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Numerical investigation of the imaging quality of diffraction lenses*

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A numerical method of determining of the light intensity distribution in the aberration spot which is produced by means of the diffraction lens has been presented. The procedure is based on modifications of the method applied in a determination of the light intensity distribution for the holographic imaging. The imaging quality for a choice of diffraction lenses has been estimated taking, particularly, into account the aplanatic and collimating diffraction lenses.

1. Introduction

The image quality, obtainable by using a diffraction lens, has been the subject of recent investigations [1]. By the diffraction lens we mean a type of the zone plate or, in other words, a synthetic holographic lens with a rectangular modulation of transmittance. Therefore, when we are looking the means for correcting the aberration of diffraction lenses, we may use the whole method of analysis of the imaging quality which is commonly applied in the case of holographic imaging. This approach has been used in [2] in the aberration analysis of the diffraction lens, where the image quality of the selected diffraction lenses was investigated for a one-dimensional case. It is well known that the image quality of the holographic lens can be improved by a proper choice of the position of the entrance pupil or, alternatively, by applying a lens made on a curved surface, [3]–[7]. In the case of the diffraction lens the improvement can be attained for plane lenses, provided that the distribution of fringes is modified to have the aberration characteristics corresponding to the one for a holographic lens made on a curved surface. An algorithm for determination of the form of the modified fringes has been reported in[2] and we shall not present it here. In this paper the intensity distribution of the diffraction spot has also been found numerically for selected diffraction lenses in a one-dimensional case. The results which stand for the one-dimensional case cannot be however generalized to the case of an actual, two-dimensional lens. Nevertheless, this manner might be quite justified while tackling the problem of the holographic lens: it reduces time-consuming calculations, whereas the aberration-spot intensity distributions in the meridional

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2. Method of determining of the aberration-spot intensity distribution

In this paper, we present a method of determination of the intensity distribution yielded by two-dimensional diffraction lenses. In the following, this method will be applied to investigation of the imaging quality of the selected types of the lens. The algorithm used is the modified one for the investigation of the holographic imaging [8], [9] and we will briefly present only some essential differences due to the specifity of the diffraction lens. The means of numerical calculations at the stage of constructing the diffraction lens is analogous to that for the holographic lens. In the former, the rectangular modulation (rather than the sinusoidal one) of the lens transmittance is taken into account and the eventual modifications of the fringe distribution are made to have the aberration characteristics of the diffraction lens corresponding to the one for a lens made on a sphere [2].

Numerical reconstruction of the image should be made in a different way. The assumption of this method requires the same energy of light "ray" under calculation, in the case of a holographic lens this condition is fulfilled when the crossing density betwen the rays of the object wave and the lens plane is constant. The calculated intensity distribution in the image plane will be consistent with the true one, if a sufficient number of image-constituting rays are included in calculation. The accuracy of this kind of calculation has been discussed in [8]. As regards the diffraction lens, in order the condition of the same energy be fulfilled for each "ray", the crossing density between the traces of the image-constituting rays and the diffraction-lens plane should be constant on each fringe rather than on the entire plane. This is illustrated in Fig. 1. The point object is denoted by $P_C(x_C, y_C, z_C)$, and

