Optica Applicata, Vol. XII, No. 3-4, 1982

Numerical method for the calculation of the light intensity distribution in the holographic image*

JERZY NOWAK, MAREK ZAJĄC

Institute of Physics, Technical University of Wrocław, Wybrzeże Wyspiańskiego 27, 50-370 Wrocław, Poland.

The imaging quality may be estimated by the evaluation of the aberration spot being an imperfect image of the point object. In classical optics the aberration spot shape is calculated with the help of the "ray tracing" method. In holographic imaging it should be taken into account, however, that the light used for image reconstruction is coherent. In this paper an algorithm enabling us to calculate the light intensity distribution in the holographic image of a point object is presented and the accuracy of the results obtained is discussed.

1. Introduction

In the holographic imaging the image quality assessment is a problem of major importance. This quality may be assessed via examination of the coefficients describing particular aberrations [1-3], similarly as it is usually done in classical optics. Some goal may be achieved by calculating the wave aberration [4] instead of the geometrical ones. The holographic image quality can be also evaluated by means of the "ray tracing" calculations [5, 6]. In [6] an attempt has been made to estimate the intensity distribution in the image plane based on the "ray tracing" algorithm. The latter investigations have shown, however, that the method proposed there leads sometimes to uncorrect results. The results of calculations based on the "ray tracing" algorithm presented in [6] depend in a substantial degree on the manner in which the computations are conducted, being moreover ununiquely related to the parameters of holographic recording and reconstruction geometry.

* This work has been carried on under the Research Project M.R. I.5, and presented at the XXVII Congress of the Polish Physical Society in Lublin, 21-25 September, 1981.

The new method of calculations of the light intensity distribution in the image plane which seems to be accurate enough and to give univocal results is presented herein.

2. Method for the calculations of the light intensity in the image plane

The geometry of hologram recording adopted here is presented in the Fig. 1. $P(x_1, y_1, z_1)$ and $R(x_R, y_R, z_R)$ denote the point object and the reference wave source, respectively. The hologram is located in the $(x', 0, y')$ plane, $O(0, 0, 0)$ denotes the hologram centre, and $H(x', y', 0)$ – an arbitrary point on its surface.

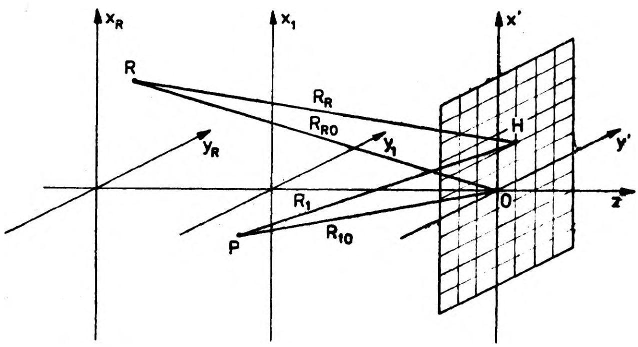

Fig. 1. Geometry of hologram recording

The interference pattern of the object and reference waves is recorded and forms a hologram. The phase difference between the waves originating from the object point and the reference source on the hologram is described by

$$\varphi_1(H) = \frac{2\pi}{\lambda_1}(R_1 - R_R), \tag{1}$$

where λ_1 stands for the wavelength of the light used during recording, R_1 and R_R are the distances $\overline{PH}$ and $\overline{RH}$, respectively.

In order not to operate with great numbers it is convenient to normalize the phase difference by subtracting from φ_1 the phase difference corresponding to the middle-point of the hologram

$$\varphi_2(H) = \frac{2\pi}{\lambda_1}\left[(R_1 - R_R) - (R_{10} - R_{RO})\right], \tag{2}$$

where R_{10}, R_{RO} are the distances $\overline{PO}$ and $\overrightarrow{RO}$, respectively.

