r}_{I0}(x_{I0}, y_{I0}, z_{I0})$ determine the point of intersection of this plane with the principal ray reflected at the reference point of the object surface. Further, let us choose the Cartesian coordinate system with the axis z perpendicular to the plane of the interference pattern, and let

$$u = x_I - x_{I0}, \quad v = y_I - y_{I0}. \tag{13}$$

Then the curvilinear coordinates u, v of the object surface are simulataneously Cartesian coordinates in the plane of the interference pattern. For the sake of brevity we introduce the vector $\boldsymbol{g}(u, v, 0)$. This vector may also be interpreted as two-dimensional position vector in the plane of the interference pattern. Then

$$\boldsymbol{g} = \boldsymbol{r}_I - \boldsymbol{r}_{I0} \tag{14}$$

holds. If $V = V(u, v)$ is the distance along a principal ray from a surface point (x_s, y_s, z_s) of the object to a point (u, v) of the plane of the interference pattern, then the position vector $\boldsymbol{r}_s$ can be expressed in the following way:

$$\boldsymbol{r}_s = \boldsymbol{g} + \boldsymbol{r}_{I0} + V\boldsymbol{s}. \tag{15}$$

The distance $V(u, v)$ can be found by solving the equation

$$F(\boldsymbol{g} + \boldsymbol{r}_{I0} + V\boldsymbol{s}) = 0, \tag{16}$$

if

$$F(\boldsymbol{r}_s) = 0 \tag{17}$$

is the equation of the geometric surface averaging the microstructure of the illuminated part of the object. Substituting (15) into (13) and writting explicitly the vector $\boldsymbol{g}$ after a suitable rearrangement we get the following equation of the interference fringe of k-th order in the plane of the interference pattern

$$\begin{aligned} &u^2(Cq - Bm)p_{i0} + v^2(Am - Cp)q_{i0} + uv(Cq - Bm)q_{i0} + (Am - Cp)p_{i0} + \\ &\quad + u\{([\boldsymbol{O}\boldsymbol{r}_{I0}\boldsymbol{s}] + \boldsymbol{t}\boldsymbol{s} - k\lambda)p_{i0} - a + C(y_i + qM) - B(z_i + mM)\} + \\ &\quad + v\{([\boldsymbol{O}\boldsymbol{r}_{I0}\boldsymbol{s}] + \boldsymbol{t}\boldsymbol{s} - k\lambda)q_{i0} - b + A(z_i + mM) - C(x_i + pM)\} + \\ &\quad + ([\boldsymbol{O}\boldsymbol{r}_{I0}\boldsymbol{s}] + \boldsymbol{t}\boldsymbol{s} - k\lambda)M + [\boldsymbol{O}\boldsymbol{r}_{I0}\boldsymbol{r}_i] + \boldsymbol{t}(\boldsymbol{r}_i - \boldsymbol{r}_{I0}) + \\ &\quad + uV(u, v)(Cq - Bm)\boldsymbol{s}\boldsymbol{s}_{i0} + vV(u, v)(Am - Cp)\boldsymbol{s}\boldsymbol{s}_{i0} + \\ &\quad + V(u, v)\{([\boldsymbol{O}\boldsymbol{r}_{I0}\boldsymbol{s}] + \boldsymbol{t}\boldsymbol{s} - k\lambda)\boldsymbol{s}\boldsymbol{s}_{i0} - [\boldsymbol{O}\boldsymbol{r}_i\boldsymbol{s}] - \boldsymbol{t}\boldsymbol{s}\} = 0, \end{aligned} \tag{18}$$

where the notation

$$M = R_{i0} + (\boldsymbol{r}_{I0} - \boldsymbol{r}_{s0})\boldsymbol{s}_{i0} \tag{19}$$

has been used for brevity.

Additional analysis can be made if the concrete spahe of the object surface is given. The simplest case is that when the object surface is a plane. Hereafter we shall investigate only this case.

Let equation (17) of the object surface be

$$(\boldsymbol{r}_s - \boldsymbol{r}_{s0})\,\boldsymbol{n} = 0, \tag{20}$$

where $\boldsymbol{n}(p_n, q_n, m_n)$ is the unit normal to the plane surface of the object. Then the distance $V(u, v)$ has to obey the relation

$$(\boldsymbol{g}+\boldsymbol{r}_{I0}+V\boldsymbol{s}-\boldsymbol{r}_{s0})\,\boldsymbol{n} = 0 \tag{21}$$

for each point (u, v) of the plane of the interference pattern. From (21) it follows

$$V(u, v) = (\boldsymbol{r}_{s0}-\boldsymbol{g}-\boldsymbol{r}_{I0})\,\boldsymbol{n}/\boldsymbol{sn}. \tag{22}$$