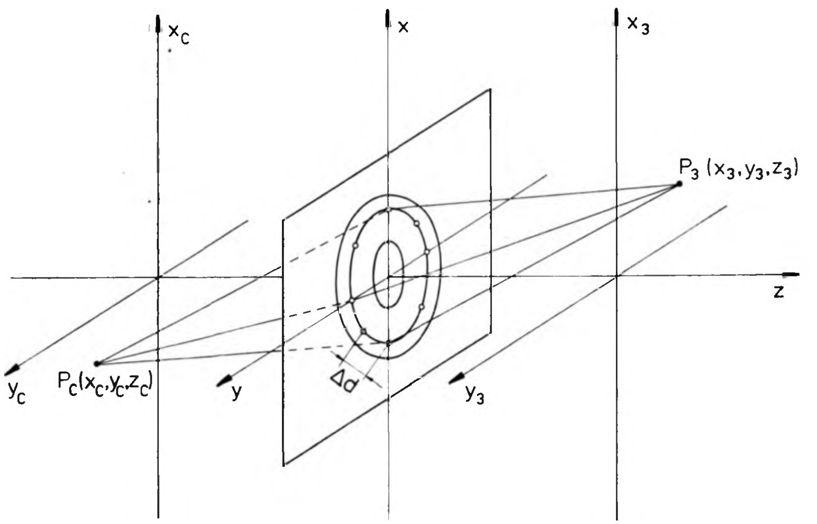


Fig. 1. Geometry of ray tracing through a diffraction lens

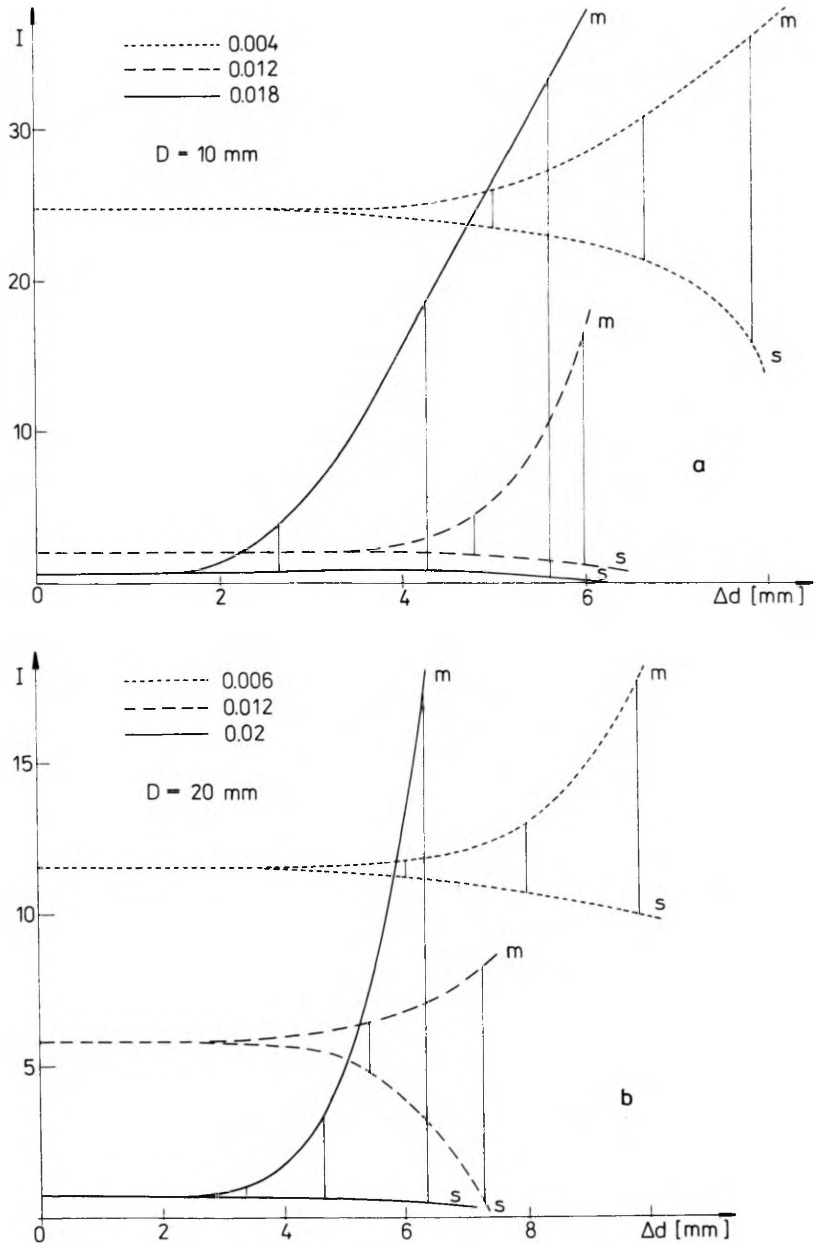


Fig. 2. Calculated light intensity in two choosen points equally distant from the aberration spot centre vs distance of rays in the differenction lens pupil (**a** — 10 mm, **b** — 10 mm)

