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# Influence of the entrance pupil position on the hologram aberration correction*

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In this paper the formulae for the coefficients determining the hologram aberrations have been derived under assumption of arbitrary position of the entrance pupil. The aberrational spots of confusion have been examined numerically for typical cases of hologram recording and reconstruction, at the given position of the entrance pupil.

## 1. Introduction

The position of the entrance pupil is one of the parameters which define the quality of holographic imaging. In the majority of works dealing with the holographic imaging it is assumed that the plane of the entrance pupil coincides with the hologram plane. The influence of the entrance pupil position upon the hologram aberrations, under assumption that the object and the source of the reference wave are positioned on the hologram axis (perpendicular to the hologram plane) assumed to be the $z$ axis of a Cartesian coordinate system, has been discussed in [1], [2]. The examinations of astigmatism for the case of entrance pupil located in front of the hologram were carried out also by SMITH [3]. The purpose of this work is to analyze the influence of the pupil position on all the third-order aberrations of hologram for arbitrary parameters of hologram recording and reconstruction.

## 2. Third-order aberrations of hologram

In the figure 1 the setup for recording and reconstruction of holograms is shown. The points $P$, $R$, $C$, and $P'$ denote the sources of object wave, reference wave, reconstructing wave, and the Gaussian image of the object, respectively; $t$ — denotes the position of the entrance pupil, $2\varrho$ — denotes the diameter of the entrance pupil, $2\bar{\varrho}$ — is the diameter of the active region of the hologram, $\bar{y}$ — is the centre of this active region.

For the sake of simplicity let us assume that the object, the reference wave source and the reconstructing wave source (and hence also the Gaussian image) lie in the $yz$ plane (the $z$ axis being conventional axis of the hologram). If the entrance pupil of diameter $2\varrho$ lies at the distance $t$ from the hologram, the respective radius and the centre of the active region of the hologram are expres-

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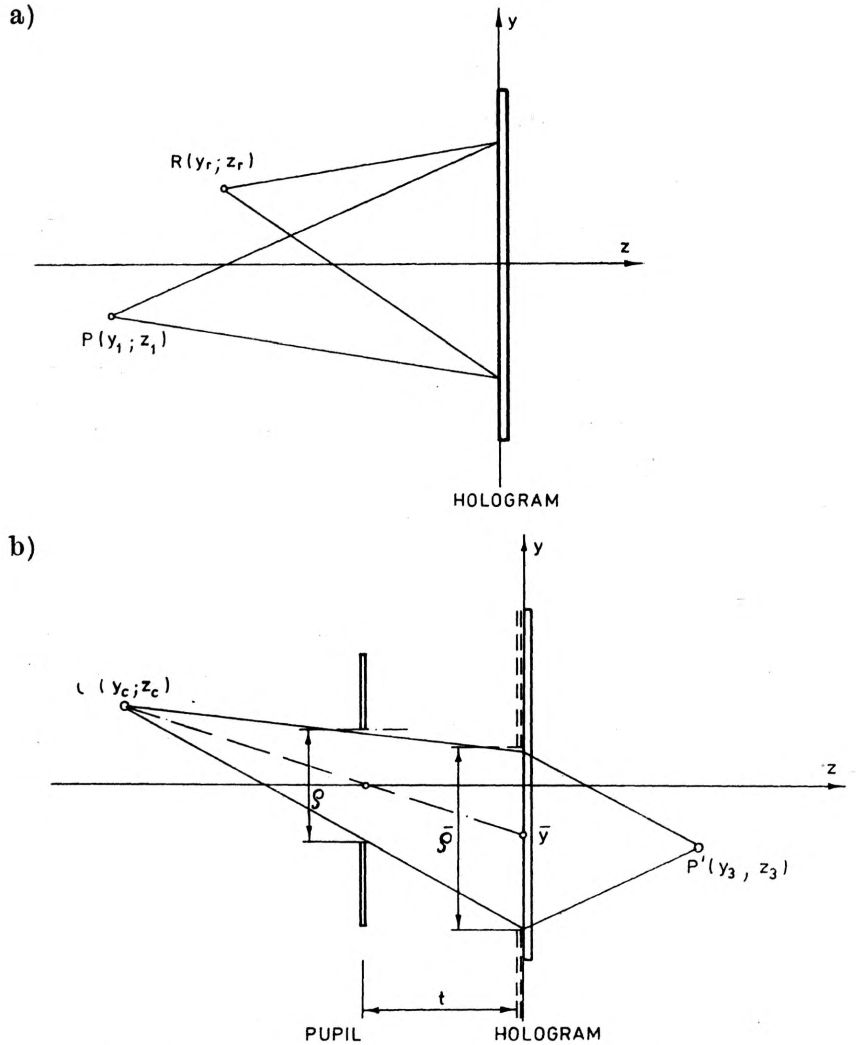