If substituting the expression (22) into the equation (18) we should obtain the equation of second order with respect to the variables u, v for the interference fringe in the plane of the interference pattern. If the object surface is a plane then it is suitable to identify this surface with the plane of the interference pattern, because in this case the interference fringes, localized on the object surface, are localized simultaneously on the plane of the interference pattern. Then $V(u, v) = 0$ holds. When $\boldsymbol{r}_{I0} = \boldsymbol{r}_{s0}$ then also holds $M = R_{i0}$. Substituting $V = 0$ and $M = R_{i0}$ into (18) we get the following relation for the interference fringe of k-th order lying in the plane of the interference pattern

$$\begin{aligned} &u^2(Cq-Bm)p_{i0}+v^2(Am-Cp)q_{i0}+uv[(Cq-Bm)q_{i0}+(Am-Cp)p_{i0}]+\\ &+u\{([\boldsymbol{O}\boldsymbol{r}_{I0}\boldsymbol{s}]+\boldsymbol{ts}-k\lambda)p_{i0}-a+C(y_i+qR_{i0})-B(z_i+mR_{i0})\}+\\ &+v\{([\boldsymbol{O}\boldsymbol{r}_{I0}\boldsymbol{s}]+\boldsymbol{ts}-k\lambda)q_{i0}-b+A(z_i+mR_{i0})-C(x_i+pR_{i0})\}+\\ &+([\boldsymbol{O}\boldsymbol{r}_{I0}\boldsymbol{s}]+\boldsymbol{ts}-k\lambda)R_{i0}+[\boldsymbol{O}\boldsymbol{r}_{I0}\boldsymbol{r}_i]+\boldsymbol{t}(\boldsymbol{r}_i-\boldsymbol{r}_{I0}) = 0. \end{aligned} \tag{23}$$

This equation is of second order with respect to the variables u, v of the following type

$$a_{11}u^2+a_{22}v^2+a_{12}uv+a_{13}u+a_{23}v+a_{33} = 0. \tag{24}$$

The interference fringes described by such equations are conic sections.

Before starting an analysis of the equation (23) let us turn some attention to a simpler case when the wave illuminating the object is a plane instead of spherical one. Then R_{i0} tends to infinity and $\boldsymbol{s}_i = \boldsymbol{s}_{i0}$ =constant. The equation for an interference fringe can be obtained by dividing the equation (23) by R_{i0} and putting $R_{i0} = \infty$. Hence we get the linear equation

$$\begin{aligned} &u[C(q-q_i)-B(m-m_i)]+v[A(m-m_i)-C(p-p_i)]+\\ &+[\boldsymbol{O}\boldsymbol{r}_{I0}(\boldsymbol{s}-\boldsymbol{s}_i)]+\boldsymbol{t}(\boldsymbol{s}-\boldsymbol{s}_i)-k\lambda = 0. \end{aligned} \tag{25}$$

This equation represents straight parallel interference fringes. We immediately see that of the investigated displacement of the object was a pure translation, i.e. $A = B = C$ then the interference fringes would disappear and the whole field of view be covered with constant intensity.

Now let us come back to the equation (23) which is valid for a spherical illuminating wave. From this equation it easily follows that the interference fringes become straight lines, i.e. the equation (23) becomes a linear equation, if at least one of the three following cases occurs:

(i) The rotation vector $\boldsymbol{O}(A, B, C)$ is a zero vector.

(ii) The vectors $\boldsymbol{O}(A, B, C)$ and $\boldsymbol{s}(p, q, m)$ are collinear i.e. the axis of the object rotation is parallel to the direction of view.

(iii) The components p_{i0} and q_{i0} of the vector $\boldsymbol{s}_{i0}(p_{i0}, q_{i0}, m_{i0})$ are equal to zero. It is the case when the point source positioned on a normal to the object surface at the reference point illuminates the plane surface.

