Optica Applicata, Vol. XI, No. 1, 1981

Theory and applications of the far field double diffraction on the progressing spatial phase modulation and stationary amplitude grating

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Time dependent irradiance distributions in the three lowest diffraction orders in the far field of stationary amplitude and moving phase gratings of the same spatial period are theoretically investigated. General formulae are developed for separated gratings placed in an arbitrary order on the optical axis. They allow an optimization of the amplitude of time-dependent harmonics in 0 and ± 1 double diffraction orders by a proper choice of the separation distance between the gratings. The differences in optimization conditions for 0 and ± 1 diffraction beams are related to the Talbot distance. Practical applications of theoretical investigations are discussed.

1. Introduction

The phenomenon of double diffraction on periodic structures has recently gained the considerable interest due to the widespread use of laser radiation. Its theoretical analyses and practical applications proposed can be divided into three main groups. The first one is concerned with the use of a pair of diffraction structures in various types of double-grating interferometers [1]. The second group of investigations deals with the Fresnel diffraction field of dual gratings [2], whereas the third one is concerned with the far field theory of double diffraction and its applications [3].

This work deals with theoretical aspects of the far field double diffraction phenomenon. Until now, its properties were presented from the point of view of its particular application, e.g. displacement measurement [4, 5], absolute position monitoring [6], ad pattern alignment [3, 7, 8]. Another type of application deals with the phenomena present in double diffraction on a stationary amplitude grating and moving periodic spatial phase modulation generated, for example, by a progressing acoustical wave [9, 10]. In this case the temporal irradiance dependence in the far field diffraction orders was proposed for the light modulation and for the measurement of the amplitude of phase modulation. However, the analyses presented in the referenced works [9, 10] were conducted for special configurations of double diffraction setup, e.g. the fixed distance between the phase grating placed before the amplitude grating and for the spatial coincidence of two periodic modulations. The goal of this work is to present a general analysis of the irradiance distributions in the three lowest diffraction orders 0 and ± 1 in the far field of a progressing phase modulation and stationary amplitude grating arbitrarily situated in space. The three lowest diffraction orders are the most important ones from the point of view of practical applications of the phenomenon and, moreover, its analysis explains all major characteristics of the far field double diffraction. The theoretical analysis will show important differences between zero and first order irradiance modulation curves depending on the gap distance. The results lead to suggestions as to the accuracy improvement of the method of measurement of very small amplitudes of phase modulations, and constitute general guidelines to experiments on double diffraction and considering light modulation techniques.

2. Theoretical analysis

The complex amplitude transmittance of a progressing sinusoidal phase grating will be expressed as

$$\exp\left\{iB\cos\left(2\pi\frac{x}{d}+\beta\right)\right\} = \sum_{m=-\infty}^{\infty} i^m J_m(B)\exp\left\{im\left(2\pi\frac{x}{d}+\beta\right)\right\}, \tag{1}$$

where B designates the amplitude of phase modulation, d is the spatial period of the grating, x is the direction perpendicular to grating lines, and β describes a temporal change of phase imposed by a displacement Δx of the gratings in the x direction

$$\beta = 2\pi\frac{\Delta x}{d} = 2\pi\frac{V_a t}{d} = \Omega t, \tag{2}$$

where V_a is the velocity of wave generating the progressing phase grating, Ω designates its radial frequency, and t is a time parameter. In eq. (1) the Jacobi–Anger formula was used and $J_m(B)$ is the m-th order Bessel function. In the following analysis two types of stationary diffraction gratings will be considered: single-frequency and general amplitude transmittance gratings.

2.1. The case of sinusoidal amplitude diffraction grating

2.1.1. Stationary amplitude grating placed behind progressing phase grating

Single frequency amplitude diffraction grating will be expressed as

$$C+A\cos 2\pi\frac{x}{d}. \tag{3}$$

The two gratings of the same spatial period d are assumed to be of infinite extent, mutually parallel and normally illuminated by a plane, λ quasi-monochromatic spatially coherent wavefront. By using the approach of angular spectrum of plane waves [11] the amplitude of light field in the plane of amplitude grating is calculated as

$$E(x, z) = \left(C + A\cos 2\pi \frac{x}{d}\right) \sum_{m=-\infty}^{\infty} i^m J_m(B) \exp i\left\{m\left(2\pi\frac{x}{d} + \beta\right) - \pi m^2 \frac{\lambda z}{d^2}\right\}. \quad (4)$$

