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A non-reciprocal optical effect in optical gyroscope*

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In a fibre-optic rotation sensor a non-reciprocal phase shift modulation is usually introduced to determine the difference of phase shifts produced by rotation of the sensor. To prevent lock-in in ring laser gyroscopes some frequency bias technique with non-reciprocal propagation properties should be also used. Faraday effect, polar and transverse magnetooptical Kerr effects and optical activity of quartz rotator employed in a four frequency differential laser gyro and magnetic mirror laser gyro, can be described by a generalized dielectric tensor containing off-diagonal and symmetrical conjugate complex elements.

This paper deals with the basic form of the wave-vector surface and normal mode of non-reciprocal gyrotropic dielectric tensor. The non-reciprocal effects have been discussed and their applications to laser gyro presented. Some errors coming from the non-reciprocal elements employed have also been analysed theoretically.

1. Introduction

When a ring plane rotates in inertial space with a rotation rate Ω perpendicular to the laser cavity plane, a frequency difference between two counterpropagating travelling waves CW and CCW due to Sognac effect will be generated and can be expressed by

$$\Delta\nu = \nu_{\mathrm{CW}} - \nu_{\mathrm{CCW}} = \frac{4S}{\lambda L}\Omega$$

where S is the enclosed area, L — the optical length of RLG, and λ is the optical wavelength. This is the principal formula to describe the fact that the ring laser is used as a rotation sensor. The gain process of active medium in the laser cavity makes the bandwidth of laser light much narrower than that of the empty cavity. Thus, the measuring error of frequency difference $\Delta\nu$ has been reduced significantly. At present the random drift of the output signal of a laser gyro is about 10^{-3} deg/h (sample time $\tau = 1$h), which is nearly equal to the accuracy limited by quantum noise from spontaneous radiation.

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Due to the backscattering and nonhomogeneous losses in the ring laser cavity a coupling between two counterpropagating travelling waves will exist, hence the output signal will vanish for the low rotating rate $|\Omega| < \Omega_L$, Ω_L being the lock-in threshold about 10^3–10^4 deg/h. This is a well-known lock-in effect shown in Fig. 1. Thus, to prevent laser gyro from working in the lock-in region it is necessary to use the bias technology. There are three main means of bias, namely: mechanical dithering, magnetic mirror (using the transverse magneto-optic Kerr effect) and four-frequency differential laser gyro (using

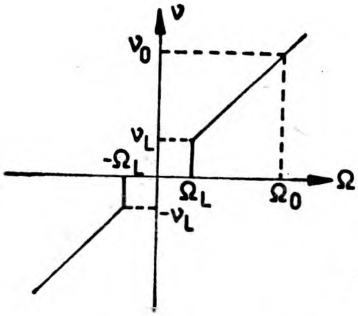

Fig. 1. Principle curve and lock-in region in laser gyro

the Faraday effect). They all use the effects based on one of non-reciprocal effects for two counterpropagating travelling waves. Due to the non-reciprocal effect there occurs a non-reciprocal phase shift $\Delta\Phi$ between two travelling waves, resulting in their frequency difference, $\nu_0 = \Delta\Phi C/2\pi L$ where C is the light velocity. If this non-reciprocal effect is strong enough, the produced bias frequency ν_0 may be much greater than lock-in threshold ν_L $\left(\nu_L = \frac{4S}{\lambda L}\,\Omega_L\right)$, see Fig. 1, hence, application of bias technology in a laser gyro is of a great importance.

In a four-frequency laser gyro [1, 2] two left and right circularly polarized laser gyros operate in the same ring cavity, in which four different waves are travelling. A 90° quartz rotator (see Fig. 2) is used to provide the difference in

