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Some experiments in the Fresnel region of double diffraction systems

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The contrast change of the intensity distribution in the Fresnel diffraction field of two axially separated linear gratings with lines mutually rotated is experimentally observed and physically explained. Spatially coherent plane wave illumination is used. The effect is due to spatial superposition of the multiple of Fresnel diffraction fields of one of the gratings without observing high frequency interference patterns.

1. Introduction

The pure diffraction-interference imaging process of periodic objects is well known as the self-imaging phenomenon [1–4]. Object replicas or its modified versions are formed along the illumination direction without using any optical elements. Optical properties of diffraction images depend on the wavefront curvature of the spatially coherent light beam, object period, light wavelength and the observation plane localization. Amongst the general class of self-imaging objects [5] the linear diffraction grating plays the most important role, because of its possible applications.

In the studies of the self-imaging phenomenon the single beam illumination is usually assumed. However, some attempts have been made to investigate properties of the Fresnel diffraction field of the so-called double-diffraction systems [6–8]. These systems are composed of two diffraction gratings separated in space along the illumination direction. The configurations with grating lines mutually parallel [6, 7] or slightly inclined [8] have been considered. Since the general mathematical description and physical interpretation in the Fresnel region behind the second structure are very complex, only some special cases have been discussed [6–8].

In this paper we would like to present some properties of the Fresnel field of double-diffraction systems not yet described in the literature. In contrast to the former studies [6–8], where gratings with parallel or slightly inclined lines have been assumed, we investigate the change of the Fresnel field parameters as a function of a large inclination angle between the lines of two gratings. The obtained effect of spatial superposition of multiple self-imaging of one grating is physically explained. Two cases are treated separately, namely: the, first grating illuminated by the light beam with respect to the second stationary and vice versa.

2. Experimental work and physical interpretation

The optical configuration under investigation is shown schematically in Fig. 1. Spatially coherent plane wave illuminates the first diffraction grating $G1$. The second grating $G2$ is placed at the finite distance z_1 from $G1$, the observation is performed in the Fresnel diffraction plane located at the distance z_2

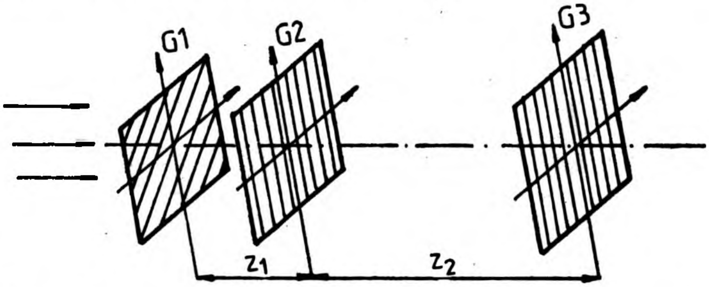

Fig. 1. Geometry of the double-grating diffraction system. $G1$ and $G2$ – linear rulings form ming the double-diffraction configuration, $G3$ – linear ruling for detecting the Fresne field behind the grating $G2$

from the grating $G2$. Grating $G3$, placed in the observation plane, will be called the detection grating, since it facilitates visual observation of the diffraction field (at the distance z_2) by means of the Moiré fringe technique. All gratings used in our experiments were of amplitude type binary rulings with the same spatial period. The possible variations of the gratings parameters will be discussed below.

2.1. Rotation of the first grating $G1$

The detection grating $G3$ was placed in one of the self-image planes of the grating $G2$, and a very slight tilt between the grating lines introduced. Grating $G1$ was not yet placed in the optical system. In this way as the beat product between the self-image of $G2$ and grating $G3$ the Moiré fringes have been obtained. They are shown in Fig. 2. Additionally, the spatial frequency intensity distribution (harmonic content) has been observed in the back focal plane of a lens placed behind the detection grating $G3$. Figure 3 shows the intensity distribution corresponding to the situation just mentioned. Monitoring of the changes occurring in this plane as a function of the changes of mutual position of gratings in the double diffraction system (gratings $G1$ and $G2$) will allow us to explain easily the system performance.

Thereupon, the grating $G1$ was inserted in front of $G2$ at the finite distance z_1. In the following, the cases when the lines of gratings $G1$ and $G2$ are parallel or nearly parallel will not be studied, as the properties of Moiré fringes produced in these cases have been discussed in detail in paper [8]. Our observations were made for larger inclination angles at which the moire patterns, analysed in paper [8], cannot be distinguished.