Therefore, the amplitude of double diffracted beam of spatial frequency $1/d$ and forming $+1$st diffraction order in the far field is

$$E_{+1} = \frac{A}{2} J_0(B) + iCJ_1(B) \exp i(\beta - \pi\lambda z/d^2) - \frac{A}{2} J_2(B) \times \exp i(2\beta - 4\pi\lambda z/d^2). \quad (5)$$

Its irradiance is calculated as

$$I_{+1} = C^2 J_1^2(B) + \frac{A^2}{4}[J_0^2(B) + J_2^2(B)] - ACJ_0(B)J_1(B) \times \sin(\beta - \pi\lambda z/d^2) - ACJ_1(B)J_2 \sin(\beta - 3\pi\lambda z/d^2) - \frac{A_2}{2} J_0(B)J_2(B)\cos(2\beta - 4\pi\lambda z/d^2). \quad (6)$$

It can be seen from eq. (6) that the time-dependent irradiance is composed of the three temporal frequencies described by terms with $\beta = 0, 1\beta$, and 2β. After further simplifications the constitutive temporal harmonics are described as

$$I_{+1}^0 = C^2 J_1^2(B) + \frac{A^2}{4}[J_0^2(B) + J_2^2(B)],$$

$$I_{+1}^\beta = -ACJ_1(B)\left[J_0^2(B) + J_2^2(B) + 2J_0(B)J_2(B)\cos\left(2\pi\frac{\lambda z}{d^2}\right)\right]^{1/2} \times \sin\left[\beta - \arctan\frac{J_0(B)\sin\left(\pi\frac{\lambda z}{d^2}\right) + J_2(B)\sin\left(3\pi\frac{\lambda z}{d^2}\right)}{J_0(B)\cos\left(\pi\frac{\lambda z}{d^2}\right) + J_2(B)\cos\left(3\pi\frac{\lambda z}{d^2}\right)}\right], \quad (7)$$

$$I_{+1}^{2\beta} = -\frac{A^2}{2} J_0(B)J_2(B)\cos(2\beta - 4\pi\lambda z/d^2).$$

Similar calculations can be performed for the $-1(I_{-1})$ and $0(I_0)$ diffractions orders. The results are quoted below.

$$I_{-1}^{0} = I_{+1}^{0},$$

$$I_{-1}^{\beta} = ACJ_1(B)\left[J_0^2(B)+J_2^2(B)+2J_0(B)J_2(B)\cos\left(2\pi\frac{\lambda z}{d^2}\right)\right]^{1/2}$$

$$\times\sin\left[\beta+\arctan\frac{J_0(B)\sin\left(\pi\frac{\lambda z}{d^2}\right)+J_2(B)\sin\left(3\pi\frac{\lambda z}{d^2}\right)}{J_0(B)\cos\left(\pi\frac{\lambda z}{d^2}\right)+J_2(B)\cos\left(3\pi\frac{\lambda z}{d^2}\right)}\right], \tag{8}$$

$$I_{-1}^{2\beta} = -\frac{A^2}{2}J_0(B)J_2(B)\cos(2\beta+4\pi\lambda z/d^2),$$

and

$$I_0^0 = C^2J_0^2(B)+\frac{A^2}{2}J_1^2(B),$$

$$I_0^{\beta} = 2ACJ_0(B)J_1(B)\sin(\pi\lambda z/d^2)\cos\beta, \tag{9}$$

$$I_0^{2\beta} = \frac{A^2}{2}J_1^2(B)\cos 2\beta.$$

The following properties of irradiance distributions in $+1, 0$, and -1 double diffraction orders can be deduced from eqs. (7) and (8):

1. The amplitudes of the bias and second temporal frequencies are independent of the separation distance z between the gratings. For $z = Md^2/\lambda$ (d^2/λ) is the so-called Talbot distance known from the theory of the self-imaging [11]), and $z = (M+1/2)d^2/\lambda$ the second harmonics in all discussed diffraction orders are mutually in phase, M is a positive integer including 0 (corresponding to the case of coincidence of phase and amplitude gratings).

