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Aberration coefficients method for evaluation of optimal refractive index distribution in rotationally symetric GRIN materials*

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A method for evaluation of the optimal refractive index distribution in rotationally symmetric GRIN materials has been proposed. The method is based on the rms spot size as the quality measure. This rms spot size was approximated by its analytical expression using aberration coefficients of third and fifth orders. The proposed method has been exemplified by numerical results.

1. Introduction

In conventional optical systems refractive index within each optical component is assumed to be constant. Materials whose index of refraction varies continuously with the material are called gradient index materials (GRIN). Of course, the quality of optical devices composed of GRIN elements depends upon the type of gradients of refractive index.

There are three main types of refraction index gradients: axial, spherical and radial (cylindrical) ones.

The cylindrical type of gradient varies continuously outward from the optical axis, so that the surface of constant index is formed by cylinders whose common axis coincides with the optical axis of optical element.

Optical devices composed of GRIN components may be designed in two different ways. One of them is based on the ray tracing. However, this way is very time consuming, even for computer, due to the fact that the ray path within GRIN material is not rectilinear. The other way is based on the aberration theory. This theory for GRIN materials has been developed for both third and fifth order aberration coefficients having relatively simple analytical form for GRIN materials with cylindrical gradients [1], [2]. As the quality criterion the rms spot size will be used. This criterion expressed in terms of Buchdahl aberration coefficients has been proposed by Robb [3]. The main advantage of the results obtained by Robb is their independence upon grids of rays computed by ray tracing through the optical system. An additional advantage of Robb's proposition appears when applied to GRIN materials with radial gradient, since in this case the aberration coefficients are of analytical form.

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Thus our task is to calculate refractive index distribution in cylindrical GRIN materials and the optimal, in the sense of rms spot size. For a better clarity we present also the main points of Robbs proposition.

2. Theory

In this paper we consider rotationally symmetric refractive index distribution of the form

$$n(u) = n_0[1-1/2(\alpha^2 u)+1/2\beta(\alpha^2 u)^2+\gamma(\alpha^2 u)^3] \tag{1}$$

where n_0 is the refractive index at the axis of symmetry, u is the squared distance of any point from this axis, α, β and γ are constants of second-, fourth- and sixth-orders, respectively. For the refractive index of the form (1) the aberration theory has been developed for both the third- and fifth-order aberrations. The explicit forms of the corresponding third-order aberrations have been derived by THYAGERAJAN et al. [1]:

$$y_3(\alpha,\beta,\varrho,\theta,H) = S_1\varrho^3\cos\theta+(3S_3+S_4)H^2\varrho\cos\theta, \tag{2a}$$

$$x_3(\alpha,\beta,\varrho,\theta,H) = S_1\varrho^3\cos\theta+(S_3+S_4)H^2\varrho\sin\theta \tag{2b}$$

where ϱ, β are polar coordinates of the ray hitting the exit pupil and H is a measure of the field coordinate. We see that the only aberration coefficients different from zero are S_1, S_3 and S_4, i.e.:

$$S_1 = z_1\alpha^3(5/16-3\beta/4),\qquad S_3 = 1/4z_1\alpha^3(\beta+1/4),\qquad S_4 = 1/2z_1\alpha^3 \tag{3}$$

where z_1 denote the paraxial image locations ($z_1 = m\frac{\pi}{\alpha}$, $m = 1, 2,\ldots$).

An analysis of fifth-order aberrations for inhomogeneous media of the form (1) was performed a few months later by GUPTA et al. [2]. They obtained the following results:

$$y_5 = (\alpha,\beta,\gamma,\varrho,\theta,H)A\varrho^5\cos\theta+(B_1+B_2\cos 2\theta)H\varrho^4+H^2\varrho^3(C_1+C_2\cos^2\theta)$$
$$+H^3\varrho^2(D_1+D_2\cos 2\theta)+H^4\varrho E\cos\theta+FH^5, \tag{4a}$$

$$x_5(\alpha,\beta,\gamma,\varrho,\theta,H) = A\varrho^5\sin\theta+H\varrho^4 G\sin 2\theta+H^2\varrho^3\sin\theta(H_1+H_2\cos^2\theta)$$
$$+H^3\varrho^2 P\sin 2\theta+H^4\varrho Q\sin\theta. \tag{4b}$$