$P_3(x_3, y_3, z_3)$ denotes an image point. Several bright fringes are drawn in the lens plane and the traces of the crossing of the image-constructing rays are marked. To have the same energy for each "ray", the condition of Δd = const has to be fulfilled for all the fringes. Strictly speaking, the intensity distribution obtained in such a way will be related to a fringe lens which constitutes only a particular case of the diffraction lens. We are interested, however, in the aberration characteristics rather than in the problem of the diffraction efficiency, that is why such an *ad hoc* approach seems to be adequate.

The number of including "rays" is of great importance, particularly, when the calculation is made with a microcomputer. It was useful to examine the effect of the number of the included rays on the accuracy of the results. The programme tests have been made for two diffraction lenses of diameter 10 mm (Fig. 2**a**) and 20 mm (Fig. 2**b**), respectively, with a point object located axially. The calculated values of the light intensity are shown in the figures for the chosen pairs of image points lying at the same distance from aberration spot centre, but in its different, meridional and sagittal cross-sections (denoted by subscripts m and s, respectively) – as a function of the distance Δd. Because of the symmetry of the spot the light intensity in both cross-sections should be the same for a given image point and must be independent of the distance Δd. In this way, we could find tolerable maximum distances between the included rays in the lens plane, namely $\Delta d = 1$ mm ($\sim$4200 rays) and $\Delta d = 1.5$ mm ($\sim$11000 rays) for the diameters under consideration.

3. Numerical results and final conclusions

The conditions for the recording of aplanatic holographic lenses and their imaging have been given in [10]. If the aberration characteristics of the diffraction lens can be assumed to be analogous to those of the holographic lens, the corresponding diffraction lenses should be aplanatic, too. The geometry of the "recording" of selected diffraction lenses and imaging which were investigated in this work is given in Tab. 1. In this table, z_1 and z_R are the locations of two sources of waves

Table 1. Geometry of the "recording" and imaging with the selected diffraction lenses

Lens No.	z_1	z_R	z_C	z_3	ϱ	t	D	μ
I–1	200	−200	−200	200	∞	0	20	1
I–2	100		−200	200	∞	0	20	2
I–3	916.12	−183.28	−200	200	∞	0	20	1.5275
I–4	183.28	−916.12	−200	200	∞	0	20	1.5275
II	150	−300	−300	150	∞	0	15	1
III–1	100	∞	∞	100	∞	0	10	1
III–2	100	∞	∞	100	∞	−50	10	1
III–3	100	∞	∞	100	100	0	10	1

"constituting" the diffraction lens, ϱ is an equivalent "curvature" of the surface of the diffraction lens (related to the required modification of the fringes), t is the location of the entrance pupil, D is the diameter of the lens, μ is the ratio of the light wavelengths used for the imaging and for the "recording", respectively, and z_C and z_3 are the same as in Fig. 1. All the relevant magnitudes in the table and throughout this paper are given in millimeters. Lenses denoted by I–1, ..., I–4 are aplanatic lenses according to the relations in [10].

Figure 3 shows the distribution of light intensity of the aplanatic lenses for three different field angles. The values of parameters of the imaging quality, such as the