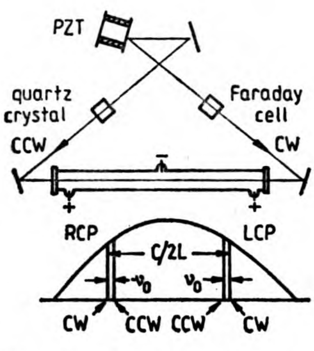

Fig. 2. Principle scheme of four-mode laser gyro

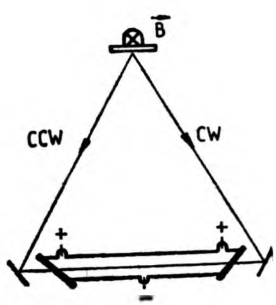

Fig. 3. Principle scheme of magnetic mirror laser gyro

phase shift between the left and right circularly polarized (LCP and RCP) waves due to their non-reciprocity. Hence, the frequencies of LCP and RCP laser gyro are split from each another in a half of mode separation ($C/2L$). A Faraday cell with longitudinal magnetic field can also be used, as its non-reciprocity provides left and right circularly polarized gyros with a bias $\pm\nu_0$, equal in magnitude and opposite in sign. Then the left and right polarized gyros shift off symmetrically from the lock-in region. At this time the frequencies of output signals of the left and right circularly polarized gyros are given by $\nu_L = \nu_0 + \frac{4S}{\lambda L}\Omega$, $\nu_R = \nu_0 - \frac{4S}{\lambda L}\Omega$, respectively. The final differential frequency is $\Delta\nu = \nu_L - \nu_R = \frac{8S}{\lambda L}\Omega$, in this expression the term of Faraday bias is cancelled leaving only the information related to the rotation rate Ω. Because of the direct bias in four-frequency laser gyro, the measuring range of gyro is limited by the magnitude of bias ν_0. In practice, its magnitude is about 1 MHz.

As to a laser gyro with magnetic mirror bias (Fig. 3) a ferromagnetic mirror (rare earth-iron garnet, for instance) coated by a stock of dielectric thin film has been used. It provides a non-reciprocal reflective coefficient for P linearly polarized light. Hence, a non-reciprocal phase difference $\Delta\Phi = \Phi_{\mathrm{CW}} - \Phi_{\mathrm{CCW}}$ is introduced when a reflection takes place. An alternate bias occurs when the magnetic field is applied alternatively. The laser gyro with magnetic mirror usually works at an alternative bias. Hence, a bias should be greater than lock-in threshold ν_L by 1–2 orders of magnitude, i.e., ν_0 is about 50 KHz.

All the non-reciprocal effects mentioned above can be characterized by a dielectrical tensor (containing off-diagonal and symmetrical conjugate complex elements $\pm i\varepsilon_{12}$, called a non-reciprocal gyrotropic tensor

$$\overleftrightarrow{\varepsilon} = \varepsilon_1 \begin{bmatrix} \varepsilon_{11} & i\varepsilon_{12} & 0 \\ -i\varepsilon_{12} & \varepsilon_{22} & 0 \\ 0 & 0 & \varepsilon_{33} \end{bmatrix} \tag{1}$$

where ε_0 is the permittivity of the vacuum (MKS units) in the case of the magnetic field or crystal axis running along the z-axis. As it is well-known, the energy loss due to absorption is in nonisotropic medium proportional to $\sum\limits_{i,j=1}^{3} (\varepsilon_{ij}^* - \varepsilon_{ij}) E_j^* E^*$. Therefore, if there is no absorption in a non-reciprocal medium this gyrotropic tensor $\overleftrightarrow{\varepsilon}$ must be of Hermitian type, i.e., $\varepsilon_{ij} = \varepsilon_{ji}^*$. It means that all the diagonal elements must be real, i.e., $\varepsilon_{ii} = \varepsilon_{ii}^*$, and all the off-diagonal elements $\pm i\varepsilon_{12}$ must be purely imaginary ones, i.e., ε_{12} is a real number. The elements of gyrotropic tensor $\overleftrightarrow{\varepsilon}$ of the medium with absorption ε_{ii} and ε_{12} are complex. Their imaginary parts are related to the coefficient of absorption. In this case, tensor $\overleftrightarrow{\varepsilon}$ may be resolved in the sum of Hermitian and anti-Hermitian tensors (anti-Hermitian tensor satisfies $\varepsilon_{ij} = -\varepsilon_{ji}^*$).

The Faraday cell used in a laser gyro is made of a sort of glass with isotropic characteristic. But for the rare earth-iron garnet crystal, which belongs to the cubic crystal system, it keeps $\varepsilon_{11} = \varepsilon_{22} = \varepsilon_{33} = N_o^2$. The quartz rotator is a single-axis crystal, hence, $\varepsilon_{11} = \varepsilon_{22} = N_o^2$ and $\varepsilon_{33} = N_e^2$. In the sequel we can treat it as a single-axis crystal. For cubic crystal, it suffices only to put $N_e = N_o$.