Aberration coefficients A, $B_1,\ldots$, depend upon the parameters α, β, γ as follows:

$$A = z_1\alpha^3(159/128+15\gamma-69\beta/16-33\beta^2/8)/8,$$

$$B_1 = 2\alpha^6 z_1[z_1/64(-3\beta^2+\beta^3/2-35/16)+1/2\zeta(15\beta^2-5\beta^8/4+55/16)], \tag{4c}$$

with $\zeta = (2m-1)\pi/(2\alpha)$; the explicit forms of the remaining coefficients are written in paper [2]. Further, coordinates of the interception point of an arbitrary ray with the paraxial image plane, in the fifth-order approximation, are:

$$y(\alpha,\beta,\gamma,\varrho,\theta,H) = y_0+y_3(\alpha,\beta,\varrho,\theta,H)+y_5(\alpha,\beta,\varrho,\theta,H), \tag{5a}$$

$$x(\alpha, \beta, \gamma, \theta, H) = x_3(\alpha, \beta, \gamma, \theta, H) + x_5(\alpha, \beta, \gamma, \varrho, \theta, H) \tag{5b}$$

where y_0 denotes the height of the ideal image point. To calculate the standard deviation in y and x directions it is necessary to evaluate the following integrations over the aperture:

$$\sigma_y^2(\alpha, \beta, \gamma, \varrho_m, H) = 1/(\pi \varrho_m^2) \int_0^{\varrho_m} \int_0^{2\pi} [y(\alpha, \beta, \gamma, \varrho, \theta, H) - y_c]^2 \varrho \, d\varrho \, d\theta, \tag{6a}$$

$$\sigma_x^2(\alpha, \beta, \gamma, \varrho_m, H) = 1/(\pi \varrho_m^2) \int_0^{\varrho_m} \int_0^{2\pi} [x(\alpha, \beta, \gamma, \varrho, \theta, H)]^2 \varrho \, d\varrho \, d\theta, \tag{6b}$$

where

$$y_c = 1/(\pi \varrho_n^2) \int_0^{\varrho_m} \int_0^{2\pi} y(\alpha, \beta, \gamma, \varrho, \theta, H) \varrho \, d\varrho \, d\theta. \tag{6c}$$

From Equation (6c) we see that y_c is the centroid of the spot diagram. Finally, the rms radius of the spot diagram is defined as

$$\sigma_r(\alpha, \beta, \gamma, \varrho_m, H) = \sqrt{\sigma_x^2(\alpha, \beta, \gamma, \varrho_m, H) + \sigma_y^2(\alpha, \beta, \gamma, \varrho_m, H)}. \tag{7a}$$

Integrations of (6a) and (6c) after the respective factorization give

$$\sigma_y^2(\alpha, \beta, \gamma, \varrho_m, H) = a_0 + a_1 H^2 + a_2 H^4 + a_3 H^6 + a_4 H^8 \tag{7b}$$

where

$$\begin{aligned}
a_0 &= A^2 \varrho_m^{10}/12 + AS_1 \varrho_m^8/5 + S_1^2 \varrho_m^6/8, \\
a_1 &= \varrho_m^8 (B_2^2/10 + 4B_1^2/45 + 3/20\, AC_2) + \varrho_m^6 (3S_1 C_2/16 + S_1 C_1/4 \\
&\quad + AS_4/4 + 3AS_3/4) + \varrho_m^4 (S_3 S_1 + S_4 S_1), \\
a_2 &= \varrho_m^6 (AE/4 + D_1 B_1/6 + D_2 B_2/4 + 3C_1 C_2/16 + C_1^2/8 + 5C_2^2/64) + \varrho_m^4 (ES_1/3 \\
&\quad + S_3 C_1 + 3S_3 C_2/4 + S_4 C_1/3 + S_4 C_2/4) + \varrho_m^2 (3S_3 S_4/2 + 9S_3^2/4 + S_4^2/4), \\
a_3 &= \varrho_m^4 (EC_1/3 + EC_2/4 + D_1^2/12 + D_2^2/6) + \varrho_m^2 (3ES_3/2 + ES_3/2 + ES_4/2), \\
a_4 &= \varrho_m^2 E^2/4.
\end{aligned} \tag{7c}$$

In similar way the standard deviation σ_x^2 may be evaluated

$$\sigma_x^2(\alpha, \beta, \gamma, \varrho_m, H) = b_0 + b_1 H^2 + b_2 H^4 + b_3 H^6 + b_4 H^8 \tag{8a}$$

where:

$$\begin{aligned}
b_0 &= AS_1 \varrho_m^8/5 + A^2 \varrho_m^{10}/12 + S_1^2 \varrho_m^6/8, \\
b_1 &= AS_3 \varrho_m^6/4 + AS_4 \varrho_m^6/4 + AH_1 \varrho_m^8/5 + AH_2 \varrho_m^8/20 + S_1 S_3 \varrho_m^4/3 + S_1 S_4 \varrho_m^4/3 \\
&\quad + S_1 H_1 \varrho_m^6/4 + S_1 H_2 \varrho_m^6/16 + S_3^2 \varrho_m^8/10,
\end{aligned}$$