These three conditions can be expressed shortly:

$$\text{(i)} \qquad \psi = 0.$$

$$\text{(ii)} \qquad A = p\psi, \quad B = q\psi, \quad C = m\psi. \tag{26}$$

$$\text{(iii)} \qquad p_{i0} = q_{i0} = 0.$$

By applying any from these three conditions to the equation (23) we obtain in each case a linear equation of the type

$$u(d_1 + h_1 k) + v(d_2 + h_2 k) + d_3 + h_3 k = 0, \tag{27}$$

where for brevity we have used new coefficients $d_1, h_1, d_2, h_2, d_3, h_3$. From this equation it is seen that the straight interference fringes create the bundle of lines converging at the point (u_T, v_T). Using the well-known relations of analytical geometry we can derive for these coordinates

$$u_T = (d_2 h_3 - h_2 d_3)/(d_1 h_2 - h_1 d_2), \tag{28}$$
$$v_T = (h_1 d_3 - d_1 h_3)/(d_1 h_2 - h_1 d_2).$$

If

$$d_1 h_2 = h_1 d_2 \tag{29}$$

holds, and simultaneously, the numerators in (28) are not equal to zero, e.g. when $p_{i0} = q_{i0} = 0$, then the convergence point of the bundle of the lines lies at the infinity and the interference fringes are parallel. It is to be noted, that the interference fringes can also be detected as parallel straight lines if the convergence point of their bundle is sufficiently far beyond the field of view of the observing optical system.

If the interference fringes are not straight lines then the small discriminant of the conic section (24) defined by

$$D' = \begin{vmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{vmatrix} \tag{30}$$

becomes

$$D' = -[(Cq - Bm)^2 q_{i0}^2 + (Am - Cp)^2 p_{i0}^2 + (Cq - Bm)(Am - Cp) p_{i0} q_{i0}] \tag{31}$$

in the case of the conic section (23). Since

$$(Cq - Bm)^2 q_{i0}^2 + (Am - Cp)^2 p_{i0}^2 \geqslant 2(Cq - Bm)(Am - Cp) p_{i0} q_{i0} \tag{32}$$

holds always the investigated small discriminant D' is always non-positive, becoming equal to zero in certain case leading to straight interference fringes. If the discriminant D

of the conic section (23), defined by

$$D = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{12} & a_{22} & a_{23} \\ a_{13} & a_{23} & a_{33} \end{vmatrix}, \tag{33}$$

is not equal to zero then the negative value of small discriminant D' means that the observed interference fringes are hyperboles. If the discriminant D is equal to zero for some interference fringe then this interference fringe degenerates into two intersected straight lines.

The previous analysis discovers the connection between a general rigid body displacement and the interference pattern. Let us consider the question how to use one interference pattern of the the interference fringes described by the equation (23) to evaluate a general rigid body displacement, which is fully given by the vectors $\boldsymbol{t}(a, b, c)$, $\boldsymbol{O}(A, B, C)$. Before looking for the answer let us underline that the coefficients of the interference fringe equation (23) are linear function of the interference order k and of the quantities a, b, c, A, B, C.

In the first place let us consider the case when the interference fringes are hyperboles. It is well-known in analytical geometry that a conic section is given unambiguously by five points lying in a plane if no three of these points lie on a straight line. If any three points are lying on a straight line then the conic section is singular. If the position of some five points of a conic section is known then it is given by the equation

$$\begin{vmatrix} u^2, & uv, & v^2, & u, & v, & 1 \\ u_1^2, & u_1 v_1, & v_1^2, & u_1, & v_1, & 1 \\ u_2^2, & u_2 v_2, & v_2^2, & u_2, & v_2, & 1 \\ u_3^2, & u_3 v_3, & v_3^2, & u_3, & v_3, & 1 \\ u_4^2, & u_4 v_4, & v_4^2, & u_4, & v_4, & 1 \\ u_5^2, & u_5 v_5, & v_5^2, & u_5, & v_5, & 1 \end{vmatrix} = 0, \tag{34}$$

where u_1, v_1 up to u_5, v_5 are the coordinates of the given five points. Inserting the numerical values of the coordinates of the five points, suitably chosen in a concrete interference fringe of k-th order, into the equation (34) and then comparing this equation with the equation (24) we obtain the numerical values of the coefficients a_{ik} of the equation (24) valid for the investigated interference fringe. Without loss of generality we may assume that the coefficient a_{11} differs from zero. It can always be ensured by a suitable rotation of the coordinate system round its axis z. When the coefficient a_{11} differs from zero the equation (24) with the concrete values of the coefficients a_{ik} may be divided by the coefficient a_{11}. The new coefficients — let us denote them $b_{ik} = (a_{ik}/a_{11})$ — may be compared with their analytical expression obtained from the equation (23) being divided by $(Cq-Bm)p_{i0} \neq 0$. In this way five linear homogeneous equations are obtained usually for the seven unknowns a, b, c, A, B, C, k. From the interference pattern we can get one more equation bringing a new information about relations among the interference fringes. To find this sixth equation the system of the five linear homogeneous equations should be derived also for the interference fringe of $(k+n)$-th order. For this purpose the first equation, valid for the interference fringe of k-th order is substrated from the first one, valid for the interference fringe of $(k+n)$-th order, and this subtraction is repeated also for the second to fifth couples of equations. Finally we get

$$\begin{aligned} b_{22}^{(k+n)} &= b_{22}^{(k)}, \\ b_{12}^{(k+n)} &= q_{12}^{(k)}, \\ b_{13}^{(k+n)} &= b_{13}^{(k)}+n\lambda/(Bm-Cq), \\ b_{23}^{(k+n)} &= b_{23}^{(k)}+n\lambda q_{i0}/(Bm-Cq)p_{i0}, \\ b_{33}^{(k+n)} &= b_{33}^{(k)}+n\lambda R_{i0}/(Bm-Cq)p_{i0}. \end{aligned} \tag{35}$$