2. Propagation of light in crystal characterized by a gyrotropic tensor – normal mode and wave vector

The dielectric tensor $\overleftrightarrow{\varepsilon}$ is expressed as

$$\overleftrightarrow{\varepsilon} = \varepsilon_0 \begin{bmatrix} N_o^2 & i\varepsilon_{12} & 0 \\ -i\varepsilon_{12} & N_o^2 & 0 \\ 0 & 0 & N_e^2 \end{bmatrix}. \tag{2}$$

The general wave equation, which is satisfied by the electrical vector of light wave, may be written as

$$\nabla \times (\nabla \times \vec{E}) + \frac{1}{C^2}\frac{\partial^2 \vec{E}}{\partial t^2} = -\frac{1}{C^2}\overleftrightarrow{\chi}\frac{\partial^2 \vec{E}}{\partial t^2} \tag{3}$$

where χ is susceptibility tensor. We introduce a displacement vector, written as $\vec{D} = \varepsilon_0(1+\overleftrightarrow{\chi})\vec{E} = \overleftrightarrow{\varepsilon}\vec{E}$. By substituting this expression into (3), we obtain

$$\nabla \times (\nabla \times \vec{E}) + \frac{1}{C^2}\frac{\overleftrightarrow{\varepsilon}}{\varepsilon_0}\frac{\partial^2 \vec{E}}{\partial t^2} = 0. \tag{4}$$

If a monochromatic plane $\vec{E}e^{i(\omega t - \vec{k}\vec{r})}$, whose wave vector direction is $\vec{K} = \vec{k}/k = \{a_1, a_2, a_3\}$, can maintain its polarization mode, while travelling through a non-reciprocal crystal characterized by a gyrotropic tensor, then this wave is called normal mode in $\vec{K}$ direction of the crystal. In order to determine the polarizatin of the normal mode electrical vector $\vec{E}$ and the corresponding refraction index n or the wave-number $k = k_0 n$ we substitute the monochromatic plane wave into (4) and obtain and expression related to $\vec{E}$

$$\vec{k}(\vec{k}\vec{E}) - k^2\vec{E} + k_0\frac{\overleftrightarrow{\varepsilon}}{\varepsilon_0}\vec{E} = 0 \tag{5}$$

where $k_0 = \omega/C$ is the wave-number in vacuum of this monochromatic wave, expression (5) can be written in matrix form

$$\begin{bmatrix} k_0^2 N_o^2 - k^2(1-a_1)^2 & k^2 a_1 a_2 + ik_0^2\varepsilon_{12} & k^2 a_1 a_3 \\ k^2 a_1 a_2 - ik_0^2\varepsilon_{12} & k_0^2 N_o^2 - k^2(1-a_2^2) & k^2 a_2 a_3 \\ k^2 a_1 a_3 & k^2 a_2 a_3 & k_0^2 N_e^2 - k^2(1-a_3^2) \end{bmatrix}\begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = 0, \tag{6}$$

and get linear homogeneous equations of E_x, E_y and E_z. If the determinant of coefficients vanishes, there exists a nontrivial solution, then we obtain an equation in fourth order of wavenumber k of normal mode, i.e., a Fresnel wave-vector surface equation. There will be two values of k^2 in any direction of wave-vector $\vec{K}$. Putting these values back into (6) we obtain the polarization mode of the corresponding normal mode, that will be discussed in detail in the following Sections.

3. Quartz-rotator effect and Faraday effect of magneto-rotator

Both the mentioned above cases are similar, since the wave-vector $\vec{K}$ follows the light axis or the direction of magnetic field i.e., z-axis. Hence, $a_1 = a_2 = 0$, $a_2 = 1$, by substituting it into (6) we obtain

$$\begin{bmatrix} k_0^2 N_o^2 - k^2 & ik_0^2 \varepsilon_{12} & 0 \\ -ik_0^2 \varepsilon_{12} & k_0^2 N_o^2 - k^2 & 0 \\ 0 & 0 & k_0^2 N_e^2 \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = 0, \tag{7}$$

$E_z = 0$ means that the electrical vector is perpendicular to $\vec{K}$, i.e., that the normal modes are transverse waves. The condition for the existence of nontrivial solution is $k_\pm^2 = k_0^2(N_o^2 \pm \varepsilon_{12})$, or $k_\pm = k_0(N_o^2 \pm \varepsilon_{12})^{1/2}$, if only its positive values are taken. Putting it back into equation (7), we find out the correspondent polarization to be $\{1, \pm i, 0\}$. It means that the normal modes are left and right circular polarized light and that they both are perpendicular to each other. The difference in their refraction indices is

$$\Delta n = n_+ - n_- = (N_o^2 + \varepsilon_{12})^{1/2} - (N_o^2 - \varepsilon_{12})^{1/2} \doteq \varepsilon_{12}/N_o.$$