Fig. 1. Geometry of holographic recording (a) and reconstruction (b)

sed by the relations

$$\bar{\varrho} = \varrho \frac{z}{z_c - t}, \tag{1}$$

$$\bar{y} = y_c \frac{t}{t_c - z_c}. \tag{2}$$

Any change in the pupil position corresponds to a shift of the corresponding hologram active region centre to the point $y = \bar{y}$, thus, the influence of the pupil position may be described by changing properly the $y$ coordinates of the object and the sources of reference and reconstructing waves.

According to MEIER [4] the coefficients determining the particular third-order aberrations of hologram are the following:

$$S_1 = \frac{1}{z_c^3} \pm \left(\frac{\mu}{m^4}\right)\left(\frac{1}{z_1^3} - \frac{1}{z_r^3}\right) - \frac{1}{z_3^3}, \tag{3}$$

$$S_2 = \frac{y_c}{z_c^3} \pm \left(\frac{\mu}{m^3}\right)\left(\frac{y_1}{z_1^3} - \frac{y_r}{z_r^3}\right) - \frac{y_3}{z_3^3}, \tag{4}$$

$$S_3 = \frac{y_c^2}{z_c^3} \pm \left(\frac{\mu}{m^2}\right)\left(\frac{y_1^2}{z_1^3} - \frac{y_r^2}{z_r^3}\right) - \frac{y_3^2}{z_3^3}, \tag{5}$$

$$S_5 = \frac{y_c^3}{z_c^3} \pm \left(\frac{\mu}{m}\right)\left(\frac{y_1^3}{z_1^3} - \frac{y_r^3}{z_r^3}\right) - \frac{y_3^3}{z_3^3}. \tag{6}$$

where $m$ denotes the scaling factor of the hologram, and $\mu = \lambda_2/\lambda_1$ ($\lambda_2$ — the wavelength used for reconstruction, $\lambda_1$ — the wavelength used for recording).

If it is assumed that the pupil does not lie in the hologram plane (or lies in this plane but is shifted with respect to the $z$ axis) then the third order aberration coefficients can be calculatated by replacement the respective $y$-coordinates by $y - \bar{y}$. Namely

$$y_1 \to y_c - \bar{y},$$
$$y_r \to y_r - \bar{y},$$
$$y_c \to y_c - \bar{y},$$
$$y_3 \to y_3 - \bar{y}.$$

Now, the aberration coefficients have the form:

$$S_1' = S_1, \tag{7}$$

$$S_2' = S_2 - \bar{y}\left[\frac{1}{z_c^3} \pm \left(\frac{\mu}{m^3}\right)\left(\frac{1}{z_1^3} - \frac{1}{z_r^3}\right) - \frac{1}{z_3^3}\right], \tag{8}$$

$$S_3' = S_3 - 2\bar{y}\left[\frac{y_c}{z_c^3} \pm \left(\frac{\mu}{m^2}\right)\left(\frac{y_1}{z_1^3} - \frac{y_r}{z_r^3}\right) - \frac{y_3}{z_3^3}\right] + \bar{y}^2\left[\frac{1}{z_c^3} \pm \left(\frac{\mu}{m^2}\right)\left(\frac{1}{z_1^3} - \frac{1}{z_r^3}\right) - \frac{1}{z_3^3}\right], \tag{9}$$

$$S_5' = S_5 - 3\bar{y}\left[\frac{y_c^2}{z_c^3} \pm \left(\frac{\mu}{m}\right)\left(\frac{y_1^2}{z_1^3} - \frac{y_r^2}{z_r^3}\right) - \frac{y_3^2}{z_3^3}\right] + 3\bar{y}^2\left[\frac{y_c}{z_c^3} \pm \left(\frac{\mu}{m}\right)\left(\frac{y_1}{z_1^3} - \frac{y_r}{z_r^3}\right) - \frac{y_3}{z_3^3}\right] - \bar{y}^3\left[\frac{1}{z_c^3} \pm \left(\frac{\mu}{m}\right)\left(\frac{1}{z_1^3} - \frac{1}{z_r^3}\right) - \frac{1}{z_3^3}\right]. \tag{10}$$