The geometry of image reconstruction is presented in the Fig. 2. The points $C(x_C, y_C, z_C)$ and $P'(x_3, y_3, z_3)$ denote the reconstructing wave point source and the investigated point in the image, respectively. The hologram plane is $(x, 0, y)$, the Gauss image plane being $(x_3, 0, y_3)$.

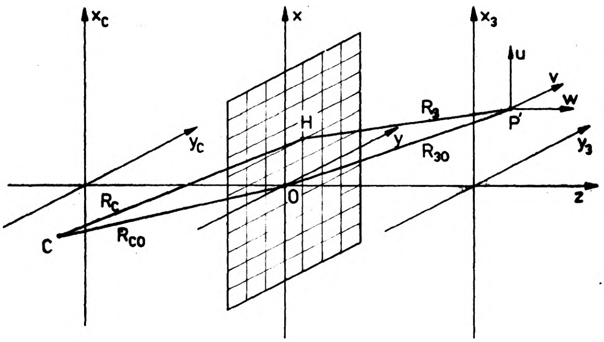

Fig. 2. Geometry of image reconstruction

Since the possibility of the hologram scaling should be admitted the following relation is assumed

$$x/x' = y/y' = m. \tag{3}$$

If the wavelength of the light used during image reconstruction (λ_2) differs from that used when hologram recording (λ_1) then their ratio is denoted by

$$\mu = \lambda_2/\lambda_1. \tag{4}$$

While travelling along the path CH the phase of the reconstructing wave changes by φ_3

$$\varphi_3(H) = \frac{2\pi}{\lambda_2}(R_C \pm R_3), \tag{5}$$

where the upper (+) and lower (–) signs refer to the real and imaginary image, respectively.

This sign depends on whether the image-forming light wave is convergent (real image placed on the right-hand side of the hologram) or divergent (the imaginary image on the left-hand side of the hologram), being independent of the fact whether the image is primary or secondary.

Similarly as during recording we will operate with a "normalised" phase difference

$$\varphi_4(H) = \frac{2\pi}{\lambda_2}\left[(R_C \pm R_3) - (R_{CO} \pm R_{3O})\right]. \tag{6}$$

The distances R_C, R_3, R_{CO}, R_{3O} are as follows:

$$R_C = \overline{CH}, \quad R_3 = \overline{HP}', \quad R_{CO} = \overline{CO}, \quad R_{3O} = \overline{HO}.$$

Let us now consider a ray originating from the reconstructing point source R, piercing the hologram in the point H, and striking the image plane in the point P′. To this ray a phase may be ascribed

$$\varphi(H, P') = \varphi_2 + \varphi_4, \tag{7}$$

and an amplitude, dependent on the angle α describing the inclination of the ray with respect to the hologram plane attached

$$A(H, P') = \sqrt{z_C/R_C}. \tag{8}$$

To calculate the amount of light in the given point of the image it is enough to sum-up the rays that intersect the hologram in a number of points and fall onto this point. To this end we construct a rectangular lattice in the hologram plane $(x', 0, y')$ (as well as in the plane $(x, 0, y)$ - but properly scaled), and calculate the phase $\varphi(H,P')$ and amplitude $A(H, P')$ corresponding to the rays passing the nodes of this lattice.

The resultant light intensity in the image plane is obtained by "coherent" summing-up the constructions from all the rays

$$I'(x_3,y_3,z_3) = \left(\sum_{i=1}^{n_x}\sum_{j=1}^{n_y} A_{ij}\cos\varphi_{ij}\right)^2 + \left(\sum_{i=1}^{n_x}\sum_{j=1}^{n_y} A_{ij}\sin\varphi_{ij}\right)^2, \tag{9}$$

where subscripts $i = 1, \ldots, n_x$, $j = 1, \ldots, n_y$ mark the nodes of the lattice constructed on the hologram. It is convenient to operate with a normalized intensity obtained by dividing the value calculated from the formula (9) by the square of total number of rays taken into account

$$I(x_3, y_3, z_3) = I'(x_3, y_3, z_3)/(n_x\, n_y)^2. \tag{10}$$

Such normalization allows us to receive in the centre of aberration-free image the value of light intensity equal to 1.