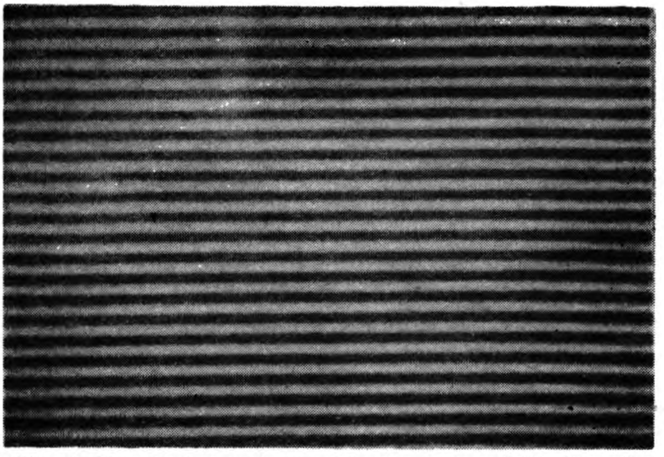

Fig. 2. Moiré fringes observed in the plane of the detection grating $G3$ due to the beat phenomenon between the self-image of $G2$ and grating $G3$

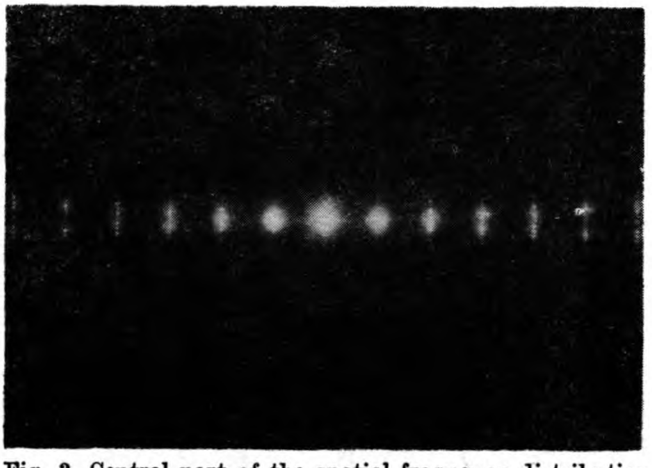

Fig. 3. Central part of the spatial frequency distribution of the pattern shown in Fig. 2

By rotating gradually the grating $G1$ (Fig. 1) we observed the change in contrast of Moiré fringes detected in the observation plane $G3$ (see Fig. 2). The fringes vanish and appear again retaining their spatial period and lateral position. This means that the contrast of the self-image of $G2$ is detected by the changes of $G3$ as a function of the rotation angle of $G1$. The effect observed is independent of the axial distance z_1 between G_1 and $G2$, i.e., the variation of z_1 does not introduce any change into the moire fringe contrast determined by the chosen rotation angle of $G1$.

To explain the physical origin of the effect the self-suggesting approach is to treat the Fresnel field behind the grating $G2$ as a coherent superposition of many Fresnel fields, generated by each diffraction order of $G1$ playing the role of the illuminating beam. Rotation of $G1$ changes the plane incidence of the illuminating orders with respect to the plane normal to the lines of $G2$. The analysis of the second structure in the Fresnel region would lead to complex mathematical calculations, as their final stage would require a proper procedure for the selection of terms.

Here we will present a simple heuristic explanation of the effect using the approach mentioned above, our argumentation being, however, based on the observations made in the spatial frequency plane. These observations are sufficient for interpretation of the system performance. For this purpose, spatial frequency distributions for the rotation angles: 30°, 60° and 90° of $G1$ (with respect to the stationary gratind $G2$) are shown in Fig. 4. Let these photographs be compared with that in Fig. 3. It may be comprehended that the presence of $G1$ results in the multiplication of Fresnel fields behind $G2$. The number of fields is, of course, much greater than that of diffraction orders of $G1$ illuminating the grating $G2$, because of the cross-products appearing due to the interaction of the periodic illumination with the periodic object structure. The multiplication is visualized by the multiplied number of groups of the optical diffraction spots corresponding to Moiré fringe frequencies (see Figs. 3 and 4). All constitutive Fresnel patterns (self-images) form Moiré fringes in the plane of $G3$. Their lateral disposition, however, depends on the incidence angle of the illuminating beams [9]. In our system these angles are determined by the angle of rotation of $G1$. This last statement is confirmed by the photographs in Fig. 4. Vertical distance between the groups of Moiré diffraction spots (harmonics) changes with the rotation of $G1$. We can expect, therefore, that for some values of the rotation angle of $G1$ the spatial coincidence of self-images will occur at the selected distance z_2 of self-image observation. In such a case well-defined Moiré fringes will be observed in the plane of grating $G3$, for other angular positions of $G1$ the Moiré fringe contrast will be degraded.