3.1. Rotationability of quartz crystal

Dielectrical tensor of this crystal can be expressed by (2), where N_o and N_e are the respective refraction indices of O light and E light in quartz crystal. ε_{12}^Q, N_o and N_e are all real because the absorption is negligible. If $\lambda = 0.63$ µm, $N_o = 1.544$ $N_e = 1.553$ and $\varepsilon_{12}^Q = 1.0 \times 10^{-4}$ (the upper index Q represents the quartz crystal for the left and right rotation quartz crystal, their signs of ε_{12}^Q are opposite). When a beam of the light polarized linearly travels through the crystal along its light axis, the light will be resolved into the left and right circularly polarized normal modes, given by

$$E_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{E_0}{2} \begin{bmatrix} 1 \\ i \end{bmatrix} + \frac{E_0}{2} \begin{bmatrix} 1 \\ -i \end{bmatrix}.$$

Both the beams travel in the crystal independently with different phase velocities shown as $\Delta n = n_R - n_L = \varepsilon_{12}^Q / N_o$. As they cover a certain length in the crystal, then the resulting phase difference will be proportional to it. In other words, the polarization plane of the resultant linearly polarized light will rotate by an angle proportional to this length. We define specific rotatory power δ, as the angular shift of polarization plane when a light beam travels through a unit length. For the quartz crystal $\delta_Q = \pi\varepsilon_{12}^Q/\lambda N_o$ (rad/mm). In the case of $\lambda = 0.63$ μm, $\varepsilon_{12}^Q = 1.0 \times 10^{-4}$, we obtain $\delta_Q = 18.7$ deg/mm. We shall use now the thickness of 4.81 mm cut normally to the light axis of the crystal as 90^0 rotator in a four-frequency differential laser gyro to generate a frequency split of half mode $C/2L$ between the left and the right circular polarized gyros (Fig. 2).

3.2. Faraday effect

In the case of Faraday effect ε_{12} is not constant and it behaves as an odd function of magnetic induction intensity $\vec{B}$. In the linear region it can be cosidered as $\varepsilon_{12} = \pm\beta B$. Because the wave-vector $\vec{K}$ goes along the positive direction of z-axis, the direction of $\vec{B}$ along and opposite the z-axis corresponds respectively to the "+" and "−" sings, in this expression. In practice, the Faraday cell used in a laser gyro is isotropic (as optical glasses or material of cubic crystal system). Here $N_o = N_e$. In a general practice, the Faraday cell has an obvious absorption (as in the iron-garnet crystal), i.e., N_o and ε_{12} are all complex. Let us define the complex specific rotatory power $\delta_F = \delta_F' + i\delta_F'' = \frac{\pi}{\lambda}\frac{\varepsilon_{12}}{N_0}$. Therefore, the normal modes of the left and right circularly polarized light travel in the Faraday cell with different phase speed and different absorption. As they travel through a certain length, their phases and magnitudes will differ. In other words, when a beam of linear polarized light travels in a Faraday cell with absorption, the elliptical polarized light will be formed. In general cases the absorption is weak. After the light travelled through a unit length, the ratio of short to long axes of ellipse is

$$a/b = \tanh\delta_F'' \doteq \delta_F'' = \frac{\pi}{\lambda}\mathrm{Im}(\varepsilon_{12}/N_o)$$

(Im denotes the imaginary part) and the rotating angle of polarizat ion plane denoted by a long axis of ellipse related to the polarization plane of original light, is $\delta_F' = \frac{\pi}{\lambda}\mathrm{Re}(\varepsilon_{12}/N_o)$, Re — stands for the real part. These expressions provide a definition of Faraday specific rotatory power in a general meaning. The measurement of Faraday effect δ_F' and δ_F'' to determine the saturate magnetic rotatory term ε_{12} is a very efficient method, e.g., in the laser gyro with magnetic mirror rare earth-iron garnet, having a measured value of saturated rotatory power δ_F [3] of the thin film of $(Yb, Gd, Pr, Bi)_3$, $(FeAl)_5O_{12}$, we may calculate:

$$\lambda = 0.63\ \mu\text{m},\quad \delta'_F = 1.3\times 10^4\ \text{deg/cm},\quad \varepsilon_{12} \doteq 1.0\times 10^{-2},$$

$$\lambda = 1.15\ \mu\text{m},\quad \delta'_F = 1.8\times 10^3\ \text{deg/cm},\quad \varepsilon_{12} \doteq 0.3\times 10^{-2}.$$