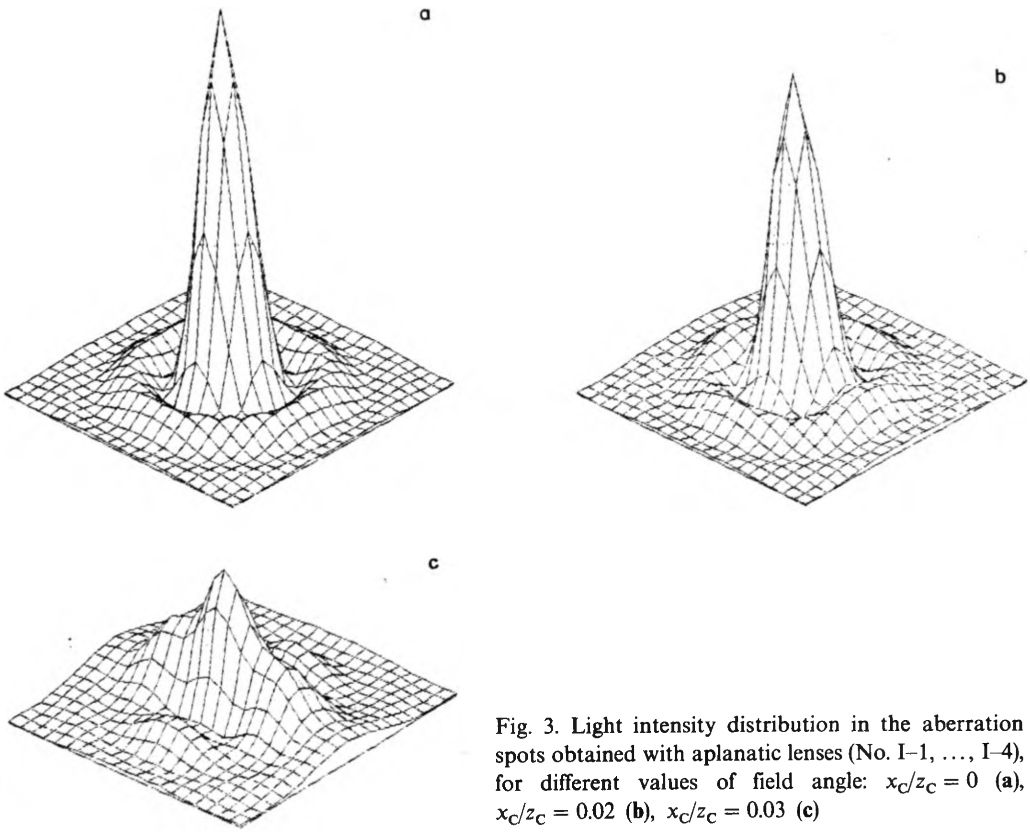


Fig. 3. Light intensity distribution in the aberration spots obtained with aplanatic lenses (No. I–1, ..., I–4), for different values of field angle: $x_C/z_C = 0$ (**a**), $x_C/z_C = 0.02$ (**b**), $x_C/z_C = 0.03$ (**c**)

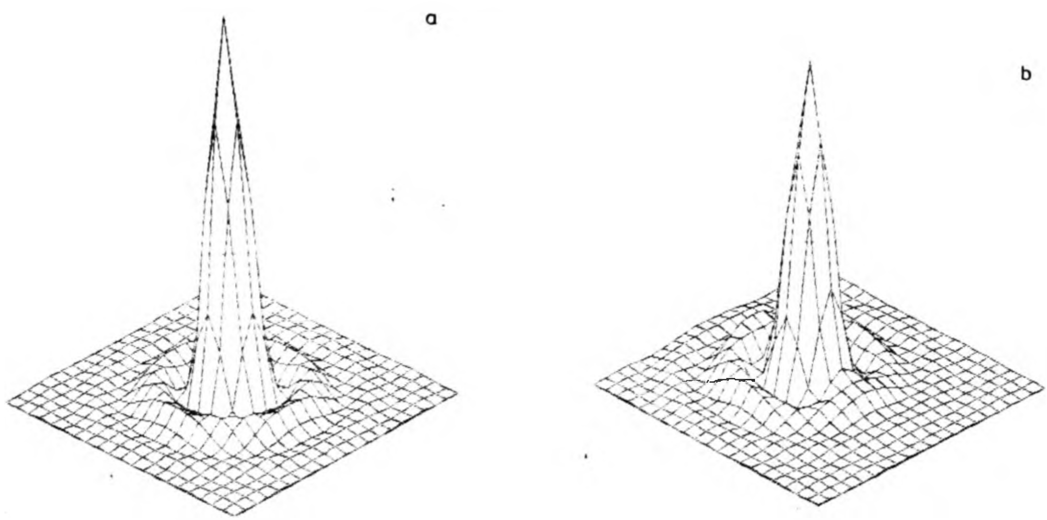
normalized intensity of the aberration spot centre (Strehl ratio), the second and the third intensity moments (for the definition, see e.g. [8]) and the 80% energy spot diameter are given in Tab. 2. Practically the same intensity distribution and the same quantitative parameters of the imaging quality correspond to all the four lenses, hence, we confine ourselves to a single example.