3. Numerical results

A number of numerical examples have been adopted to test the proposed algorithm. The light intensity distribution in the Gauss image plane has been calculated for the geometry of recording and reconstruction parameters, indicated in Table 1 (all dimensions in the table as well as in the following text are given in millimeters). To shorten the computing time the calculations have been carried out for one dimensional holograms, although the same programme remains valid for the case of a 2-d hologram in a 3-d space.

T a b l e 1. Parameter of the hologram recording and image reconstruction geometry

No.	P			R			C			P'			m/μ
	x_1	y_1	z_1	x_R	y_R	z_R	x_C	y_C	z_C	x_3	y_3	z_3	
1	0	0	-100	0	5	-100	0	5	-100	0	0	-100	1
2	0	0	-100	0	0	-200	0	30	-200	0	15	-100	1

The first example concerns a fixed hologram 2 mm wide (No. 1 in Table 1). The light intensity was determined in several points lying in the plane of primary image which is imaginary and aberration free. The coordinates of those points with respect to the Gauss image are: $v = 0$, 0.02, 0.04, 0.06, 0.08, respectively. The computations were performed for several numbers of rays taken into account. The density of the points in which those rays intersect the hologram varied from 0.48 to 0.00375, which corresponds to the number of rays $N = 5$ and $N = 534$. The results shown in Fig. 3 allow us to analyse the dependence of the stabilisation of the numerical results on the number of rays. For the sake of convenience a logarithmic scale on the abscissa has been adopted. In order not to complicate the picture the curve for $v = 0$ was not shown. It may be easily seen from this figure that in this case about 50 rays taken for calculations is a quite satisfactory number, as the calculated intensity does not differ more than by about 5% from the value which would be obtained for the infinitely many rays.

The next test should ascertain whether for the properly chosen number of rays resultant intensity in the image depends on the choice of the nodes location in the hologram. A fixed step of the hologram division was equal to $\Delta y = 0.03$, and the coordinate of the initial node was changed by the value much less than Δy. The light intensity values calculated for several points in the image are presented

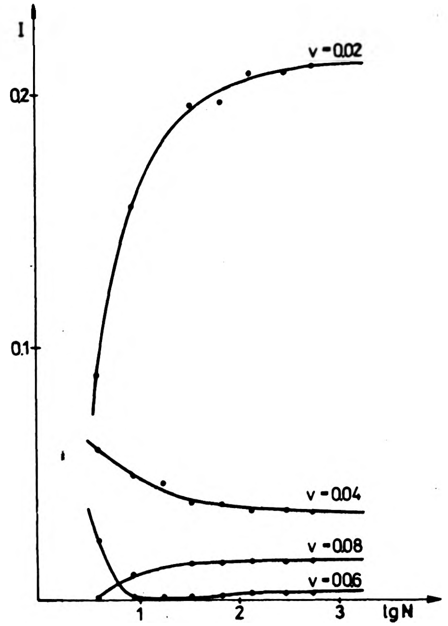

Fig. 3. The calculated light intensity vs. the number of rays taken into account - hologram No. 1