If in a four-frequency laser gyro we use K_9 glass as the Faraday cell and neglect its absorption, then its $\delta_F = VB$. In the general definition of Faraday rotatory effect $V = 180\beta/\lambda N_o$ is known as the Verdet constant (deg/Oe × cm). The Verdet constant of K_9 glass is $V_{K_9} \doteq 2\times 10^{-4}$ deg/Oe × cm. Let us assume the cavity length of a ring laser $L = 60$ cm, then the space between two longitudinal modes is $C/L = 500$ MHz. Let the length of Faraday cell of K_9 glass $d = 1$ cm, the axial magnetic field $B = 1000$ Oe. In this case $\varepsilon_{12} = \beta B = 1\times 10^{-7}$ and non-reciprocal phase shift $\Delta\Phi \doteq 0.4$ deg, then the bias frequency is

$$\Delta\nu_b = \frac{VBd}{180}\frac{C}{L} \doteq 0.56\ \text{MHz}.$$

From the descriptions found in American literature (1977) and in other reports devoted to studies on four-frequency laser gyro in America, we know that the quartz rotator is also used as a Faraday cell in their systems. This system can minimize the number of optical elements in cavity at the same time, however, provides a lot of errors. We shall analyse it in the following way. When a longitudinal magnetic field is applied to the quartz crystal, then $\varepsilon_{12} = \varepsilon_{12}^{Q} \pm \beta B$. In this case, the refraction index of CW and CCW travelling waves of the left and right circularly polarized gyro may be expressed as

$$n_R^{\text{CW}} = (N_o^2 + \varepsilon_{12}^{Q} + \beta B)^{1/2},$$

$$n_R^{\text{CCW}} = (N_o^2 + \varepsilon_{12}^{Q} - \beta B)^{1/2},$$

and

$$\Delta n_R = n_R^{\text{CW}} - n_R^{\text{CCW}} = \beta B(N_o^2 + \varepsilon_{12}^{Q})^{-1/2} = \beta B/n_R$$

for RCP gyro, and as

$$n_L^{\text{CCW}} = (N_o^2 - \varepsilon_{12}^{Q} + \beta B)^{1/2},$$

$$n_L^{\text{CW}} = (N_o^2 - \varepsilon_{12}^{Q} - \beta B)^{1/2},$$

and

$$\Delta n_L = n_L^{\text{CW}} - n_L^{\text{CCW}} \doteq -\beta B(N_o^2 - \varepsilon_{12}^{Q})^{-1/2} = -\beta B/n_L$$

for LCP gyro. It may be seen that the bias frequencies of the left and the right gyro are not equal, i.e., $\nu_0^L \neq \nu_0^R$ (Fig. 2). The output differential frequency $\Delta\nu = (\nu_0^L - \nu_0^R) + \dfrac{8S}{\lambda L}\Omega$, i.e., in this output, there is a term of $\Delta\nu(B) = \nu_0^L - \nu_0^R$ related to the applied magnetic field, which will introduce a systematic error. This term of null shift may be evaluated by $\Delta\nu(B)/\nu_0 = (n_R - n_L)/N_o = 4\times 10^{-5}$.

If $\nu_0 = 1\times10^6$ Hz, then $\Delta\nu_0 = 40$ Hz will be given. This is a systematic null shift, which may be cancelled by its foreword measurement in operation of laser gyro. The random drift of the applied magnetic field and laser beam will, however, cause the harmful drift of bias frequencies, because of ν_0^L and ν_0^R the bias frequencies will not be cancelled completely and their random drifts will be added to the output of laser gyro.

4. Transverse Kerr effect

4.1. Normal mode

A non-reciprocal effect occurs when the light reflects on the surface of iron-magnetic material. As show in Fig. 4, $\vec{B}$ is perpendicular to the plane of incidence and the coordinates are selected as before, i.e., $\vec{B}$ is along the z-axis. In

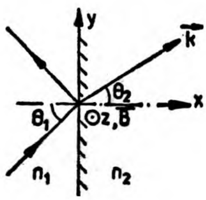

Fig. 4. Diagram showing transverse Kerr effect

this case, the dielectrical tensor takes the form of (1) and the wave-vector lies in x-y plane. We have $\vec{K} = \{a_1, a_2, 0\}$, $a_1 = \cos\Theta_2$, $a_2 = \sin\Theta_2$, and $N_o = N_e$, because the iron garnet crystal is a cubic crystal, thus the dielectrical tensor may be simplified to

$$\overleftrightarrow{\varepsilon} = \varepsilon_0 N_o^2 \begin{bmatrix} 1 & iQ & 0 \\ -iQ & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{8}$$

where $Q = \varepsilon_{12}/N_o^2$, and Eq. (6) may be simplified as

$$\begin{bmatrix} k_0^2N_o^2 - k^2(1-a_1^2) & k^2a_1a_2 + ik_0^2N_o^2Q & 0 \\ k^2a_2a_2 - ik_0^2N_o^2Q & k_0^2N_o^2 - k^2(1-a_2^2) & 0 \\ 0 & 0 & k_0^2N_o^2 - k^2 \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = 0. \tag{9}$$