The analysis of the imaging quality of the aplanatic diffraction lens shows that it is corrected relatively well only for a small field of $x_C/z_C \leqslant 0.02$, furthermore, it is burdened with astigmatism and the field curvature. Both aberrations confine seriously the usefulness of those lenses for larger field angles. Therefore, we investigated lenses other than the aplanatic ones, i.e., those having uncorrected coma, which proved to have a smaller astigmatism and to eventually yield a better imaging.

Two such exemplary lenses have been chosen and denoted by II and III–1 in Tab. 1, where also relevant "recording" and imaging parameters can be found. The latter lens is a collimating lens. The result of investigation of the imaging quality of these lenses is presented in Figs. 4 and 5, and in Tab. 2.

A brief examination of the figures shows that both the lenses yield a better imaging possibility in comparison with the aplanatic lenses. The collimating lens yields adequate imaging for a slightly larger field of $x_C/z_C \leqslant 0.03$, and this lens was chosen for further modification to obtain still larger field angle.

The next steps were to find the optimum location of the entrance pupil (III–2) and the optimum "curvature" of the lens surface (III–3). The results of analysis of the imaging quality are shown in Figs. 6 and 7, and collected in Tab. 2. A considerable improvement of the imaging quality has been attained with the entrance pupil shifted by 50 mm before the lens plane where a large field angle of $x_C/z_C \leqslant 0.06$ can be admitted. This effect was brought about by correction of astigmatism (with coma having been left and the spot remaining asymmetric), however, a smaller effect was



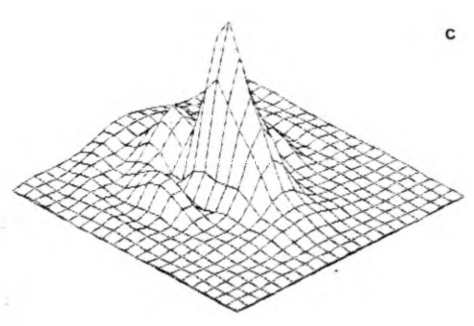


Fig. 4. Light intensity distribution in the aberration spots obtained with the lens No. II, for different values of field angle: $x_C/z_C = 0$ **(a)**, $x_C/z_C = 0.02$ **(b)**, $x_C/z_C = 0.03$ **(c)**