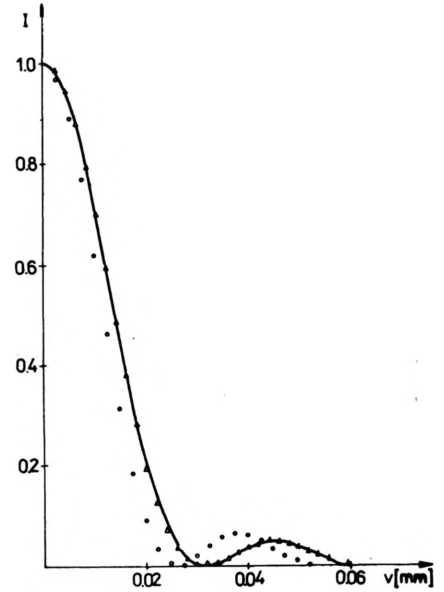

Fig. 4. Light intensity distribution in the image calculated for different numbers of rays taken into account - hologram No. 1 (oooo - $\Delta y = 0.48$, $N = 5$; —— $\Delta y = 0.00375$, $N = 534$; ▲▲▲ $\Delta y = 0.06$, $N = 34$)

in Table 2. The difference between the results being of order of 10^{-3}%, may be neglected.

Then the intensity distribution in the Gauss image for the same hologram as in the first two cases has been calculated, taking the numbers of rays equal to 5, 34, and 534. The results are plotted in the Fig. 4. It is seen that the results corresponding to $N = 34$ and $N = 534$ practically coincide. This allows us to believe that the necessary number of rays obtained on the base of the Fig. 3 may be even reduced.

Table 2. Light intensity distribution calculated for the same number of rays, but differently distributed on a hologram

v = 0.02	v = 0.04	v = 0.06	v = 0.08
0.1963956	0.03792369	0.00114285	0.01436014
0.1963946	0.03792335	0.00114276	0.01436023
0.1963942	0.03792323	0.00114281	0.01436014
0.1963948	0.03792342	0.00114278	0.01435997

In the following test the effect of hologram size increasing at the constant density of rays has been investigated. The image analysed being aberration free, the light intensity distribution in this image should depend only on the diffraction of the hologram

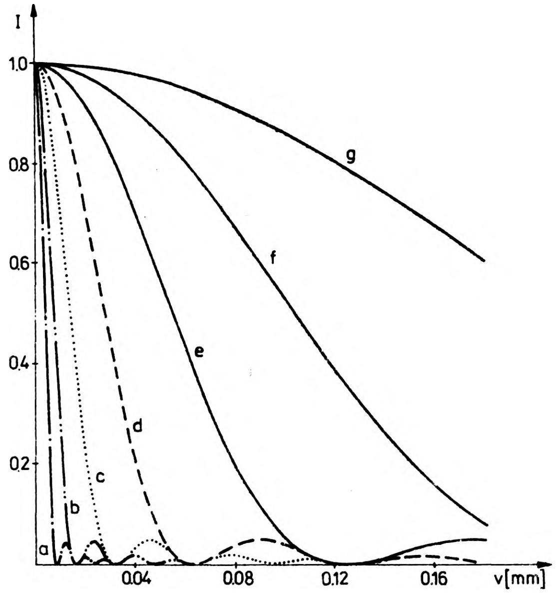

Fig. 5. Light intensity distribution in the image calculated for holograms of different width - hologram No. 1. Hologram width: a - 8 mm, b - 4 mm, c - 2 mm, d - 1 mm, e - 0.5 mm, f - 0.25 mm, g - 0.125 mm

aperture. In our case (one-dimensional hologram) this distribution should be described theoretically by a $(\text{sinc})^2$ function. Figure 5 shows that the numerical results are in good consistence with this prediction. For example, the dark fringes in a Fraunhoffer diffraction pattern of an empty slit 2 mm wide should be placed in the points $v_1 = 0.03164$, $v_2 = 0.06328$, $v_3 = 0.09492$, etc. which is in a perfect agreement with the positions of minima in the calculated light intensity distribution in the image obtained from 2 mm wide hologram (curve c). As it should be expected, with increasing hologram size the image is getting more similar to δ Dirac function (point image).

In the following analysis a second example from the Table 1 is employed. The hologram size is 4 mm.