The interpretation given above is supported by additional experimental observations. For example, the following experiment has been performed. In order to let only some selected Moiré-spot groups spatial filter was placed in the frequency plane. The following features have been observed:

i) When the filter lets through one of the stationary groups (not changing its position with the rotation of $G1$, these groups correspond to the ones shown in Fig. 3 and are formed by the zero diffraction order of $G1$, they constitute the central, almost horizontal *line* of Moiré-spots in Fig. 4), then the Moiré fringes do not change their contrast and position in the observation field with the rotation of the grating $G1$.

ii) When the filter lets through one of the Moiré-spot groups lying above the *line* of stationary Moiré groups, mentioned in i), then the fringes move laterally with the rotation of $G1$. The same occurs when letting through the group lying under the *line*. In this case, however, the direction of fringe movement is opposite. The same is observed when all the frequencies occupying one half of the frequency plane and lying above or under the stationary groups pass through the filter.

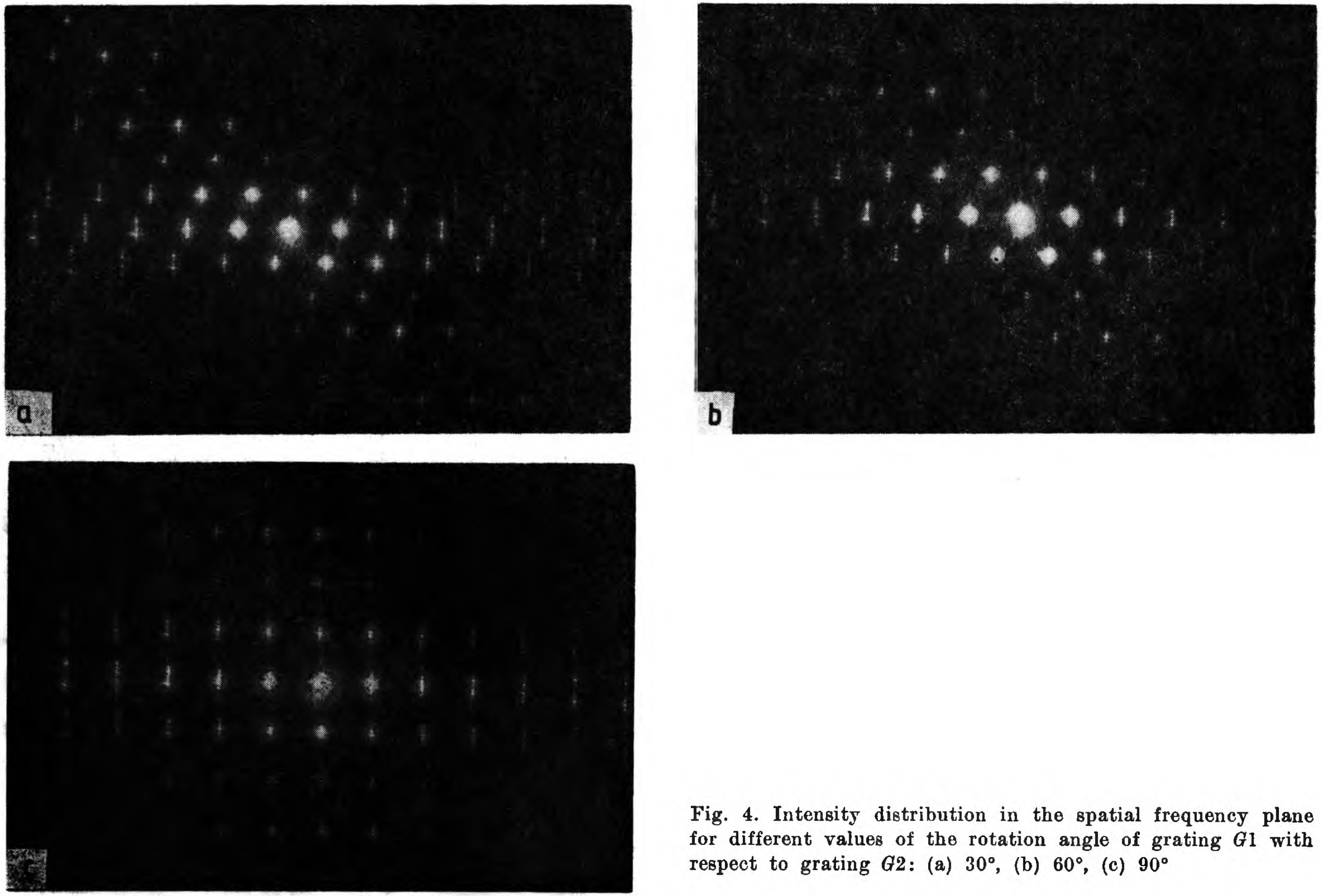

Fig. 4. Intensity distribution in the spatial frequency plane for different values of the rotation angle of grating $G1$ with respect to grating $G2$: (a) 30°, (b) 60°, (c) 90°

iii) The passage of one stationary and one moving group with the rotation of $G1$ leads to stationary Moiré fringes with their contrast changing as a function of the rotation angle. It seems that the stationarity is imposed by a higher intensity of the stationary moire group, whereas the contrast change is due to the mutual lateral displacement of the stationary and moving patterns. The same behaviour is observed, when one stationary and two moving groups lying on the opposite sides of the stationary one are let through. When only the two latter ones are let through, two fringes fields moving in opposite directions are observed. Similar features are noted when the number of groups let through the filter is appropriately increased.

The described observations confirm the concept of spatial superposition of Fresnel patterns introduced before. With the rotation of $G1$ the incidence angles of the beams illuminating $G2$ are resulting in the lateral displacement of multiple Fresnel fields (with exception of the field generated by the zero diffraction order of $G1$ and producing the stationary Moiré-spot groups). For some positions of $G1$ the patterns are overlapping and when the detection grating $G3$ is in the self-image plane of $G2$, good contrast Moiré fringes are observed. Obviously, when $G3$ is placed in the middle between the self-image plane (the plane of uniform intensity, in the case of single beam illumination of the Ronchi type binary amplitude grating used in our experiment) no well-defined Moiré fringes can be seen, irrespectively of the rotation angle of $G1$. It follows from the above discussion that the conditions of spatial superposition of Fresnel fields in a given observation plane are influenced only by the incidence angles of the illuminating beams. This is why the change of the distance z_1 between gratings $G1$ and $G2$ does not influence the observed effect. Moreover, the grating $G1$ does not have to be of the same spatial period as grating $G2$; when it is of a higher spatial frequency the change of contrast with the rotation angle occurs faster.