If we assume that $m = 1$, which is the most common case in holography, then

$$\bar{S}_1 = S_1, \tag{11}$$

$$\bar{S}_2 = S_2 - \bar{y}S_1, \tag{12}$$

$$\bar{S}_3 = S_3 - 2\bar{y}S_2 + \bar{y}^2 S_1, \tag{13}$$

$$\bar{S}_5 = S_5 - 3\bar{y}S_3 + 3\bar{y}^2 S_2 - \bar{y}^3 S_1. \tag{14}$$

The coefficient determining the field curvature $S_4$ (and respectively $\bar{S}_4$) is identically equal to $S_3$ (and respectively to $\bar{S}_3$).

The total third order aberration may now be expressed as follows

$$W(\bar{\varrho}, \bar{y}) = \frac{2\pi}{\lambda}\left(-\frac{1}{8}\bar{S}_1\bar{\rho}^4 + \frac{1}{2}\bar{S}_2\bar{\varrho}^3 \sin\theta - \frac{1}{2}\bar{S}_3\bar{\varrho}^2 \sin^2\theta - \frac{1}{4}\bar{S}_4\bar{\varrho}^2 + \frac{1}{2}\bar{S}_5\bar{\varrho}\sin\theta\right). \tag{15}$$

From the formulae (11)–(13) it follows that all the aberrations except for spherical aberration may be compensated by manipulating the position of the pupil.

It may be seen that the coma may be compensated if

$$\bar{y}_C = S_2/S_1, \quad \text{for} \quad S_1 \neq 0. \tag{16}$$

The astigmatism may be compensated by the pupil position

$$\bar{y}_A = S_3/2S_2, \quad \text{for} \quad S_1 = 0 \tag{17}$$

or

$$\bar{y}_A = \frac{S_2}{S_1} \pm \frac{1}{S_1}\sqrt{S_2 - S_3S_1}, \quad \text{for} \quad S_1 \neq 0. \tag{18}$$

In the second case the inequality $S_2 \geqslant S_3S_1$ must hold.

The distortion disappears if

$$\bar{y}_D = \frac{3S_3 \pm \sqrt{9S_3^2 - 4S_5(3S_2 - S_1)}}{6S_2 - 2S_1}. \tag{19}$$

This is possible only under the following conditions:

$$S_1 \neq 3S_2,$$

$$S_3 \geqslant \frac{2}{3}\sqrt{S_5(3S_2 - S_1)}. \tag{20}$$

## 3. Analysis of selected holographic systems

### 3.1. Recording and reconstruction when using the plane reference and reconstructing waves

As in this case no spherical aberration exists, only the coefficients $S_3$ and $S_5$ depend upon the position of the entrance pupil. The conditions for correcting the astigmatism will be found when using the formulae (5) and (7). Then

$$\bar{y}_A = y_1 + \frac{z_1}{2}\left(\frac{y_c}{z_c} - \frac{y_r}{z_r}\right). \tag{21}$$

Here, it has been additionally assumed that $m = \mu = 1$, and only the primary image was taken into account.

The conditions for the distortion correction may be found taking into accouut of (6) and (19). We obtain

$$\bar{y}_{D1} = y_1 - z_1\frac{y_r}{z_r} \tag{22a}$$

or

$$\bar{y}_{D2} = y_1 + z_1\frac{y_c}{z_c}, \tag{22b}$$

To illustrate the results obtained we will find the shape of the aberration spot (see [5]) for the chosen position of the entrance pupil. The table contains the coordinates of the object, reference wave source, reconstructing wave source and the coefficients characterizing the particular aberrations (all the dimensions are given in mm). The line No. 1 corresponds to the hologram discussed.

Table

| No. | $y_1$ | $z_1$ | $y_r$ | $z_r$ | $y_c$ | $z_c$ | $S_1$ | $S_2$ | $S_3$ | $S_5$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | −100 | −0.2* | | 0.2* | | 0 | 0.00004 | 0.0016 | −0.048 |
| 2 | 0 | −100 | 10 | −100 | 30 | −100 | 0 | 0 | −0.0004 | −0.0180 |
| 3 | 0 | −100 | 0 | −200 | 30 | −200 | 0 | 0.00001125 | 0.0001125 | 0 |