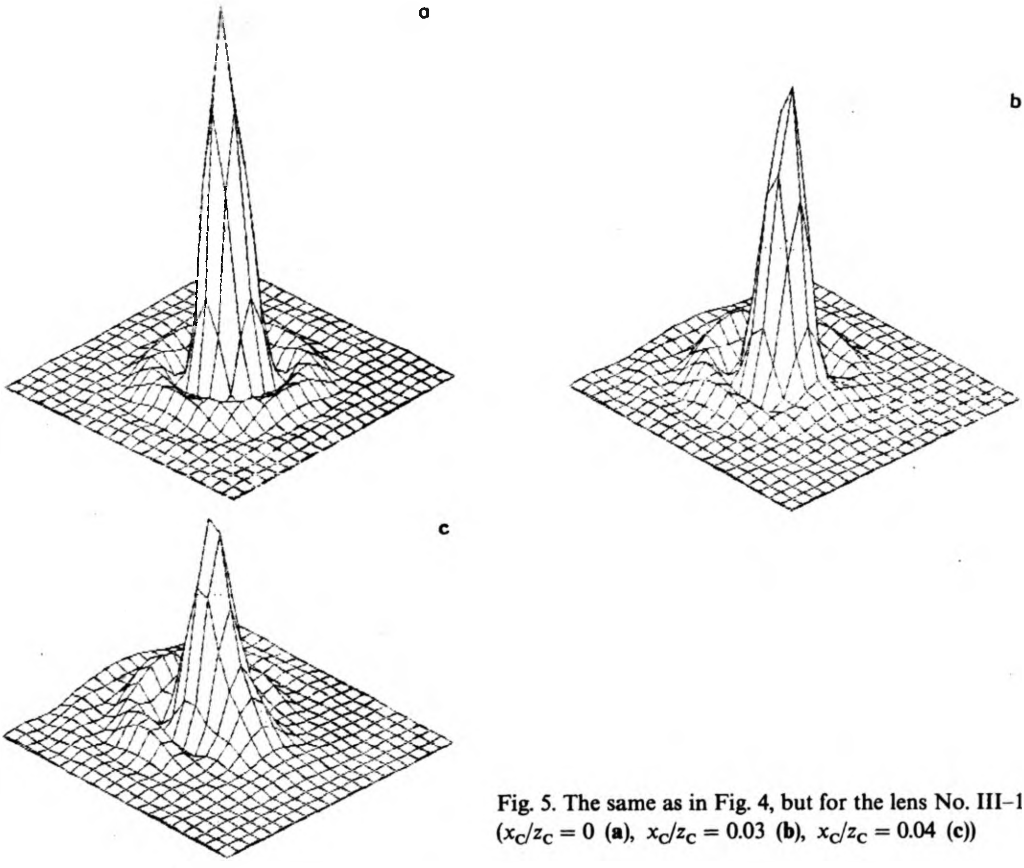


Fig. 5. The same as in Fig. 4, but for the lens No. III–1 ($x_C/z_C = 0$ (**a**), $x_C/z_C = 0.03$ (**b**), $x_C/z_C = 0.04$ (**c**))

Table 2. Some parameters of the aberration spots with the investigated diffraction lenses

Lens No.	x_C/z_C	I_{max}	M_{2x}	M_{2y}	M_{3x}	M_{3y}	$d_{0.8}$
	0	1	1.9×10^{-5}	1.9×10^{-5}	0	0	.01
I	.02	.825	3.3×10^{-5}	2.3×10^{-5}	0	0	.011
	.03	.366	8.0×10^{-5}	4.4×10^{-5}	0	0	.014
	0	1	$1,3\times10^{-5}$	1.3×10^{-5}	0	0	.008
II	.02	.836	2.2×10^{-5}	1.7×10^{-5}	4.4×10^{-8}	0	.009
	.03	.436	4.8×10^{-5}	2.2×10^{-5}	7.1×10^{-8}	0	.011
	0	1	1.4×10^{-5}	1.4×10^{-5}	0	0	.008
III–1	.03	.78	2.2×10^{-5}	1.8×10^{-5}	5.2×10^{-8}	0	.009
	.04	.54	3.5×10^{-5}	2.0×10^{-5}	6.4×10^{-8}	0	.010
	0	1	1.4×10^{-5}	1.4×10^{-5}	0	0	.008
III–2	.04	.871	1.6×10^{-5}	1.4×10^{-5}	6.8×10^{-8}	0	.008
	.06	.797	2.1×10^{-5}	2.3×10^{-5}	1.1×10^{-7}	0	.008
	0	1	1.3×10^{-5}	1.3×10^{-5}	0	0	.008
III–3	.03	.854	1.5×10^{-5}	2.2×10^{-5}	0	0	.009
	.04	.604	2.1×10^{-5}	3.8×10^{-5}	0	0	.010