More general remark is worthy to be mentioned. All plane beams present behind the grating $G2$ interfere with each other due to the spatially coherent illumination. However, only the low spatial frequency patterns can be visually observed — these are the Fresnel diffraction patterns of $G2$ (being displayed in the form of Moiré fringes in the plane of grating $G3$). Other interference patterns formed, for example, by the interference of harmonics belonging to the adjacent Moiré groups are of high spatial frequency and cannot be resolved. This is why the described effect can be treated as a multiple superposition of self-imaging under spatially coherent illumination. Another mechanism of multiple superposition of self-imaging, using spatially incoherent illumination of the double-diffraction system, has been recently described [10].

Finally, it can be assumed that the amplitude type grating $G1$ can be replaced by a phase grating, since its only role is to provide a spatially periodic illumination of $G1$. Moreover, because of the observed spatial superposition of multiple Fresnel fields without, as explained above, interference effects (to be expected under spatially coherent illumination) it can be inferred that spatially incoherent periodic illumination may be used, as well. This can be done, for example, by moving laterally the grating $G1$ or by employing ultrasonically produced progressive phase grating [7, 11, 12].

The grating $G2$ can be of the amplitude or of phase type; in the latter case the best visibility diffraction patterns are observed in the planes lying in the middle between the self-image planes of amplitude grating [10].

2.2. Rotation of the second grating $G2$

Next, the following experiment has been performed. The grating $G3$ was placed in one of the self-image planes of the grating $G1$ (the grating $G2$ being removed). The lines of $G1$ and $G3$ were set almost parallel, the moire fringes observed by us were similar to the ones shown in Fig. 2. Next, the grating $G2$ was inserted between the gratings $G1$ and $G3$. It has been noted that irrespectively of the axial localization of $G2$ (i.e., when setting arbitrary values of the distances z_1 or z_2, Fig. 1) the Moiré fringes detected in the plane $G3$ change their contrast with the rotation of the grating $G2$. Moiré fringes retain their spatial period and lateral localization depending on the rotation angle. On the other hand, when the angular position of $G2$ is fixed then the Moiré fringe contrast changes with the axial displacement of this grating. If the grating $G3$ is not placed in one of the self-image planes of $G1$, the maximum contrast Moiré fringes can never be obtained, irrespectively of the axial and angular positions of the grating $G2$. Other experimental observations with spatial filtering in the frequency plane coincide with the ones described in the previous section.