In order to clearly understand the polarization mode of normal modes, we rotate the coordinate through an angle Θ_2 around its z-axis, to make the x'-axis coincident with $\vec{K}$, as in Fig. 5. It is equivalent to a transform

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = A \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad \text{and} \quad A = \begin{bmatrix} a_1 & a_2 & 0 \\ -a_2 & a_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Therefore, in these new coordinates (x, y, z) Eq. (9) transforms into

$$\begin{bmatrix} k_0^2N_o^2 & ik_0^2N_o^2Q & 0 \\ -ik_0^2N_o^2Q & k_0^2N_o^2-k^2 & 0 \\ 0 & 0 & k_0^2N_o^2-k^2 \end{bmatrix} \begin{bmatrix} E_{x'} \\ E_{y'} \\ E_{z'} \end{bmatrix} = 0. \tag{10}$$

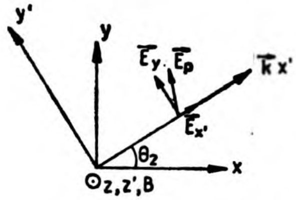

Fig. 5. Coordinate transformation and longitudinal and transverse component of P-wave

It is obvious, that the polarization mode of the normal mode corresponding to $k_S = k_0N_o$, is $\{0, 0, 1\}$ which represents the linearly polarized light. Its electrical vector $\vec{E}$ following z-axis direction is an S-polarized wave related to the interface (Fig. 4). Its refractive index $N_e = N_o$ being not related to the magneto-optic parameter Q, shows that S-polarized wave will not be modulated in refraction by the magnetic field. The value of k_S is not related to the direction angle Θ_2; it means that the locus of wave-vector $\vec{k}_S$ is a circle with radius k_0N_o, as

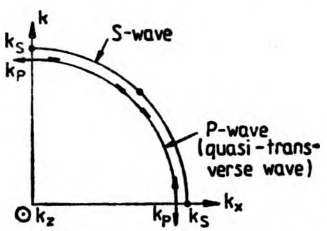

Fig. 6. Wave-vector in x-y plane

shown in Fig. 6. From Eq. (10) we obtain another solution, according to which normal mode polarization is (x', y') plane, i.e., a P-polarized light. We have

$$k_P = k_0n_P = k_0N_o(1-Q^2/2), \quad E_{x'}/E_{y'} = -iQ,$$

which represent the polarization mode of the normal mode, being $\{-iQ, 1, 0\}$. It means that the electrical vector of P-wave is not perpendicular to the direction of wave-vector (Fig. 5) and that it is a quasi-transverse wave. The ratio of small longitudinal component $E_{x'}$ to transverse component $E_{y'}$ is equal to $-iQ$ ($Q \ll 1$). The complex Q is an odd function of $\vec{B}$, so that both magnitude and phase of longitudinal component will be modulated by $\vec{B}$. The appearance of longitudinal component in P-wave causes non-reciprocal reflection. Hence, the transverse Kerr effect used in magnetic mirror layer gyro is invariably related to P-wave. In (k_x, k_y) plane the locus $\vec{k}_P$ is a circle with the radius of k_P, as shown in Fig. 6.

4.2. Transverse Kerr effect

In the following the reflection of P-polarized light on the interface of dielectric and iron-magnetic materials will be discussed. Let the wave-vectors of incidence, and the reflective and refractive waves be represented by $\vec{k}^i$, $\vec{k}^r$, $\vec{k}^t$, respectively. Their directions are $\vec{K}^i = \{\cos\Theta_1, \sin\Theta_1, 0\}$, $\vec{K}^r = \{-\cos\Theta_1, \sin\Theta_1, 0\}$, $\vec{K}^t = \{\cos\Theta_2, \sin\Theta_2, 0\}$ and $n_2 = n_0(1-Q^2)^{1/2}$. The electrical vectors are $\vec{E}^i$, $\vec{E}^r$ and $\vec{E}^t = \vec{E}^t_T + \vec{E}^t_L$, their positive directions being defined as in Fig. 7. $\vec{E}^t_T$ and $\vec{E}^t_L$ are transverse and longitudinal components of electrical vector $\vec{E}^t$ in gyrotropic dielectric medium, and $E^t_L = -iQE^t_T$. From the continuity of tangential components of electrical vector in both sides of interface we get