2. For $z = Md^2/\lambda$ the fundamental harmonics of the time-dependent irradiance in orders ± 1 have equal amplitudes

$$I_{\pm 1}^{\beta}(z = Md^2/\lambda) = \mp ACJ_1(B)[J_0(B)+J_2(B)]\sin\beta, \tag{10}$$

but are mutually out of phase. For small values of B it follows from the properties of Bessel functions that the condition $z = Md^2/\lambda$ coincides with the condition of maximum amplitude of the fundamental harmonic of irradiance changes in the ± 1 diffraction orders. At the same time the fundamental harmonic in the zero order vanishes.

3. For $z = (M+1/2)d^2/\lambda$ the fundamental harmonic in ± 1 double diffraction orders have co-phasial and equal amplitudes

$$I_{\pm 1}^{\beta}\left(z = \left(M+\frac{1}{2}\right)d^2/\lambda\right) = -ACJ_1(B)[J_0(B)-J_2(B)]\cos\beta. \tag{11}$$

Simultaneously, the fundamental temporal component in 0 order attains the value

$$I_0^\beta\left(z=\left(M+\frac{1}{2}\right)d^2/\lambda\right) = \pm 2\, ACJ_0(B)J_1(B)\cos\beta, \tag{12}$$

with the upper sign for M even, and the lower one for M odd. It is interesting to note that the amplitude of fundamental harmonic in eq. (12) is larger than that described by eq. (10). Practical consequences of this fact will be discussed below.

2.1.2. Single-frequency stationary grating placed in front of progressing phase grating

Now let us discuss for comparison the case of identical amplitude transmittance sinusoidal grating placed in front of progressing phase modulation. The complex amplitude of the light field in the plane of phase grating is given by

$$E'(x,z) = \left\{C+\frac{A}{2}\exp\left(-i\pi\frac{\lambda z}{d^2}\right)\cos 2\pi\frac{x}{d}\right\}$$
$$\times\sum_{m=-\infty}^{\infty} i^m J_m(B)\exp i\left\{m\left(2\pi\frac{x}{d}+\beta\right)\right\}. \tag{13}$$

By performing similar calculations to those presented above the following formulae for the harmonics of the time-dependent irradiance in the symmetrical +1 and −1 double diffraction orders are obtained

$$I'^0_{+1} = I'^0_{-1} = I^0_{\pm 1},$$
$$I'^\beta_{\pm 1} = \mp ACJ_1(B)\left[J_0^2(B)+J_2^2(B)+2J_0(B)J_2(B)\cos\left(2\pi\frac{\lambda z}{d^2}\right)\right]^{1/2}$$
$$\times\sin\left\{\beta\pm\arctan\left[\frac{J_0(B)-J_2(B)}{J_0(B)+J_2(B)}\tan\left(\pi\frac{\lambda z}{d^2}\right)\right]\right\},$$
$$I'^{2\beta}_{\pm 1} = -\frac{A^2}{2}J_0(B)J_2(B)\cos 2\beta. \tag{14}$$

Moreover, it can be easily shown that the expressions for temporal harmonics in the zero double diffraction order are identical with those valid for the case of amplitude grating located behind the progressing phase grating.

The comparison of eq. (14) with eqs. (7) and (8) shows no important differences between the cases of the amplitude grating placed in front of or behind the phase modulation. For the characteristic separation distances $z = Md^2/\lambda$ and $z = \left(M+\frac{1}{2}\right)d^2/\lambda$ discusssed above, the corresponding temporal harmonics in $+1$, -1, and the 0 diffraction orders take the same values in both cases. The only differences can be seen in the expression for $I^{\beta}_{\pm 1}$ and $I'^{\beta}_{\pm 1}$ in the terms depicting the phase shift of the harmonics being proportional to the values of arctan. Moreover, it is interesting to note that the second temporal harmonics, in the case of amplitude grating placed in front of the phase one, has no phase shift proportional to the separation distance z.