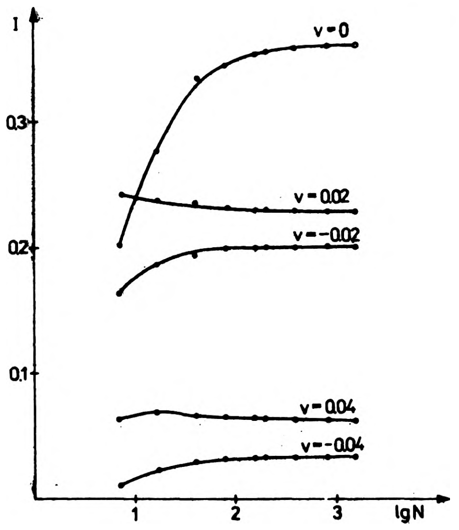

Fig. 6. The calculated light vs. the number of rays taken into account - hologram No. 2

The image of interest is also primary one; it is virtual and with substantial aberrations. The aberration coefficients [2] have the following values: $S = -3\cdot10^{-8}$, $C_y = 1.2\cdot10^{-6}$, $A_y = 2.2\cdot10^{-4}$.

In Figure 6 the results of the searching for the minimum necessary rays density are presented in the way analogical to this in the Fig. 3. The estimation of necessary number of rays gives a similar result as in the aberration-free case. The sufficient rays density seems to be equal to about $y = 0.05$ mm (for this particular geometry of recording and reconstruction). The same conclusion may be drawn from the Fig. 7

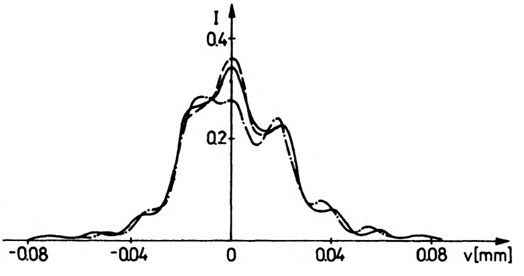

Fig. 7. Light intensity distribution in the image calculated for different numbers of rays taken into account - hologram No. 2. (- . - $\Delta y = 0.25$, $N = 17$;, —— $\Delta y = 0.05$, $N = 81$; - - - $\Delta y = 0.0005$, $N = 1601$)

in which the curves show the light intensity distribution computed for the numbers of rays $N = 17$, 81, and 1601. The last two curves practically overlap.

It can be seen in this figure that the form of image light distribution results from the aberrations and the diffraction on the hologram aperture. The aberration spot is convolved with the pure diffraction limited image as it is in real imaging.

Summing up, it may be stated that the algorithm for the calculations of the light intensity distribution in an arbitrary image plane presented here gives the satisfactory results. The number of rays necessary for computations remains within the reasonable limits and so is the computing time. The applications of this algorithm will be the subject of the next paper.

References

[1] MEIER R.W., J.Opt.Soc. Am. 55 (1965), 987.
[2] CHAMPAGNE E.B., J.Opt.Soc.Am. 57 (1967), 51.
[3] LATTA J.N., Appl.Opt. 10 (1971), 666.
[4] MILES J.F., Optica Acta 20 (1973), 19.

[5] OFFNER A., J.Opt.Soc. Am. 56 (1966), 1509.
[6] NOWAK J., ZAJĄC M., Optik 55 (1980), 93.

Received November 14, 1981

ЧИСЛЕННЫЙ МЕТОД РАСЧЕТА РАСПРЕДЕЛЕНИЯ ОСВЕЩЕННОСТИ В ГОЛОГРАФИЧЕСКОМ ИЗОБРАЖЕНИИ

Качество отображения можно оценить, определяя форму аберрационного пятна, являющегося несовершенным изображением точного предмета. В классической оптике форма аберрационного пятна определяется, применяя метод "слежения хода лучей". В голографическом отображении следует учесть факт, что в реконструкции изображения используется когерентный свет. В работе предложен алгоритм, позволяющий определить распределение интенсивности света в голографическом изображении точечного предмета и обсуждена точность достигаемых результатов.