$$\cos\Theta_1(E^i+E^r) = (\cos\Theta_2 - iQ\sin\Theta_2)E^t_L. \tag{11}$$

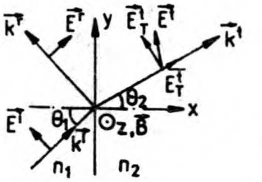

Fig. 7. Transverse magneto-optic scattering coefficients of P-wave

From $\vec{H} = n(\vec{K}\times\vec{E})$, the directions of three wave-vectors are all parallel to z-axis. From the continuity of tangential components of magnetical vector we get (notice $\vec{K}^t \times \vec{E}^t_L = 0$)

$$n_1E^i - n_1E^r = n_2E^t_T. \tag{12}$$

Eqs. (11) and (12) yield

$$r_P = \frac{E^r}{E^i} = \frac{\eta_1(1-iQ')-\eta_2}{\eta_1(1-iQ')+\eta_2}, \tag{13}$$

and

$$t_P = \frac{E^t_r}{E^i} = \frac{2(n_1/n_2)\eta_2}{\eta_1(1-iQ')+\eta_2} = \frac{n_1}{n_2}(1-r_p) \tag{14}$$

where

$$\eta_1 = n_1/\cos\Theta_1, \quad \eta_2 = n_2/\cos\Theta_2, \quad Q' = Q\tan\Theta_2, \tag{15}$$

if $B = 0$, then $Q = 0$, and

$$r^0_P = \frac{\eta_1-\varphi_2}{\eta_1+\eta_2}, \quad t^0_P = \frac{n_1}{n_2}(1-r^0_P).$$

Because Q' is a small quantity, the expression (13) may be extended to a series and taken to first order

$$r_P = r_P^0 + \Delta r, \quad \Delta r = -\frac{iQ}{2}(t_P^0)^2 \tan\Theta_1. \tag{16}$$

As Q is an odd function of $\vec{B}$, then the reflective coefficients of P-polarized CW and CCW travelling-wave electrical vectors on the magnetic mirror r_P^{CW} and r_P^{CCW} are

$$r_P^{CW} = r_P^0 + \Delta r(B),$$

$$r_P^{CCW} = r_P^0 + \Delta r(-B) = r_P^0 - \Delta r(B),$$

respectively. That causes a non-reciprocal reflectance as shown in Fig. 8. $\Delta\Phi(B)$ is the phase difference introduced by this non-reciprocal reflection, and it is required to get a bias in the laser gyro. In practice, a controlled layer of dielectric thin film is expected to nullify the non-reciprocal amplitude difference introduced by reflection. Unequal intensities CW and CCW of travelling waves will be produced since their amplitudes are different. There will be a drift of differential losses in gyro's operation.

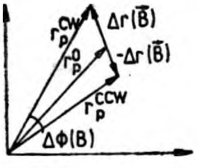

Fig. 8. Non-reciprocal reflection in transverse Kerr effect

5. Normal modes in longitudinal and polar Kerr effects

In these cases, the wave-vector $\vec{k}$ is in (x, y) or (y, z) plane. Because the wave-vector surface in cubic system is rotatory symmetrical around z-axis and the magnetic field is applied along z-axis, it remains only to discuss the fact occurring in any section plane including z-axis. Let $\vec{K} = \{0, \sin\varphi, \cos\varphi\}$, it means that the wave-vector is in (y, z) plane and its intersection angle with z-axis

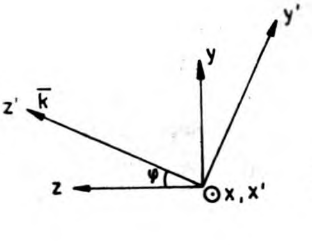

Fig. 9. Coordinate transformation

is φ. Eq. (6) is simplified as follows

$$\begin{bmatrix} N_o^2-n^2 & iN_o^2Q & 0 \\ -iN_o^2Q & N_o^2-n^2\cos^2\varphi & n^2\sin\varphi\cos\varphi \\ 0 & n^2\sin\varphi\cos\varphi & N_o^2-n^2\sin^2\varphi \end{bmatrix}\begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = 0 \tag{17}$$

where $k = k_0 n$.