2.1.3. Discussion

Let us discuss the practical implications of the properties of double diffraction field in the ± 1 and 0 orders developed above. They can be analysed, for example, from two points of view. The first one concerns the method of selective detection of the fundamental component of time-dependent irradiance in one of the first diffraction orders for noise-free measurement of the amplitude B of phase modulation. In the case of spatial coincidence of phase and amplitude modulations [10], knowing the amplitude grating parameters A and C and measuring the diffraction efficiency of the fundamental temporal harmonic of irradiance, the amplitude B can be estimated. However, it follows from the analysis presented above that the same results are obtained for the separation distances $z = Md^2/\lambda$ between the stationary amplitude grating and investigated phase modulation, irrespectively of the order of placing the two modulations on the optical axis. This fact is very important in practical investigations when the amplitude grating cannot be located in the plane of progressing phase grating [9, 12]. Due to our proposal no additional lens [12] used for imaging the phase modulation onto the amplitude grid is required. The exact fulfillment of the condition $z = Md^2/\lambda$ can be checked by observing the moment of vanishing of the fundamental temporal harmonic in the zero double diffraction order.

Moreover, even the higher accuracy of the method can be obtained by performing the measurements in the zero far field order. However, in this case the separation distance z should be $(M+1/2)d^2/\lambda$, see eq. (12). The fulfillment of the condition $z = (M+1/2)d^2/\lambda$ is realized by maximizing the basic temporal harmonic in the zero order or by checking the in-phase relationship between basic harmonics in $+1$ and -1 diffraction orders.

Additionally, if the technique using the detection of the difference in irradiances between $+1$ and -1 orders [7] is applied to selectively filtered fundamental harmonics I^{β}_{+1} and I^{β}_{-1}, then for the planes $z = Md^2/\lambda$ the detected irradiance difference

$$I^{\beta}_{-1}(z = Md^2/\lambda) - I^{\beta}_{+1}(z = Md^2/\lambda) = 2ACJ_1(B)[J_0(B) + J_2(B)] \sin\beta, \tag{15}$$

has the amplitude twice as high as in the case of single $+1$ or -1 diffraction order detection, eq. (10). This is due to the out-of-phase relationship between the harmonics I^{β}_{+1} and I^{β}_{-1} at those planes. It follows from the above discussion that the conditions for optimalization of temporal basic harmonics of irradiance changes in ± 1 and 0 double diffraction orders are basically different considering the separation distance between the progressing phase grating and a stationary amplitude grid. The choice of the detection mode should be dictated by the actual experiment conditions.

The other field, where the above developed properties of double diffraction phenomenon find an application, is light modulation. The doubly diffracted -1, $+1$, and 0 order beams can be used for that purpose. The amplitudes of fundamental or second temporal harmonics expressing the depth of modulation can be optimized for the particular values of parameters B, A, and C by making use of the equations developed above.

At the end of this chapter some additional remarks seem to be necessary. In the foregoing analysis the pure amplitude transmittance single-frequency grating was assumed. In practice, however, interference gratings produced photographically with high contrast values are always accompanied by higher harmonics due to the nonlinearities of film reiording process and the phase-relief effects. The relief-free nonlinearity effects provoke the consideration of a general amplitude transmittance grating instead of a sinusoidal one, this will be done in the following. Moreower, the arbitrary contrast single frequency amplitude modulations can be generated from binary gratings by spatial filtering process. On the other hand, the phase relief effects cause the complex amplitude transmittance of an interference type grating. They cannot be separated by a spatial filtering. Unless the index-matching immersion technique is used, the conditions derived for maximalization of temporal harmonics well be changed. The related problem has been recently studied in the case of near Fresnel diffraction field parameters of a complex transmittance grating [13]. The detailed considerations are, however, out of scope of this paper.

2.2. The case of general amplitude transmittance stationary grating

The transmittance of a general amplitude grating is expressed in the form of Fourier series

$$\sum_{n=-\infty} A_n \exp i2\pi n \frac{x}{d}, \tag{16}$$

where d designates, as before, the grating period, and A_n are Fourier coefficients describing the amplitudes of diffraction orders. Now, the calculations similar to the ones given in previous chapter have to be performed in order to find the time-dependent irradiance in double diffraction orders ± 1 and 0.