The physical interpretation of the performance of this system can be based, as before, on the concept of spatial superposition of Fresnel fields. The grating $G2$ acts now as the splitter (divider) and, therefore, as the multiplier of the Fresnel field of the first grating $G1$. This Fresnel field propagates behind the grating $G2$ in the directions corresponding to the directions of diffraction orders of this grating, as well as in the directions corresponding to the cross-products (beats between the diffraction orders of $G1$ and $G2$). When the field-splitter grating $G2$ rotates, the angular separation between the multiplied Fresnel fields changes (we do not show here the situation in the spatial frequency plane, since it is analogical to the one shown in Fig. 4). As the result the mutual lateral displacement of the fields in the observation plane is changed. This displacement is additionally a function of the distance z_2 between the splitter-grating $G2$ and the observation plane $G3$. Therefore, it is clear that in the observation plane the spatial coincidence of the multiple of Fresnel patterns of the first grating $G1$ can be obtained either by angular rotation or by axial shift of the grating $G2$. Well-defined patterns (and, consequently, Moiré fringes in the plane of the detection grating $G3$) are obtained when the self-images of $G1$ are overlapped. This is why our observations have been performed in the self-image plane of the first grating.

The additional observations resulting from the spatial filtering experiments explaining the stationarity and movement of Fresnel fields belonging to selected propagation direction behind $G2$ are identical to the ones decribed in Section 2.1. They provide a closer look on the origin of the effect under investigation and prove the correctness of our heuristic explanation.

As before, it is easy to understand that the rotating grating can be of amplitude or phase type, its spatial period does not have to be equal to the period of stationary (self-imaging) grating. Its only role is to provide multiple angular separation between the Fresnel fields. It follows from the last statement that the progressive grating, i.e., ultrasonically produced phase grating can be used for this purpose.

The grating $G1$ can be of amplitude as well as of phase type, in the latter case the best visibility diffraction patterns are observed in the planes lying in the middle between the self-image planes of the amplitude grating [10].

3. Conclusions

The effect of spatial superposition of multiple self-imaging in the Fresnel region of double-grating optical systems under spatially coherent plane wave illumination has been obtained and physically explained. This effect takes place when one of the gratings is rotated about the optical axis and the angle between lines of the two gratings is sufficiently large to avoid Moiré effects. The Fresnel fields of the stationary gratings are overlapped and can be visually observed.

When the grating being rotated stands in front of the stationary one, then its role is to provide the periodic beam illumination of the stationary grating with continuously changeable values of the incidence angle. For some angular positions the spatial coincidence of the Fresnel fields (in-registry condition in selected observation plane) is encountered. The axial separation distance between the two gratings does not influence the parameters of the Fresnel field.

When the axial order of the gratings in reversed then the grating being rotated acts as the optical field splitter (multiplier) of the Fresnel diffraction field of the first grating. The in-registry condition in a given observation plane depends on the rotation angle of the splitter, grating and on the axial distance between this grating and the observation distance.

From the above described roles performed by the grating being rotated it may be inferred that this grating can be of amplitude as well as of phase type. The same remark concerns the stationary grating. Spatial periods of the two gratings in the double-diffraction system under study do not have to be in a fixed ratio, in contrast to the previously described systems [6, 7]. The grating being rotated can be also replaced by the progressive spatial periodic modulation, but then the spatial coherence between its diffraction is destroyed. This fact, however, plays no role in the multiple superposition of self-imaging described, since for that purpose no interference effects are required.

All investigations presented in this paper were concerned with the plane beam illumination of double-diffraction system. They can be extended, however, in a straightforward manner to the case of spherical [3] or Gaussian beam illumination [13]. The analysis of these configurations together with the mathematical description of the presented case of plane beam illumination will be the subject of the following paper.

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Некоторые эксперименты в области Френеля с двумя дифракционными системами

Темой работы является изменение контраста распределения интенсивности в дифракционном поле Френеля двухосно раздвинутых линейных решеток с вращаемыми друг друга линиями, которое наблюдалось экспериментально и интерпретировалось физически. Применено освещение при помощи пространственно когерентной плоской волны. Эффект происходит от пространственного наложения кратности френелевых полей одной решетки, причем не наблюдались интерференционные фигуры высокой частоты.