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Shear strain mapping from moiré interferometry

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Principles of the new optical method for determining shear strain contours in the object under load investigated by moiré interferometry are presented. In the direct approach not requiring the dense fringe maps of in-plane displacements, the lateral shear interferograms of all four diffraction orders of the specimen grating are used. When properly overlapped and spatially filtered they give the map of γ_{xy}.

1. Introduction

Moiré interferometry is a high sensitivity technique for studying in-plane displacements of deformed bodies [1]. Theoretically the sensitivity higher by two orders of magnitude with comparison to the classical moiré method [2] can be obtained. This is possible due to the use of high frequency reflective type diffraction grating fixed to the specimen. The cross-type grating is illuminated by two perpendicular pairs of plane wave front beams. Each pair consists of two beams impinging symmetrically on opposite sides of the specimen grating normal at an angle equal to the first order diffraction angle. The $+1$ diffraction order from one illuminating beam and the -1 order of the other beam propagate along the specimen grating normal. Wavefront warpages of the two beams, because of line deformations of the specimen grating introduced by the load, are mutually conjugate. On the other hand, the wavefront warpages caused by out-of-plane displacements are the same in both interfering beams. They mutually subtract. The fringes obtained give a contour map of in-plane displacements with half a period sensitivity.

When the incidence angles of the illuminating beams are not exactly tuned to the first order diffraction angle of the specimen grating, the carrier fringes are introduced. When they are dense the interferograms of moiré interferometry method correspond to specimen grating lines in classical moiré method [1], [2]. Therefore, all known methods for obtaining the derivatives of in-plane displacements corresponding to normal strains ε_x and ε_y can be applied to the moiré interferometry patterns (for example, for the list of references see [3]). The same remark concerns the methods for determining the shear strain contours [4]–[8].

In this paper, the shearing interferometry approach for generating the shear strain maps [8] will be extended to moiré interferometry configuration. The principles of a direct method without registering the in-plane displacement interferograms will be presented.

Although the lateral shear interferometry was already applied to moiré interferometry configuration for giving the derivatives of in- and out-of-plane displacements [3], [9]–[11] its use for mapping the shear strain contours is new. The unique way of overlapping the interferograms and their spatial filtering leads to the whole field map of the shear strain distribution.

2. Description of the method

Figure 1 shows schematic representation of the optical configuration. One pair of plane wave front beams with propagation directions L_x and R_x in the incidence plane xz illuminates the cross-type specimen grating SG. They produce the interferogram representing the in-plane displacement $u(x, y)$. The other pair with propagation

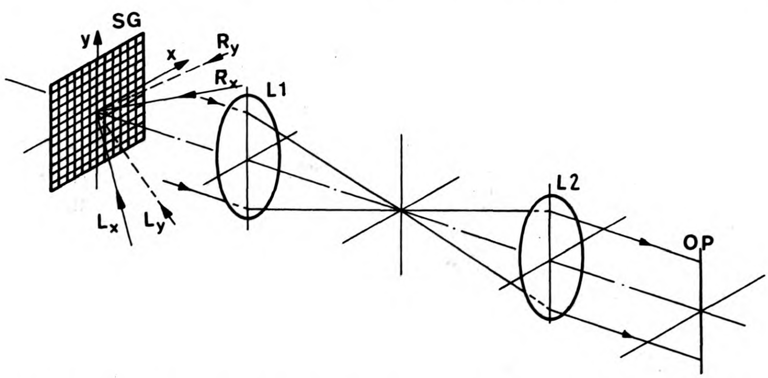

Fig. 1. Optics of the moiré interferometry method. Objectives L1 and L2 image the specimen grating plane SG in the observation plane OP. SG is illuminated by two pairs of plane wave front beams with directions L_x, R_x, L_y, R_y appropriately tuned to the grating first order diffraction angle. In general, single objective imaging optics can be used

directions L_y and R_y in the plane yz produces the interferogram of in-plane displacement $v(x, y)$. For example, when the directions L_x and R_x are at the angle equal to the first order difraction angle of SG, the amplitude of the two interfering diffraction orders in the observation plane OP can be represented as

$$E(x, y) = \exp\left\{i\left[\frac{2\pi}{d}u(x, y)+kw(x, y)\right]\right\}+\exp\left\{i\left[-\frac{2\pi}{d}u(x, y)+kw(x, y)\right]\right\} \quad (1)$$

where d denotes the period of specimen grating SG, $u(x, y)$ is the in-plane displacement along the x direction, $w(x, y)$ is the out-of-plane displacement, and $k = 2\pi/\lambda$. Amplitudes of both beams have been normalized to unity. The resulting interference pattern has the intensity

