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Analysis of power density distribution at the vicinity of focus for aberration-free focusing systems of the high speed*

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Two integral transformations describing the optical field in the vicinity of the focus of aplanatic optical systems of high speed are presented. The results of both the transformations for the field on the axis have beem compared. Focusing of the laser radiation after its passage through an axicon have been calculated for the cases of the rectangular and Gaussian entrance beams and the obtained results discussed.

1. Introduction

The knowledge of the field distribution at the vicinity of focus point of the optical system is of a great practical importance, for instance, for designers of the devices employing very high power densities, such as laser micro-processors, laser coagulator, and so on. The field distribution at the vicinity of the focus depends on the optical quality of the focusing system and on the entrance beam parameters. For the uncut Gaussian beams and optical systems of long focal length the Gaussian beam formalism [1] is usually used. Based on the scalar diffraction theory the field distribution at the vicinity of the focus is described by the Fresnel transformation [2] of the entrance field. The Wolf integral transformation [3–5], which takes account of the vector character of entrance field, is used for the optical system of high speed. The influence of the Gaussian beam apodization on the field distribution in the focus vicinity has been analysed in [5] for various numerical apertures of the lens.

In the present work the Wolf transformation has been employed to calculate the focusing of the laser beam after its passage through the axicon. The results of the Wolf transformations have been, moreover, compared with those obtained from the exact relations derived by the authors for the field on the optical axis at the focus vicinity.

2. Wolf integral transformation

Taking account of the vector character of the entrance field, Wolf gave in paper [3] the integral transformation describing the relationship (illustrated in Fig. 1)

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between the field $\boldsymbol{E}(\boldsymbol{R}_p)$ in the vicinity of the focus and the field $\boldsymbol{U}_s(\boldsymbol{S})$, given on the segment of sphere Ω, the centre of which is in the focus F and the radius equal to the focal length f, this relation may be written in the following form

$$\boldsymbol{E}(\boldsymbol{R}_p) = -\frac{i}{f\lambda}\int_{\Omega} \boldsymbol{U}_s(\boldsymbol{S})\exp\left(i\frac{2\lambda\pi}{\lambda}\boldsymbol{S}\boldsymbol{R}_p\right)d\Omega \tag{1}$$

where: λ – wavelength,
Ω – region of integration,
f – focal length of the optical system,
S – unit vector normal to the sphere

$$\boldsymbol{S}\boldsymbol{R}_p = R_p\cos\varepsilon = R_p\big(\cos\Theta\cos\Theta_p+\sin\Theta\sin\Theta_p\cos(\varphi-\varphi_p)\big).$$

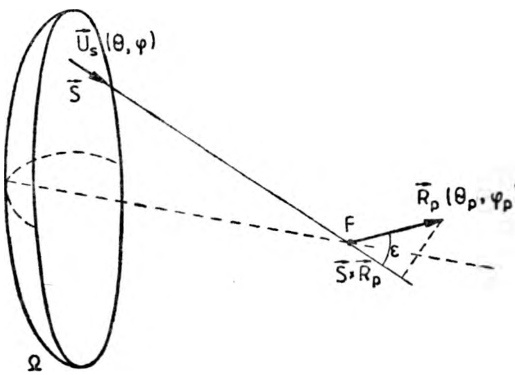

Fig. 1. Wolf transformation of the field $\boldsymbol{U}_s$, given on the sphere Ω into the field $\boldsymbol{E}$

In order to determine the dependence of $\boldsymbol{U}_s$ upon the entrance field we assume that the entrance beam is collimated and linearly polarized, of axially symmetric power density distribution. We assume, moreover, that sine condition is satisfied by the optical system. Then the dependence between the amplitude of the entrance field $p(\varrho)$ and that of the field $|\boldsymbol{U}_s|$, illustrated in Fig. 2, is given by the expression

$$a(\Theta) = |\boldsymbol{U}_s(\Theta, \varphi)| = p(f\sin\Theta)\cos^{1/2}\Theta. \tag{2}$$