* in this case $\frac{y_r}{z_r} = -0.2$ $\frac{y_c}{z_c} = 0.2$

From the formulae (21) and (22) we find that the simultaneous correction of astigmatism and distortion occurs for $y = -20$ mm, which corresponds to the position of the pupil in front of the hologram ($t = -100$ mm). In the figure 2 the shape of aberration spot is shown in the Gaussian plane for the defined position of the entrance pupil. It has been assumed that the active region of the hologram is of constant area $10 \times 10$ mm. It may be really seen that for $t = -100$ mm the aberration is the least. It is also visible that although the distortion is formally perfectly corrected, the centre of the aberration spot

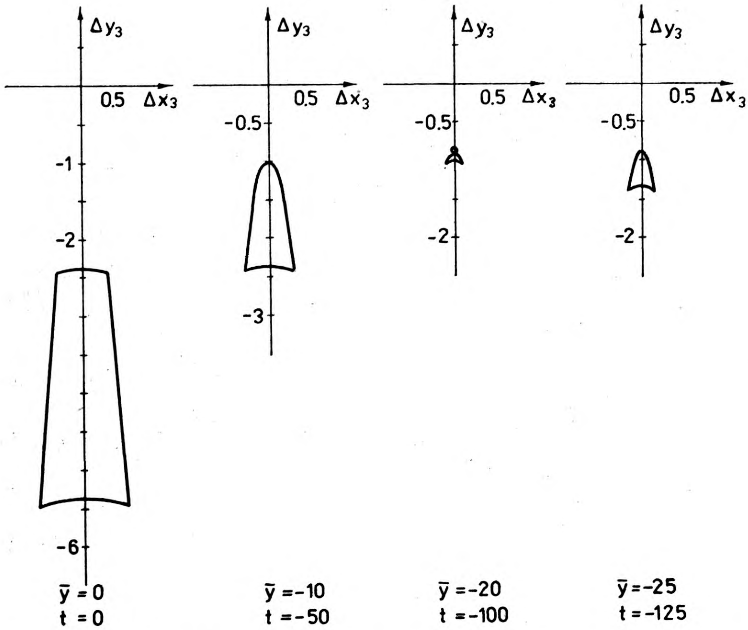


Fig. 2. The shape of aberration spot for chosen positions of the pupil for the system number 1

as well as the whole spot are shifted with respect to the aberrationless image. This is due to the fact that coma which remained unchanged is a nonsymmetrical aberration while our correction concerns only the third order aberrations.

### 3.2. Quasi-Fourier hologram

In this case $z_1 = z_r$. In the face of (5) and (17), at the same assumption as before, we obtain the following condition for astigmatism correction

$$y_A = \frac{y_c\left(\frac{z_1}{z_c}\right)^2 - \frac{y_r - y_1}{2}\left(\frac{z_1}{z_c}\right) - \frac{y_r + y_1}{2}}{\left(\frac{z_1}{z_c}\right)^2 - 1}. \tag{23}$$

If $z_1 = z_c$, astigmatism cannot be compensated, since the spherical aberration and coma disappear in accorcance with (6) and (19).

The compensation of distortion will be obtained if

$$\bar{y}_{D1} = \frac{2\left[y_c\left(\frac{z_1}{z_c}\right)^2 + (y_1 - y_c)\left(\frac{z_1}{z_c}\right)\right]}{\left(\frac{z_1}{z_c}\right)^2 - 1} \tag{24a}$$

or

$$\bar{y}_{D2} = \frac{2y_c\left[\left(\frac{z_1}{z_c}\right)^2 + (y_c - y_r)\left(\frac{z_1}{z_c}\right) + (y_1 - y_r)\right]}{\left(\frac{z_1}{z_c}\right)^2 - 1}, \text{ for } z_1 \neq z_c, \tag{24b}$$

or

$$\bar{y}_{D3} = \frac{y_c + y_1}{2}, \quad \text{for} \quad z_1 = z_c. \tag{24c}$$

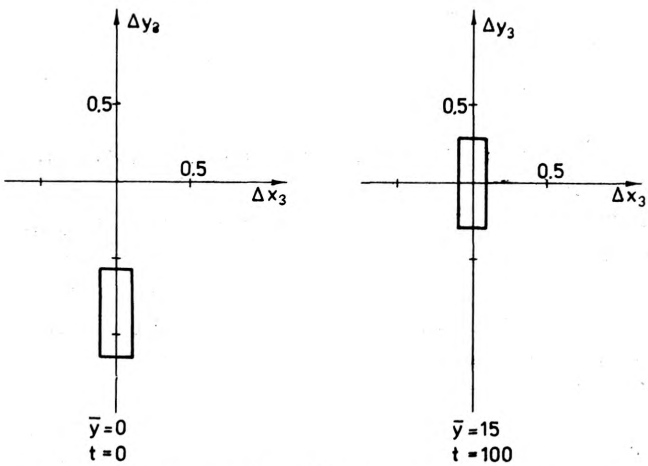


Fig. 3. The shape of the aberration spot for chosen position of the pupil for the system number 2