$$I_1(x, y) = 2\left\{1+\cos\frac{2\pi}{d}2u(x, y)\right\}, \quad (2)$$

and represents the contour map of $u(x, y)$ with half a period sensitivity. Angular mismatch of the incidence angle of the illuminating beams results in the tilt angle between the interfering diffraction orders. The intensity is

$$I_1(x, y) = 2\left\{1+\cos\left[2k\Theta_x x+\frac{2\pi}{d}2u(x, y)\right]\right\} \tag{3}$$

where $2\Theta_x$ is the angle between interfering beams. Now the information about $u(x, y)$ is in the form of the departure of fringes from straightness. When the fringes are dense the interferograms can be called the "gratings of displacements". The description of Eq. (3) can be easily extended to the other pair of illuminating beams giving the map of $v(x, y)$, i.e.

$$I_2(x, y) = 2\left\{1+\cos\left[2k\Theta_y y+\frac{2\pi}{d}2v(x, y)\right]\right\}. \tag{4}$$

Under proper conditions the lines of both interferograms are mutually perpendicular and have the same average period.

Having the interference patterns $I_1(x, y)$ and $I_2(x, y)$ we can apply any of previously reported methods [4]–[8] for generating the map of shear strains

$$\gamma_{xy} = \frac{\partial u(x, y)}{\partial y}+\frac{\partial v(x, y)}{\partial x}. \tag{5}$$

However, in the case of moiré interferometry method we can device another more direct method not requiring the "gratings of displacements" $u(x, y)$ and $v(x, y)$. On the other hand, extending the optical shearing interferometry approach [8], we propose to generate the lateral shear interferograms of each of the diffraction orders of SG used in the experiment. They should be dense carrier fringe patterns with line deformations proportional to $\partial u(x, y)/\partial y$ or $\partial v(x, y)/\partial x$. It is necessary to note that the diffraction orders of SG in the moiré interferometry configuration carry the information not only about the in-plane displacements $u(x, y)$ or $v(x, y)$ but also about the out-of-plane displacement $w(x, y)$. Because of that it is important to superimpose the "gratings of derivatives" in a special order. The way of conducting the experiment is as follows.

All four illuminating beams should be properly adjusted in space to have all four first order diffraction beams of SG propagating coaxially along the optical axis, i.e., the grating normal. Next, the specimen grating should be separately illuminated by each of the illuminating beams. The lateral shear interferogram of the diffraction order is then produced in the way described in detail in [8]. Again a linear diffraction grating can serve as the beam-splitting and shearing element. It is placed near the common focal plane of lenses L1 and L2 (Fig. 1). For example, when one of the orders of SG carrying the information about $u(x, y)$ enters the imaging optics L1–L2, the lines of the beam-splitting grating should be aligned along the x direction, i.e., be perpendicular to the lines of the grating being analysed. To ensure the maximum contrast of shearing interferograms the spatial filter set in the spatial frequency plane should transmit $+1$ and -1 orders of the shearing grating. It can be easily proved that the intensity distributions of the four derivative interferograms can be calculated as

$$I_{+1x}(x,y) \propto 1+\cos 2\left[k\Theta'_y y+\Delta y\frac{2\pi}{d}\frac{\partial u(x,y)}{\partial y}+\Delta yk\frac{\partial w(x,y)}{\partial y}\right], \tag{6}$$

$$I_{-1x}(x,y) \propto 1+\cos 2\left[k\Theta'_y y-\Delta y\frac{2\pi}{d}\frac{\partial u(x,y)}{\partial y}+\Delta yk\frac{\partial w(x,y)}{\partial y}\right], \tag{7}$$

$$I_{+1y}(x,y) \propto 1+\cos 2\left[k\Theta'_x x+\Delta x\frac{2\pi}{d}\frac{\partial v(x,y)}{\partial x}+\Delta xk\frac{\partial w(x,y)}{\partial x}\right], \tag{8}$$

$$I_{-1y}(x,y) \propto 1+\cos 2\left[k\Theta'_x x-\Delta x\frac{2\pi}{d}\frac{\partial v(x,y)}{\partial x}+\Delta xk\frac{\partial w(x,y)}{\partial x}\right] \tag{9}$$

where Δx and Δy are the shear amounts along x and y directions, respetively, $2\Theta'_x$ and $2\Theta'_y$ denote the tilt angles between the interfering beams.