The above assumptions allow us to perform integration of the expression (1) with respect to the angle φ in order to obtain (similarly as was the case in papers [4–5]) formulae convenient for numerical calculations of Cartesian components of field $\boldsymbol{E}(\boldsymbol{R}_p)$

$$\left.\begin{aligned} E_x &= -iA_0(I_0+I_2\cos\varphi_p) \\ E_y &= -iA_0I_2\sin 2\varphi_p, \\ E_z &= -2A_0I_1\cos\varphi_p \end{aligned}\right\}, \tag{3}$$

where:

$$
\left.\begin{aligned}
I_0(u,v) &= \int\limits_{\Theta_1}^{\Theta_2} p(f\sin\Theta)\cos^{1/2}\Theta\sin\Theta J_0(v\sin\Theta)\exp(iu\cos\Theta)(1+\cos\Theta)\,d\Theta\\
I_1(u,v) &= \int\limits_{\Theta_1}^{\Theta_2} p(f\sin\Theta)\cos^{1/2}\Theta\sin\Theta J_1(v\sin\Theta)\exp(iu\cos\Theta)\,d\Theta\\
I_2(u,v) &= \int\limits_{\Theta_1}^{\Theta_2} p(f\sin\Theta)\cos^{1/2}\Theta\sin\Theta J_2(v\sin\Theta)\exp(iu\cos\Theta)\,d\Theta
\end{aligned}\right\} \quad (4)
$$

$$A_0 = \pi f/\lambda, \quad u = \frac{2\pi}{\lambda} z_p, \quad v = \frac{2\pi}{\lambda}\sqrt{x_p^2+y_p^2},$$

J_0, J_1, J_2 – Bessel functions of first kind.

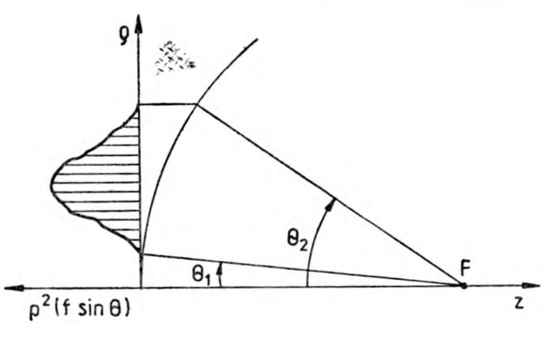

Fig. 2. Wolf transformation of the power density in the entrance beam by an ideal optical system

Next, by employing (3) we can obtain the following formulae for the time-averaged electric energy density

$$G(u, v, \varphi_p) = \frac{1}{16\pi}(\boldsymbol{E}\boldsymbol{E}^*) = \frac{A_0^2}{16\pi}\big(|I_0|^2+4|I_1|^2\cos\varphi_p$$
$$+|I_2|^2+2\,\mathrm{Re}(I_0 I_2^*)\cos 2\varphi_p\big). \quad (5)$$

The formula (1) is essentially the Fourier transformation of the field $\boldsymbol{U}_s$ given on the space $\boldsymbol{S}$ into the field $\boldsymbol{E}$ given on the space $\boldsymbol{R}_p$. For small Θ it is reduced, as shown in [3], to the commonly used Fresnel transformation for scalar field. The Wolf transformation is valid also for high numerical apertures, then however, the restrictions and approximations of the method discussed in [3] should be taken into account. Having this in mind, an attempt has been made to determine the range of its applicability by deriving rigorous expression for the field on the optical axis.

3. Field on the optical axis

Starting directly from the Kirchhoff formula the authors obtained a rigorous expression for the field on the optical axis, presented in detail in paper [6].