In the line No. 2 of the table the parameters for the selected quasi-Fourier hologram have been given. In this case ($z_1 = z_c$) only the distortion may be compensated by change of the entrance pupil position and in accordance with the formulae (24c) this should occur at $\bar{y} = 15$ mm, which corresponds to the position of the entrance pupil $t = -100$ mm. In fig. 3 the shape of aberration spot in the Gaussian plane is shown for the pupil lying in the hologram plane as well as for the pupil position $t = -100$ mm. It may be seen that the shape of the aberration spot are essentially unchanged. This is due to unchanged astigmatism, while its position for $t = -100$ mm is symmetrical with respect to the Gaussian image. The better correction of distortion in this case is due to the fact that now $S_5 = -0.018$, while previously it was $S_5 = -0.048$. Thus the distortion of higher orders is less.

### 3.3. In line hologram

Now, let us assume that $y_1 = y_r = 0$. The hologram acts as a lens of focussing power $1/F = 1/z_r - 1/z_1$. In this way no distortion occurs, while the astigmatism and coma must be compensated by the change in the pupil position.

If no spherical aberration exists ($z_c = z_r$) the condition for compensation of astigmatism has the form

$$\bar{y}_A = \frac{y_c}{2\left(2 - \frac{z_c}{z_r} + \frac{z_c}{z_1}\right)}. \tag{25}$$

In this case the coma is constant.

If, however, the spherical aberration exists, the coma may be compensated by the entrance pupil position given by the formula

$$\bar{y}_c = y_c\left[-1 + \frac{z_1 z_r + z_c}{(z_c^2 + z_c)(z_c - z_r)}\right]. \tag{26}$$

For the correction of astigmatism the respective condition is complicated and it is better to use the general formula (18).

In the line No. 3 of the table the parameters of the selected in line hologram are given. In this case the change of the entrance pupil position may improve only correction of astigmatism which in accordance with (25) disappears when $\bar{y} = 5$ mm, which corresponds to $t = 40$ mm. This particular case has been analysed in the paper [1], and the results obtained therein were analogical, though the method used was quite different.

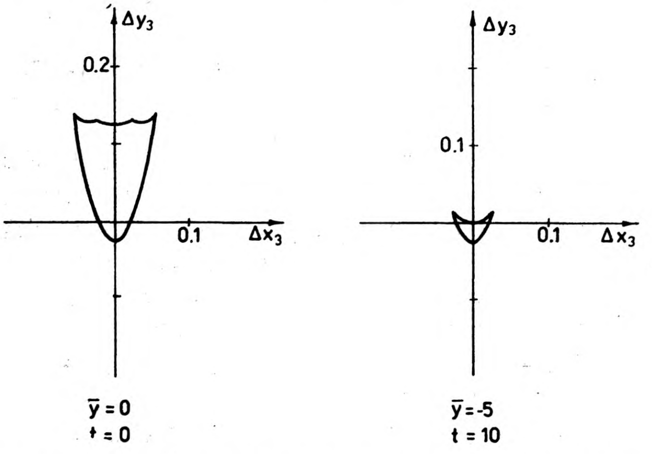


Fig. 4. The shape of the aberration spot for chosen position of the pupil for the system number 3

In the figure 4 the shape of the aberration spot in the Gaussian plane has been shown for the pupil lying in the hologram plane and the pupil lying to the right from the hologram for which $t = 40$ mm. The aberration spot for the second case is much less (the effect of astigmatism compensation for unchanged coma) and in both the case the Gaussian image occurs within the spot region (lack of distortion).

## References

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## Влияние положения входного зрачка на коррекцию аберрации голограммы

В работе выведены формулы для коэффициентов, определяющих аберрации голограммы, при до пущении произвольного положения входного зрачка. Проанализировано влияние положения зрачка на величины отдельных аберраций. Численно были исследованы аберрационные пятна для типичных случаев регистрации и реконструкции голограмм при заданном положении входного зрачка.