Similarly, we make a coordinate rotation by an angle φ around x-axis and make z'-axis coincident with $\vec{k}$ as shown in Fig. 9. In coordinate system (x', y', z') Eq. (17) transforms to

$$\begin{bmatrix} N_o^2-n^2 & iN_o^2Q\cos\varphi & iN_o^2Q\sin\varphi \\ -iN_o^2Q\cos\varphi & N_o^2-n^2 & 0 \\ -iN_o^2Q\sin\varphi & 0 & N_o^2 \end{bmatrix}\begin{bmatrix} E_{x'} \\ E_{y'} \\ E_{z'} \end{bmatrix} = 0. \tag{18}$$

The determinant of coefficients vanishes when there exists a nontrivial solution. By extending it we obtain an equation of 4-th order of refraction index n of normal mode in $\vec{K}$ direction. Since

$$n^4 - n^2N_o^2(2-Q^2\sin^2\varphi) + N_o^4(1-Q^2) = 0,$$

thus

$$n_\pm \doteq N_o^2(1 \pm 1/2\ Q\cos\varphi), \tag{19}$$

$$E_{y'}/E_{x'} = i\,\frac{N_o^2Q\cos\varphi}{N_o^2-n^2} = \pm i.$$

The longitudinal component $E_L = E_z$, and the transverse component is E_T. Their ratio is

$$E_L/E_T = \frac{iQ}{\sqrt{2}}\sin\varphi \quad \text{and} \quad E_T = (|E_{x'}|^2 + |E_{y'}|^2)^{1/2}.$$

It can be seen that in the (y, z) or (x, z) plane the normal modes are the left and right circular quasi-transverse waves with small longitudinal components E_L.

6. Wave-vector surface and normal mode of non-reciprocal magneto-optic medium

For the non-reciprocal magneto-optical medium with dielectrical tensor given by Eq. (8) and such ones as: isotropic Faraday rotatory medium, GM1, GM2, GM3 [3], YIG, $(Bi, Tm)_3 (Fe, Ga) (Fe, Ga)_5 O_{12}$ etc. of iron garnet type — recently developed magneto-optical medium, a complete diagram of wave-vector may be made in (k_x, k_y, k_z) space. If the applied magnetic field equals zero, there is a spherical surface. While applying a magnetic field, it splits into two layers of rotatory symmetrical curve surface around z-axis (not a rotatory ellipse). Because of its symmetricity, only 1/8 part of these curve surfaces and corresponding polarization mode of normal mode have been shown in Fig. 10. We have there

$$n_{\pm}(\varphi) = N_o(1 \pm Q/2 \cos\varphi),$$
$$n_{\pm}(\varphi = 90^0) = N_o(1 - Q^2/4 \pm Q^2/4),$$
$$k_S = k_0 N_o,\ k_P = k_0 N_o(1 - 1/2Q^2).$$

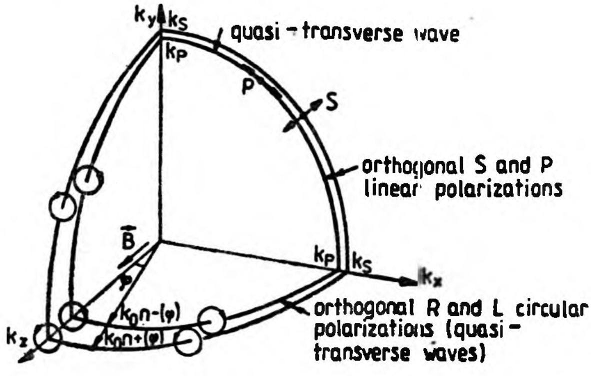

Fig. 10. Wave-vector surface and normal mode in magnetic gyrotropic medium

It should be noted that the dielectrical tensor of a single-axis quartz crystal is expressed as Eq. (2) for the light travelling along z-axis. It differs from Eq. (8) only by $N_e \neq N_o$. Hence, its wave-vector surfaces appear to be similar to those in Fig. 10. They differ only in $k_S = k_0 N_e$, in (y, z) and (x, z) planes the polarization modes of normal mode are orthogonal elliptically polarized.

7. Conclusions

The wave-vector of transverse Kerr effect used in magnetic mirror bias laser gyro lies in the k_x, k_y plane. The non-reciprocity of reflection of P-polarized normal mode is used. In four-frequency differential laser gyro all the wave-vectors of four travelling waves run along $\vec{k}_z$. As a Faraday effect, the non-reciprocity of orthogonal left and right polarized normal mode has been used; it is noticed that in four-frequency differential gyro, orthogonal quasi-circularly polarized modes in (k_x, k_z) or (k_y, k_z) plane may also be used, i.e., the non-reciprocal polar Kerr effect may be used to form a magnetic mirror instead of a Faraday cell. Thus, the optical components in cavity may be reduced and the loss of cavity and scattering decreased. The polar Kerr effect has a strong non-reciprocity. We may select the MnBi magnetic mirror with a stack of dielectric thin films to construct a cavity of the four-frequency orthogonal circularly polarized laser gyro [4, 5].

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