Using the denotations introduced in Fig. 1 the expression may be put in the form

$$E(R_p) = \frac{i}{2\lambda f}\int_{\Omega} U_s(\Theta,\varphi)\frac{1}{B(\Theta,\delta)}\left(1+\frac{1-\delta\cos\Theta}{B(\Theta,\delta)}\right)\exp\left(i\frac{2\pi}{\lambda}f\left(B(\Theta,\delta)-1\right)\right) \times \sin\Theta d\Theta d\varphi \qquad (6)$$

where R_p – distance from the focus measured along the optical axis,

$$\delta = R_p/f,\ B(\Theta,\delta) = \sqrt{1+\delta^2-2\delta\cos\Theta}.$$

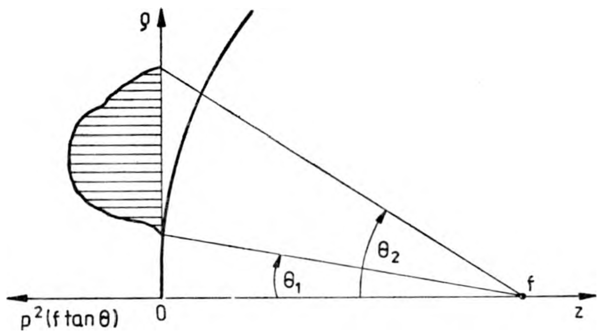

Fig. 3. Transformation of the power density in the entrance beam by an ideal optical system according to formula (7)

When accepting the assumptions concerning the entrance beam and quality of the optical focusing system, similar to those in the case of Wolf transformation, a different relationship between the entrance field $p(\varrho)$ and the field on the sphere illustrated in Fig. 3 has been proposed

$$a(\Theta) = |U_s(\Theta,\varphi)| = p(f\tan\Theta)\cos^{-3/2}\Theta. \qquad (7)$$

This relationship seems to be more advantageous for high speed optical system. Taking account of (7), the expression (6) after having beam integrated with respect to φ has been transformed into the final formula of the following form

$$E(R_p) = \frac{2if}{\lambda}\int_{\Theta_1}^{\Theta_2} p(f\tan\Theta)\frac{K(\sin\Theta)}{B(\Theta,\delta)}\left(1+\frac{1-\delta\cos\Theta}{B(\Theta,\delta)}\right) \times \exp\left(i\frac{2\pi}{\lambda}f\left(B(\Theta,\delta)-1\right)\right)\tan\Theta\cos^{1/2}\Theta d\Theta \qquad (8)$$

where

$$K(\beta) = \int_0^{\pi/2}\frac{d\varphi}{\sqrt{1-\beta^2\cos^2\varphi}}$$ – complete elliptic integral of second kind.

For small numerical apertures the expression (8) gives the results identical with those of Wolf transformation. It seems to be true for a wider interval of variability of R_p.

4. Results of calculations

By taking advantage of the above methods we have calculated the focusing of the laser beam after its passage through the axicon. We assume for it the field transformation by axicon, illustrated in Fig. 4

$$p^2(\varrho) = g^2(R-\varrho)\left|\frac{R-\varrho}{\varrho}\right| \tag{9}$$

where: $g^2(\varrho)$ – power density distribution in front of axicon,
R – radius of axicon,
$p^2(\varrho)$ – power density distribution behind the axicon.

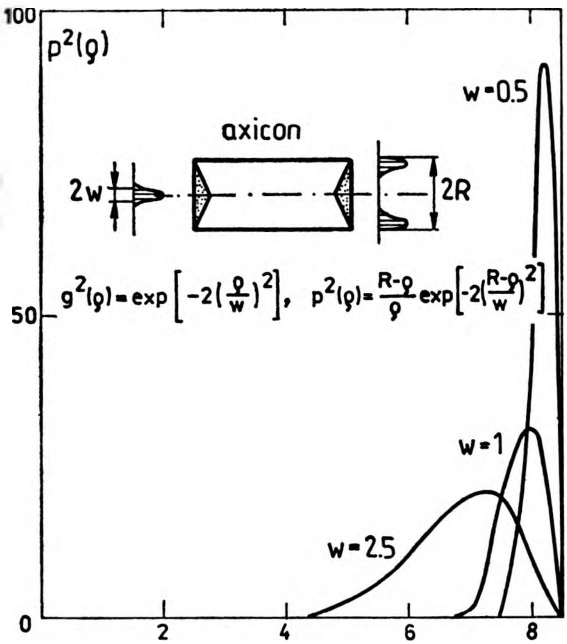

Fig. 4. Transformation of the power density in a Gaussian beam by an axicon (w – beam radius, ϱ – distance from the optical axis, R – radius of the axicon)

The calculations have been carried out for the rectangular and the Gaussian beams (Fig. 4). It has been assumed, moreover, that lens speed is $f/3$, wavelength $\lambda = 0.6328$ μm and the focal length $f = 50$ mm.