Below the results for the amplitude grating placed behind and in front of progressing phase modulation are listed up to the second temporal harmonic. It is necessary to say, however, that in the case of a general amplitude grating, different from the so-called Ronchi ruling (e.g. a square-wave grating with equal stripe and space width) higher temporal harmonics than 2 appears in diffraction orders. However, they are of less importance and we will not quote mathematical formulae describing them. The most practical case, e.g. the Ronchi ruling will be obtained as a special case of general transmittance grating. The formulae are as follows:

For the case of amplitude grating located at a distance z behind the phase grating the irradiance harmonics in the zero double diffraction order are

$$
\begin{aligned}
I_0^0 &= A_0^2 J_0^2 + 2\sum_{k=1}^{\infty} A_k^2 J_k^2,\\
I_0^\beta &= \Big\{4A_0 J_0 A_1 J_1 \sin\left(\pi\frac{\lambda z}{d^2}\right) + 4\sum_{k=1}^{\infty} A_k J_k A_{k+1} J_{k+1}\\
&\quad\times \sin\left[(2k+1)\pi\frac{\lambda z}{d^2}\right]\Big\}\cos\beta, \qquad (17)\\
I_0^{2\beta} &= \Big\{-4A_0 J_0 A_2 J_2 \cos\left(4\pi\frac{\lambda z}{d^2}\right) + 2A_1^2 J_1^2 - 4\sum_{k=1}^{\infty} A_k J_k A_{k+2} J_{k+2}\\
&\quad\times \cos\left[4(k+1)\pi\frac{\lambda z}{d^2}\right]\Big\}\cos 2\beta,
\end{aligned}
$$

and in ± 1 diffraction orders

$$
\begin{aligned}
I_{\pm 1}^0 &= A_0^2 J_1^2 + \sum_{k=1}^{\infty} A_k^2 J_{k+1}^2 + \sum_{k=1}^{\infty} A_k^2 J_{k-1}^2,\\
I_{\pm 1}^\beta &= \mp 2A_0 A_1 J_1\left[J_0 \sin\left(\beta\mp\pi\frac{\lambda z}{d^2}\right) + J_2 \sin\left(\beta\mp 3\pi\frac{\lambda z}{d^2}\right)\right]\\
&\quad\mp 2\sum_{k=1}^{\infty} A_k A_{k+1}\Big\{J_{k+1} J_{k+2} \sin\left[\beta\mp(2k+3)\pi\frac{\lambda z}{d^2}\right] \qquad (18)\\
&\quad - J_k J_{k-1} \sin\times\left[\beta\pm(2k-1)\pi\frac{\lambda z}{d^2}\right]\Big\},\\
I_{\pm 1}^{2\beta} &= 2\Big\{A_0 A_2 J_1\left[J_1 \cos 2\beta - J_3 \cos\left(2\beta\pm 8\pi\frac{\lambda z}{d^2}\right)\right]\\
&\quad - \sum_{k=1}^{\infty} A_k^2 J_{k-1} J_{k+1} \cos\left(2\beta\mp 4k\pi\frac{\lambda z}{d^2}\right) - \sum_{k=1}^{\infty} A_k A_{k+2} J_{k-1} J_{k+1}
\end{aligned}
$$

$$\times \cos\left(2\beta \mp 4k\pi\frac{\lambda z}{d^2}\right) - \sum_{k=1}^{\infty} A_k A_{k+2} J_{k+1} J_{k+3}$$
$$\times \cos\left[2\beta \mp (4k+8)\pi\frac{\lambda z}{d^2}\right]\Bigg\}, \tag{18}$$

where, for simplicity, the abbreviation $J_k(B) = J_k$ was used. In the case of amplitude grating located in front of the phase modulation the far field formulae read:

— For the 0 order double diffraction beam

$$I_0'^0 = I_0^0,$$
$$I_0'^\beta = I_0^\beta, \tag{19}$$
$$I_0'^{2\beta} = I_0^{2\beta},$$