Let us produce now cross-type gratings DG1 and DG2 superimposing interferograms (6) and (8), and (7) and (9), respectively. The multiplicative or additive superimpositions can be employed. Next, the structures obtained are inserted in the double coherent optical processor (Fig. 2). The first one is located in the front focal

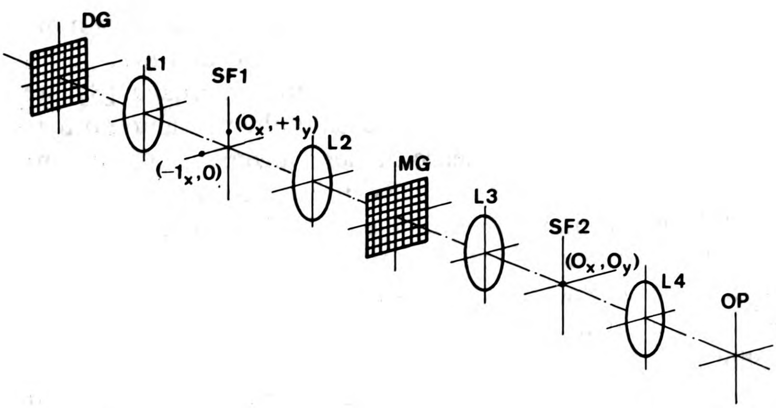

Fig. 2. Double spatial filtering system for producing the map of shear strain γ_{xy} from the two cross-derivative two-dimensional periodic structures DG1 and DG2. SF1 and SF2 – spatial filters

plane of L1 and the second one in the back focal plane of L2 (front focal plane of L3). Let us assume that the two openings of SF1 have the coordinates $(+1_x, 0)$ and $(0, -1_y)$. Numbers in the parenthesis correspond to the locations of double diffraction spots of each two-dimensional structure. A single opening of SF2 will be located on the optical axis. In this way, two beam interference pattern is formed in the observation plane OP. Its intensity distribution is

$$I(x,y) = \left|\exp\left\{i2\left[\frac{2\pi}{d}\Delta\frac{\partial v(x,y)}{\partial x}+k\Delta\frac{\partial w(x,y)}{\partial x}+\frac{2\pi}{d}\Delta\frac{\partial v(x,y)}{\partial x}-k\Delta\frac{\partial w(x,y)}{\partial x}\right]\right\}\right.$$
$$\left.+\exp\left\{i2\left[-\frac{2\pi}{d}\Delta\frac{\partial u(x,y)}{\partial y}-k\Delta\frac{\partial w(x,y)}{\partial y}-\frac{2\pi}{d}\Delta\frac{\partial u(x,y)}{\partial y}+k\Delta\frac{\partial w(x,y)}{\partial x}\right]\right\}\right|^2$$
$$= \left|\exp\left\{i4\frac{2\pi}{d}\Delta\frac{\partial v(x,y)}{\partial x}\right\}+\exp\left\{-i4\frac{2\pi}{d}\frac{\partial u(x,y)}{\partial y}\right\}\right|^2$$
$$= 2\left\{1+\cos\frac{2\pi}{d}4\Delta\left[\frac{\partial u(x,y)}{\partial y}+\frac{\partial v(x,y)}{\partial x}\right]\right\} \tag{10}$$

where $\Delta = \Delta x = \Delta y$. The interferogram obtained gives the map of shear strain γ_{xy} with the sensitivity 4 times higher than the sensitivity of the moiré of moiré method [4], [7].

Other combinations of the openings in spatial filter SF1 are possible, for example, $(+1_x, -1_y)$ and $(-1_x, +1_y)$. They give the same results but with reduced average intensity in the observation plane.

3. Conclusions

Application of the lateral shear interferometry to all four diffraction orders of the specimen grating utilized in moiré interferometry method leads to four separate fringe patterns. When properly grouped, overlapped and spatially filtered in the double coherent optical processor the contour map of the shear strain γ_{xy} in the object under load can be obtained.

The experimental verification of the principles given will be presented elsewhere.

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Изготовление карт деформаций во время сдвига методом мори

Даны принципы нового метода для определения угла сдвиговой деформации нагружаемого конструкционного элемента, исследуемого при помощи интерференционного метода мори. В непосредственном подходе, не нуждающемся в интерференционных полосах с высокой пространственной полосой перемещений в плоскости, изготовляется интерферограммы с поперечным смещением волнового фронта для всех четырёх порядков дифракции деформированной сетки. В результате их соответствующего наложения и фильтрации пространственных частот получают карту угла сдвиговой деформации γ_{xy}.