The calculation results of the field on the optical axis illustrated in Figs. 5a,b show a high similarity. It seems that for the case of optical systems having not too high speed the differences between various ways of energy transformation by the optical system (formulae (2), (7)) are insignificant.

Figure 6 illustrates the results of calculations of field distribution in the cross-sections distant from each other by 50 μm, starting from the focus point. The left and right hand side columns are referred to the rectangular and Gaussian beams, respectively. The relative maximum power density at the given cross-section with respect to the maximum power density at the focus is denoted by g. It may be seen that in both cases the prevailing part of the beam power is led to the focus by the region restricted by the broken lines and corresponding to the geometrical projection of the entrance field onto the focus point. The differences between the rectangular beam and the Gaussian beam are manifested in the position of the local maxima of the power density. For the rectangular beam

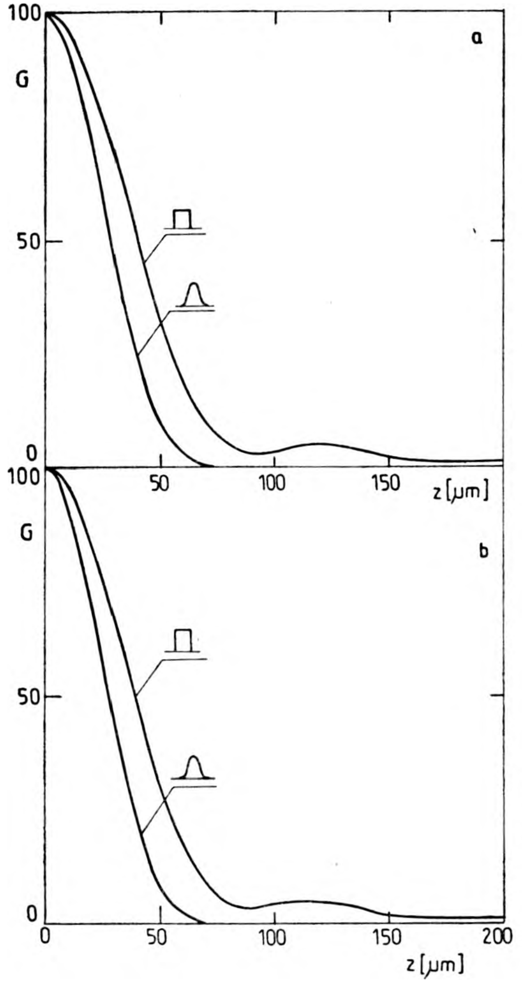

Fig. 5. Distributions of relative power density G as a function of the distance z from the focus, calculated along the optical axis for both the Gaussian and rectangular beams: a – according to the rigorous formula, b – according to the Wolf transformation

very narrow maxima are positioned on the optical axis, they however, take a negligible part in the transmission of energy to the focus. For the Gaussian beam the maxima are located in the region limited by the broken lines, and the power level observed on the optical axis is relatively low. The only exception is the cross-section at 200 μm distance from the focus, where there appears a broad maximum at the vicinity of the optical axis, which is in contrast to the corresponding situation for the rectangular beam.