$$b_2 = AQ\varrho_m^6/4+S_1Q\varrho_m^4/3+S_3S_4\varrho_m^2/2+S_3H_1\varrho_m^4/3+S_3H_2\varrho_m^4/12+S_4H_1\varrho_m^4/3$$
$$+S_4H_2\varrho_m^4/12+GP\varrho_m^6/4+H_1H_2\varrho_m^6/16+S_3^2\varrho_m^2/4+S_4^2\varrho_m^2/4+H_1\varrho_m^6/8+H_2^2\varrho_m^6/64,$$
$$b_3 = S_3Q\varrho_m^2/2+S_4Q\varrho_m^2/2+H_1Q\varrho_m^4/3+H_2Q\varrho_m^4/12+P^2\varrho_m^4/6,$$
$$b_4 = Q^2\varrho_m^2/4. \tag{8b}$$

Expression (7) and (8) depend exclusively upon the α, β, γ GRIN coefficients, the aperture ϱ_m and the field of view H.

3. Numerical results and conclusions

In the present study, we have proposed an evaluation method of optimal refractive index distribution in rotationally symmetric GRIN elements. The obtained relations have been programmed and numerical calculations carried out. The calculations were performed for different values of aperture and field of view ($\varrho_m = 0.2$, 0.5, 0.75, 1.0 and $H = 0$, 1.0). The following values of parameters were taken for calculations $a = 0.2$ and $-1 < \gamma < 0$. Under these assumptions the values of β were calculated by optimizing the corresponding rms spot size. The obtained results are presented graphically in Figs. 1 and 2. We see that for axial boundle (Fig. 1) there exists a common optimal solution ($\beta = 0.4$, $\gamma = -0.09$) irrespective of aperture. This result should be expected because (1) in an abbreviated Taylor expansion of GRIN of the form

$$n(u) = n_0 \operatorname{sech}^{-1}(\alpha\sqrt{u}), \quad \text{when } \beta = 5/12 \quad \text{and} \quad \gamma = -61/720.$$

We should not forget that the proposed method is based on two approximations:

i) assumption of the paraxial plane as the image plane,

ii) optical aberrations approximated by third- and fifth-order terms only.

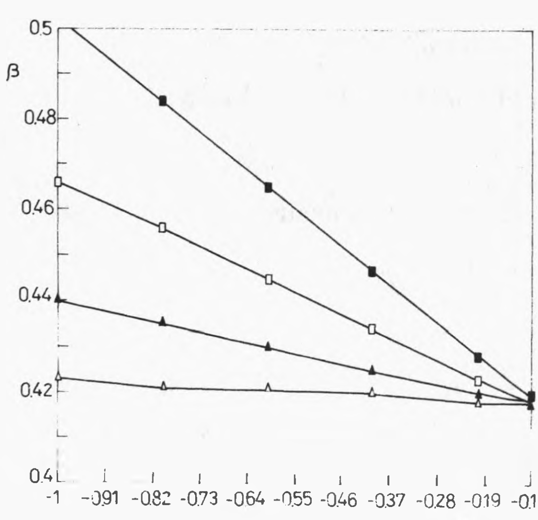

Fig. 1. Optimal GRIN parameters γ versus β for $H = 0$. and different apertures ϱ_m (△ 0.25, ▲-0.5, ◊-0.75, □-1.0)

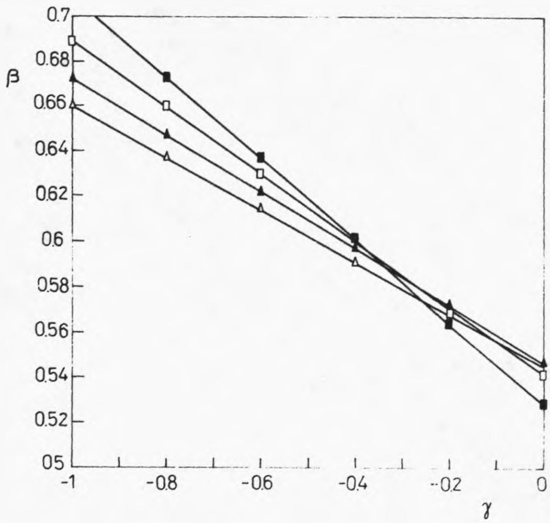

Fig. 2. The same as in Fig. 1, but for $H = 1$

Therefore, the obtained results should be understood as the first approximation in evaluations. It is obvious that, this remark is true also in the case of slightly modified method based on rms spot size being averaged over the field of view.

References

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Метод аберрационных коэффицентов оптимального решения для коэффициента преломления в ротационном симметричном GRIN материале

Предложен метод вычислений оптимального распределения коэффициента преломления в ротационном симметричном GRIN материале. Метод базирует на определении размеров изображения точки как меры качества. Изображение точки было аппроксимировано с помощью аналитических выражений с использованием аберрационных коэффициентов третьего и пятого порядков.