— For the ± 1 order beams

$$I_{\pm 1}'^0 = I_{\pm 1}^0,$$
$$I_{\pm 1}'^\beta = \mp 2A_0A_1J_1\left[J_0\sin\left(\beta \pm \pi\frac{\lambda z}{d^2}\right) + J_2\sin\left(\beta \mp \pi\frac{\lambda z}{d^2}\right)\right]$$
$$\mp 2\sum_{k=1}^{\infty} A_kA_{k+1}\Bigg\{J_{k+1}J_{k+2}\sin\left[\beta \mp (2k+1)\pi\frac{\lambda z}{d^2}\right]$$
$$-J_kJ_{k+1}\sin\left[\beta \pm (2k+1)\pi\frac{\lambda z}{d^2}\right]\Bigg\}, \tag{20}$$

$$I_{\pm 1}'^{2\beta} = 2\Bigg\{A_0A_2J_1\left[J_1\cos\left(2\beta \pm 4\pi\frac{\lambda z}{d^2}\right) - J_3\cos\left(2\beta \mp 4\pi\frac{\lambda z}{d^2}\right)\right]$$
$$-\sum_{k=1}^{\infty} A_k^2 J_{k-1}J_{k+1}\cos 2\beta - \sum_{k=1}^{\infty} A_kA_{k+2}J_{k-1}J_{k+1}$$
$$\times \cos\left[2\beta \pm 4(k+1)\pi\frac{\lambda z}{d^2}\right] - \sum_{k=1}^{\infty} A_kJ_{k+1}A_{k+2}J_{k+3}\cos\left[2\beta \mp 4(k+1)\pi\frac{\lambda z}{d^2}\right]\Bigg\}.$$

The case of Ronchi-type binary amplitude grating is obtained by inserting into eqs. (17)–(20) the condition $A_k = 0$ for k even. Therefore, if compared to the single frequency amplitude grating the fundamental temporal harmonic of the irradiances in ± 1 and 0 orders are expressed by the same expressions in these two cases, when noting the correspondences $A_0 = C$, and $A = 2A_{\pm 1}$. Thus, the whole discussion concerning the optimization of the fundamental harmonic amplitude given above for the sinusoidal grating is valid for the Ronchi ruling. These two types of amplitude gratings are most frequently used in practice.

The influence of the separation distance z on the temporal harmonics in 0 and ± 1 diffraction orders in the case of general amplitude transmittance grating can be deduced from eqs. (17)–(20). Its character is very similar to the case of sinusoidal grating discussed in detail before.

— 0 double diffraction order:

The bias components in the cases of amplitude grating placed before or behind progressing phase modulation are the same and independent of the separation distance z. The fundamental component I_0 equals zero at $z = Md^2/\lambda$ (Talbot distances) and takes maximum values at $z = \left(M+\frac{1}{2}\right) d^2/\lambda$ with opposite signs for M even and odd, respectively. Second temporal harmonic assumes identical expressions for $z = Md^2/\lambda$ and $z = \left(M+\frac{1}{2}\right) d^2/\lambda$ for all values of M.

— ± 1 double diffraction orders:

Here the bias components are also independent of the separation distance between the modulations. Fundamental harmonic I^{β}_{+1} and I^{β}_{-1} are mutually out-of-phase at the planes $z = Md^2/\lambda$ and they change sign for M even and odd. At the planes $z = \left(M+\frac{1}{2}\right) d^2/\lambda$ amplitudes of fundamental temporal harmonics in orders $+1$ and -1 are equal for a particular value of M and their sign is opposite for M even as compared to M odd. The second time-dependent harmonic takes the same value for all separation distances $z = Md^2/\lambda$.

Because of the apealing similarity of basic characteristics, just quoted, and characteristics of the case of single-frequency amplitude grating with respect to the influence of the separation distance z, their discussion from the point of view of practical implications can be reffered to Chapter 1. However, mathematical expressions relevant to the case of amplitude grating of general transmittance are much more complex. Therefore, the use of sinusoidal or Ronchi-type binary rulings as stationary amplitude gratings is preferable.

For the sake of completeness, however, it is worthy to mention at the end of this Chapter about the additional possibilities the general amplitude transmittance gratings present over the single frequency gratings. They are characterized by higher diffraction orders and, therefore, their contribution to higher double diffraction orders as well as to all temporal harmonics in every double diffraction order is to be apprehended. In the case discussed in the present work, e.g. the amplitude and phase modulations of the same spatial period higher diffraction orders require higher diffraction beams from the progressing phase grating that are being absent in the case of small amplitudes B of phase modulation. Therefore, they cannot be used for measurement of this amplitude. With respect to other application, e.g. light modulation, by the proper choice of B the light modulation depth can be of appreciable value in higher double diffraction orders. Moreover, the selective detection of higher temporal harmonics in all diffraction beams can be tried.