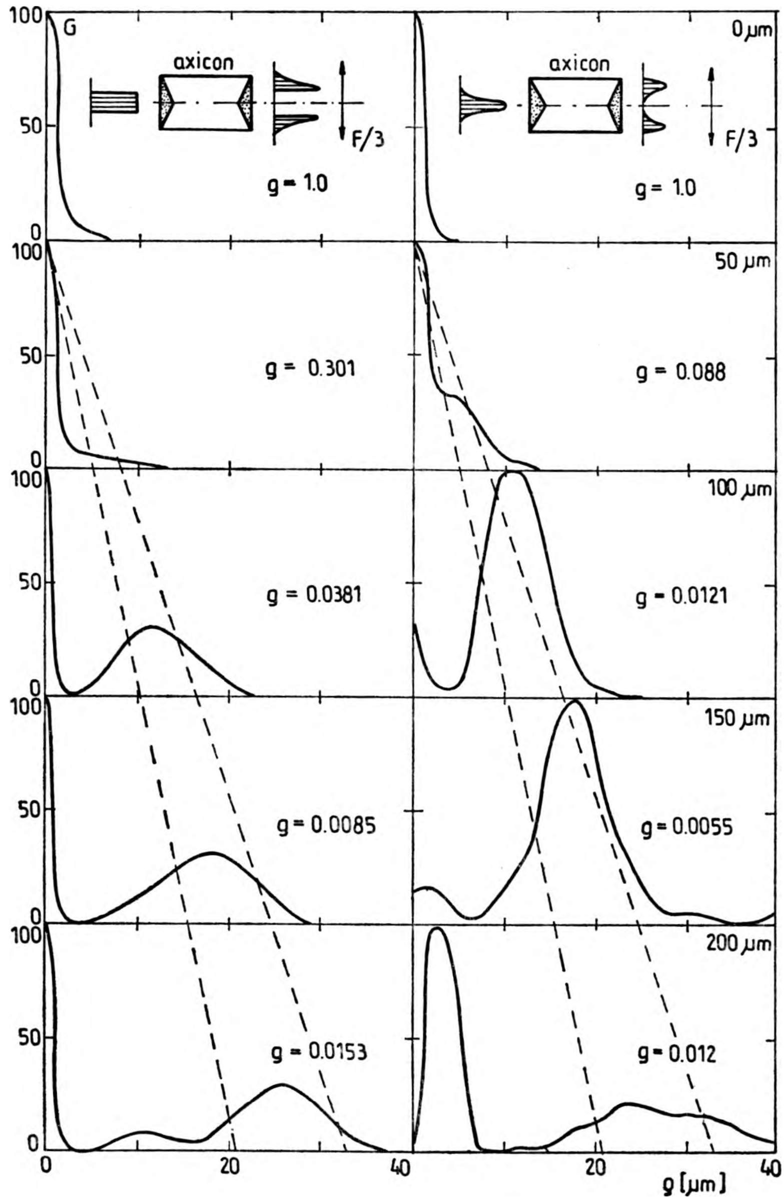

Fig. 6. Distributions of the relative power density G in the cross-sections distant by 50 µm from one another, starting from the focus vs. the distance from the optical axis: left hand column – rectangular beam, right hand column – Gaussian beam (g – ratio of the maximal power density at given cross-section to the maximal power density at the focus)

5. Final remarks

The calculations of the focusing of the laser beam after its passage through the axicon were performed in order to evaluate the effects occuring in the optical system of a laser device designed for perforation of the iris [7, 8].

The experimental evaluation of the field distribution for a rectangular beam confirmed quantitatively the correctness of the calculations, while the quantitative comparisons with experiment provided no positive result due to aberrations of axicon. The observed asymmetry of the field distribution behind the axicon and the diffraction occuring on the aperture diaphragm have caused the spread of the focus and the asymmetry of the distribution in particular cross-sections. It should be also remembered that when such transformation are applied, the aberrations of the optical system and the influence of the edges of the integration are neglected.

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Анализ распределения плотности мощности поблизости фокуса безаберрационных фокусирующих систем с большой яркостью

Представлены две интегральные трансформации, описующие поле поблизости фокуса очень ярких апланатных оптических систем. Сравнены результаты обеих трансформаций для поля на оптической оси. Проведены расчеты фокусировки лазерного излучения после перехода через аксикон (axicon) для прямоугольного или гауссового входного пучка а также обсуждены полученные результаты.