One from the last three equations of the system (35) (last seeming to be the most sutable) can serve as the sixth equation. In such a way we obtain the system of six linear unhomogeneous equations with the seven unknowns a, b, c, A, B, C, k. We assume that the difference n between the orders of interference fringes is known. Obviously, a system of six linear unhomogeneous equations with seven unknows is not unambiguously solvable. Fortunately, the case without any a priori information about the investigated position change of the object can be hardly found in practice. Usually one of the quantities a, b, c, A, B, C, k is known before the evaluation. Such cases are solvable with the help of the proposed method.

If the interference fringes are not hyperboles but they create the bundle of straight lines, then at least one of the three conditions (26) must be satisfied. Let the interferometric arrangement be chosen in such a way that the third condition (26) does not hold, i.e. $p_{i0} = q_{i0} = 0$ is not valid. It means that the interference fringes are straight lines if either $\psi = 0$, i.e. $A = B = C = 0$, or $A = p\psi$, $B = q\psi$, $C = m\psi$ holds. That is why we have now only the five unknowns a, b, c, ψ, k. Applying the second condition (26) to (23) for the bundle of straight interference fringes we get the following equation

$$\begin{aligned} &u[(\boldsymbol{ts}-k\lambda)p_{i0}-a+\psi(my_i-qz_i)]+ \\ &+v[(\boldsymbol{ts}-k\lambda)q_{i0}-b+\psi(pz_i-mx_i)]+ \\ &+(\boldsymbol{ts}-k\lambda)R_{i0}+\boldsymbol{t}(\boldsymbol{r}_i-\boldsymbol{r}_{I0})+\psi(\boldsymbol{s}\boldsymbol{r}_{I0}\boldsymbol{r}_i] = 0. \end{aligned} \tag{36}$$

Like in the case of curved interference fringes we can also find the concrete numerical values of the coefficients of the equation of the straight interference fringe of k-th order. The coordinate system can again be turned round the axis z in such a way that the coefficient at the variable u differs from zero. Dividing the equation with numerical coefficients by this coefficient and comparing the obtained equation with its analytical expression (36) being divided by $(\boldsymbol{ts}-k\lambda)p_{i0}-a+\psi(my_i-qz_i) \neq 0$ we get two linear homogeneous equations for the five unknowns a, b, c, ψ, k. Next equation can again be obtain with the help of the interference fringe of $(k+n)$-th order in the same way as in the case of curved fringes. Thus we get the system of three linear unhomogeneous equations for the five unknowns. That is why the unambiguous evaluation is conditioned again by the a priori known change of the object position. If, however, we are not interested in the magnitude of the angle ψ the interferometric arrangement can be chosen in such a way that the vectors $\boldsymbol{r}_i$ and $\boldsymbol{s}$ is collinear. Then the interference fringes will not depend on the quantity ψ as it follows from the relation (36).

The evaluation of the interference pattern caused by general change of the object position would be substantially more complicated if the shape of the object surface was not plane. Then the distance $V(u, v)$ would be a nonlinear function of u, v and the equation (18) of higher order (than the second one) with respect to the variables u, v. In such case as well as in the cases when the object surface is plane but the displacement is not unambiguously solvable with help of the previous method, it seems to be most suitable to look for the change in the object position by applying the well-known method of single or multiple hologram analysis [9] on three surface points of the object. These three points must not lie on one straight line. We shall obtain three displacement vectors Δ. With help of these three vectors Δ and with help of the relation (11) we shall be able to get just six linearly independent linear equations for the six unknowns a, b, c, A, B, C.

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Определение смещений твёрдого тела методом голографической интерферометрии

Содержанием статьи является оценка незначительных изменений положения диффузного предмета с помощью голографической интерферометрии. Приведено уравнение интерференционных линий для общих смещений твёрдого тела любой формы поверхности. Произведён подробный анализ интерференционных спектров для случая плоской предметной поверхности, а также показано, как из одной интерферограммы можно определить смещение твёрдого тела.