The numerical studies of the problem of dephasing between positive and negative double diffraction orders of the same number and relative phase relation between increasing orders irradiance distribution have been recently made [9]. Total irradiance curves including all time harmonics were calculated. However, the inspection of the fundamental harmonic is sufficient for this purpose. By using an analytical approach presented in chapter 2 the expressions for $I^{\beta}_{\pm N}$ can be derived. In the case of Ronchi-type binary ruling most frequently used in practice they assume especially simple forms

$$I^{\beta}_{\pm N} = \mp 2A_0A_1\left\{J_{N-1}J_N \sin\left[\beta \mp (2N-1)\pi\frac{\lambda z}{d^2}\right] + J_N J_{N+1}\sin\left[\beta \mp (2N+1)\pi\frac{\lambda z}{d^2}\right]\right\}, \tag{21}$$

and

$$I'^{\beta}_{\pm N} = \mp 2A_0A_1\left\{J_{N-1}J_N \sin\left[\beta \pm \pi\frac{\lambda z}{d^2}\right] + J_N J_{N+1}\sin\left[\beta \mp \pi\frac{\lambda z}{d^2}\right]\right\}. \tag{22}$$

Equations (21) and (22) relate to the cases of Ronchi ruling placed behind and in front of the progressive phase modulation, respectively. They enable a very simple interpretation of the dephasing problem.

If the modulation frequency is a deciding factor then the amplitude grating of a frequency n times higher than that of the phase grating should be used. In such a case $+1$ double diffraction order comprises the n-th diffraction beam from the phase grating and $+1$ st beam from the amplitude one, and their harmonics. The phase of the n-th order beam changes n times faster with the phase modulation displacement than in the case of 1 st order beam exploited when two gratings of the same spatial frequency are used. Therefore, the light modulation frequency will be n times higher than the frequency of progressive grating. The same conclusion based on the numerical analysis was reached in [9]. The analytical expressions for time-dependent irradiance in the far field of dual phase-amplitude gratings of unequal frequencies could be derived similarly to the expressions presented in foregoing chapters. For the both cases very important general remark is to be apprehended: when using higher number double diffraction orders the influence of the gratings separation distance z is n^2 times proportionally greater (n — the number of double diffraction order exploited) than when using ± 1 diffraction orders [3, 6].

3. Conclusions

General expressions for the time-dependent irradiance distributions in 0 and -1 far field double diffraction orders of progressing phase modulation and stationary amplitude grating of the same spatial period and arbitrarily situated in space were developed and discussed. The gratings were assumed to be of unlimited extent and normally illuminated by a plane spatially coherent quasimonochromatic wavefront. The cases of single-frequency and general amplitude transmittance gratings were studied and compared. The influence of gratings separation distance on the amplitude and phase of the temporal harmonics of irradiance in diffraction orders was emphasized. It was found to be related to the so-called Talbot distance known from the theory of self-imaging. The properties of the far field double diffraction pattern change periodically with the increasing separation distance between phase and amplitude periodical modulations, with a period d^2/λ equal to the Talbot distance.

Practical implications of theoretical analysis were presented in the cases of application of double diffraction to determination of very small phase modulations and problems of light modulation. It has been shown that by the proper choice of gratings separation distance the practically exploited parameters of 0 and ± 1 diffraction beams can be optimized.

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Received January 30, 1980

Теория и применение двойной дифракции далёкого поля на системе, состоящей из подвижной решётки с фазовой модуляцией и стационарной амплитудной решётки

Приведены теоретические рассуждения переменных во времени распределений интенсивности в трёх наинизсших порядках дифракции далёкого дифракционного поля системы двух дифракционных решёток: стационарной амплитудной и бегущей фазовой.

Выведены общие формулы для решёток, стоящих в любой очерёдности и на любом расстоянии относительно друг друга. Эти формулы позволяют оптимизировать амплитуду временных гармоний составляющих распределений интенсивности в порядках дифракции 0 и ± 1 посредством подбора межрешётчатого расстояния. Разница в условиях оптимизации связана с так называемым расстоянием Талбота. Обсуждены практические применения выведенных теоретических зависимостей.