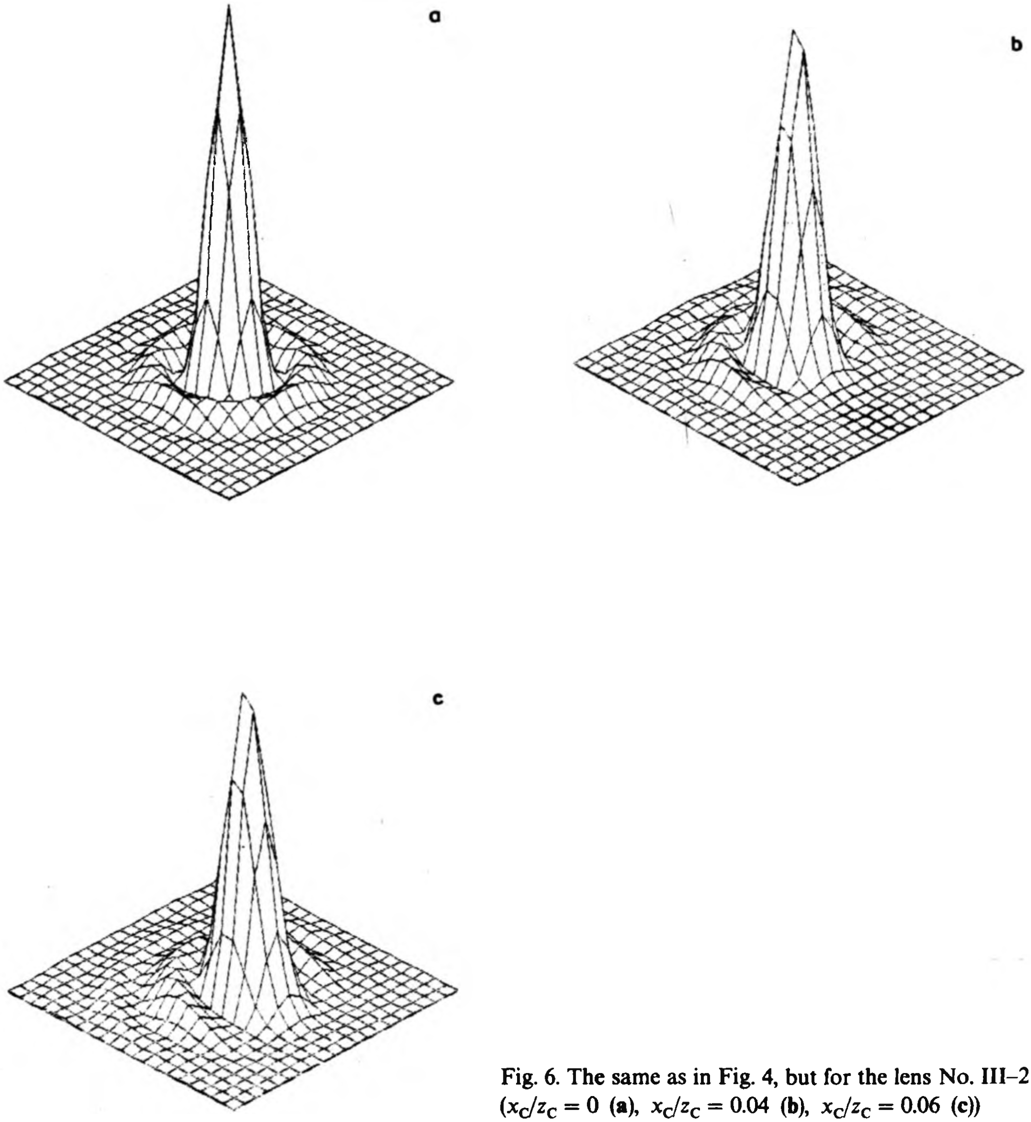


Fig. 6. The same as in Fig. 4, but for the lens No. III–2 ($x_C/z_C = 0$ (**a**), $x_C/z_C = 0.04$ (**b**), $x_C/z_C = 0.06$ (**c**))

obtained as a result of correcting coma by "curving" the surface. The latter case leads to a symmetrical spot but astigmatism is an obstacle in the improvement of the original lens (III–1).

Such evaluation of the imaging, which includes only the analysis of the aberration spot of one-point object, is certainly incomplete, however, it is very useful in preliminary estimation of the imaging quality and may be recommended as a reconnaissance procedure when dealing with diffraction lenses, in particular.

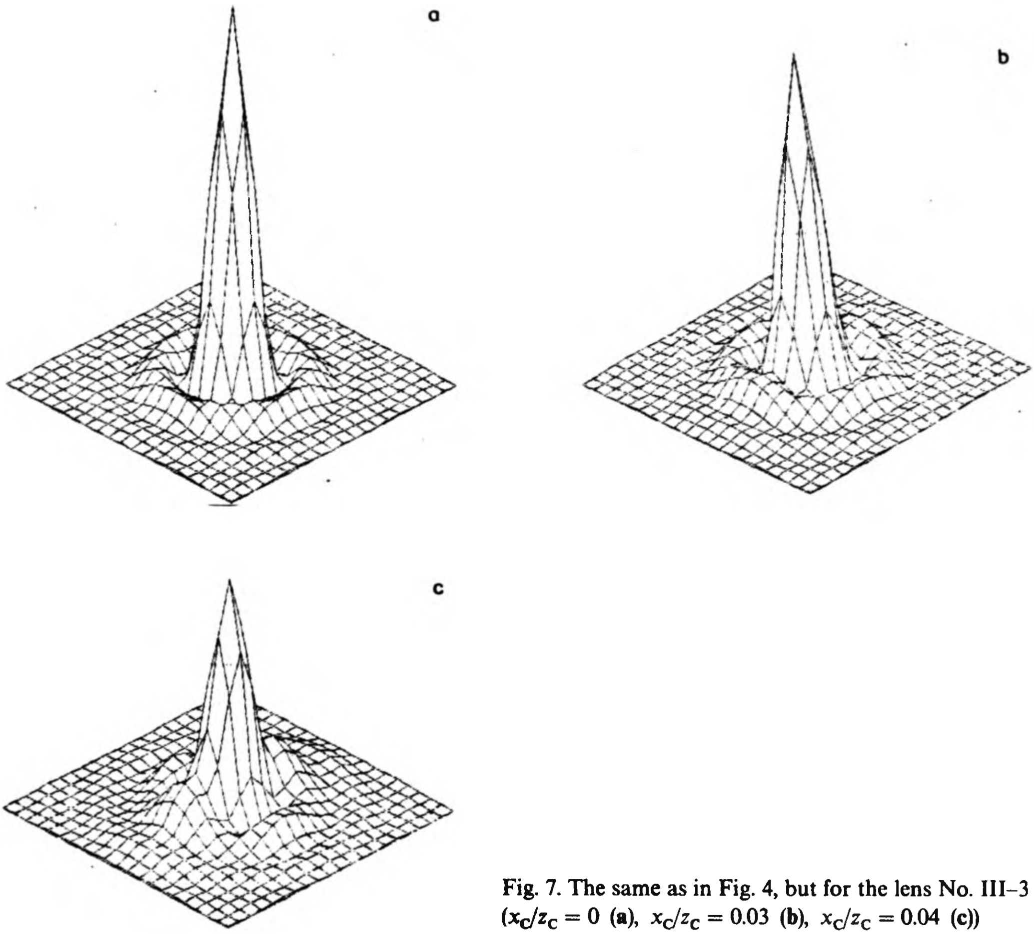


Fig. 7. The same as in Fig. 4, but for the lens No. III–3 ($x_C/z_C = 0$ (**a**), $x_C/z_C = 0.03$ (**b**), $x_C/z_C = 0.04$ (**c**))

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Численные исследования качества изображения диффракционных линз

В работе приведен численный метод определения распределения интенсивности света в аберрационном пятне, полученном от диффракционной линзы. Процедура основана на модификации метода, употребляемого раньше для численного моделирования голографического изображения. Исследовано качество изображения избранных диффракционных линз, особенно для апланатических и коллимационных апланатических линз.