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Simple minimum resolvable temperature difference model for thermal imaging systems

K. Chrzanowski

Institute of Optoelectronics, Military Academy of Technology, 00-908 Warszawa, Poland.

Thermal Imaging System (TIS) performance is described by the minimum resolvable temperature difference (MRT) which is the spatial-frequency-dependent thermal resolution. The so far derived MRT models have been developed for TIS constructors; for determining the MRT many TIS parameters unknown to the usual TIS users must be known. This paper presents an MRT model which can be used for determining MRT, based only on the typical TIS catalogue data, i.e., spatial resolution, temperature resolution and field of view. The model is a revised and simplified form of Ratches-Lawson one. The revision is based on the author's knowledge of the accuracy of Ratches-Lawson (R-L) model, topical values of normalized noise equivalent bandwidth, and on the relationship between the spatial resolution of the device and its modulation transfer function. In this work an improved MRT model is tested by comparing the calculated MRTs of three known systems to be the measured values.

1. Introduction

The process of searching the display of a thermal imaging system for a target consists of three interrelated processes: detection, recognition, and identification. Detection is the discrimination of an object from its background and its assignment to the class of potentially interesting objects. Recognition is the assignment of the detected object to a specific subclass such as men, trucks, etc. Identification is the discrimination of the recognized object as a particular member of a class of objects, e.g., tank M60, T55, etc. The probabilities of detection, recognition, and identification are used to assessment of a thermal imaging system perceptual capabilities.

MRT is one of the most important thermal imaging system parameters, and describes the system thermal resolution and its dependence on spatial frequency. It is defined as the minimum temperature difference above 300 K required by an observer viewing through the device to resolve a vertical four bar pattern of 7:1 aspect ratio. If MRT is known, the probability of detection, recognition, and identification can be determined [1], [2]. A few MRT models [1]–[4] have been derived. The models have been developed for TIS constructors; to design systems, to meet specific applications and to evaluate competitive design. For determining MRT many TIS parameters must be known. The typical TIS users usually do not know these parameters, thus, the so far derived MRT models can not be used by them to assessment of TIS perceptual capabilities.

In this work an MRT model is derived. It can be used to determine MRT based only on typical catalogue data: spatial resolution, temperature resolution and field of view, parameters known to TIS users. Thermal and spatial resolution are usually specified as follows [5]–[8]. Spatial resolution ω is the angle subtended by the observed object which is small enough to reduce the video signal of the system to one half of the maximum signal amplitude obtained for a large object. Thermal resolution ΔT is the temperature difference between two large blackbodies which gives a signal equal to the total noise amplitude of the system. This means that thermal resolution is defined as Noise Equivalent Temperature Difference (NETD). There are a few other definitions of thermal and spatial resolution. However, the definitions which have been presented above are usually found in the TIS catalogue data.

The model is a revised and simplified torm of the Ratches–Lawson model. The revision is based on the author's knowledge about the accuracy of the Ratches–Lawson model, typical values of normalized noise equivalent bandwidth and on the relationship between spatial resolution of the device and its modulation transfer function. The model developed in this paper has been tested by comparing the calculated MRTs of three known systems to the measured values. The determined MRT based on catalogue data can be used for the following purposes:

i) for military applications (determining the probabilities of detection, recognition and identification of any military target, the thermal signature of which is known),

ii) comparison of different systems when only their catalogue data are known,

iii) planning of testing,

iv) prediction of field performance through modelling.

2. Derivation of the MRT expressions

The most widely known MRT model was derived by Ratches and Lawson. It is used nowadays as the industry standard and has the following form [1], [2]:

$$\mathrm{MRT}(f) = \frac{\pi^2 \mathrm{SNR\ NETD}}{4\sqrt{14}\,\mathrm{MTF_{TIS}}(f)}\left[\frac{VB(f)f\,\mathrm{DAS}}{\Delta f F_R t_E c}\right]^{\frac{1}{2}}, \tag{1}$$

where:

SNR – threshold signal to noise ratio,

$\mathrm{MTF_{TIS}}$ – thermal imaging system MTF,

V – angular scan velocity,

f – spatial frequency generated when scanning across test pattern bars,

NETD – Noise Equivalent Temperature Difference,

Δf – electronic noise band-pass,

$B(f)$ – effective noise bandwidth of TIS including spatial integration effect of the eye,

DAS – detector angular subtense,

c – overscan ratio,
f_R – frame rate,
t_E – eye integration time.
SNR, NETD, t_E, V, Δf, DAS, F_R, c are independent of spatial frequency f. Therefore MRT equation can be written as follows:

$$\mathrm{MRT}(f) = A\frac{f\sqrt{K_{SP}(f)}}{\mathrm{MTF_{TIS}}(f)} \tag{2}$$

where:

$$K_{SP} = \frac{B(f)}{f}, \tag{3}$$

K_{SP} – normalized TIS noise equivalent bandwidth, A – MRT factor.

The R–L model accuracy is really good in the middle and high-frequency ranges, but the calculated MRT values tend to be overoptimistic (lower) in the low-frequency range [2]. Typical MRT values in the low-frequency range are usually approximately equal to the device temperature resolution value (defined as Noise Temperature Equivalent Difference). Thus, for MRT calculation, when only the catalogue data are known, it can be assumed that MRT is equal to temperature resolution ΔT in the low-frequency range, and that in the middle- and the high-frequency ranges, the reasonable values of MRT can be obtained using the R–L model.

Assume that the limit between the low- and the middle-frequency range is the frequency f_1 and that

$$\mathrm{MTF_{TIS}}(f_1) = 0.6 \tag{4}$$

where $\mathrm{MTF_{TIS}}$ is TIS modulation transfer function.

The Fourier transform of the thermal imaging system line spread function (LSF) is the optical transfer function (OTF). This definition is an appropriate special case of the more general definition of the optical transfer function as the complex Fourier transform of the point spread function. The line spread function analysis isolated one dimension which is convenient for thermal imaging systems. For modelling purposes the OTF is usually approximated by its absolute value, which is the modulation transfer function (MTF). The MTF is sine wave amplitude response function . The value of this function is normalized to unity at or near zero spatial frequency by convention. The assumption (4) is based on the author's knowledge about MRT (measured and determined using R–L model) and MTF of some TIS. Therefore MRT can be given by equation

$$\mathrm{MRT}(f) = \begin{cases} \Delta T & \text{for } f \leqslant f_1, \\ A\dfrac{f\sqrt{K_{SP}(f)}}{\mathrm{MTF_{TIS}}(f)} & \text{for } f \geqslant f_1. \end{cases} \tag{5}$$

For $f = f_1$ MRT is equal to the temperature resolution ΔT. Therefore

$$\Delta T = A\frac{f_1\sqrt{K_{SP}(f_1)}}{\mathrm{MTF_{TIS}}(f_1)}. \tag{6}$$

Thus, MRT factor has the form

$$A = \frac{0.6\Delta T}{f_1\sqrt{K_{SP}(f_1)}}. \tag{7}$$

After employing Eq. (7) MRT has the form

$$\mathrm{MRT} = \begin{cases} \Delta T & \text{for } f \leqslant f_1, \\ \dfrac{0.6\Delta T f\sqrt{K_{SP}(f)}}{f_1\sqrt{K_{SP}(f_1)}\,\mathrm{MTF_{TIS}}(f)} & \text{for } f \geqslant f_1. \end{cases} \tag{8}$$

To calculate $\mathrm{MRT}(f)$ from Equation (8) the $\mathrm{MTF_{TIS}}$, f_1 and K_{SP} are required. As they can not be taken from typical catalogue data, they can be derived according to the procedure described below.

The total system MTF is approximately the product of all component modulation transfer functions. The TIS components are: optic, detector, electronics, display and eye. Thus the TIS modulation transfer function has the form

$$\mathrm{MTF_{TIS}} = \mathrm{MTF_{OPT}}\mathrm{MTF_{DET}}\mathrm{MTF_{ELECT}}\mathrm{MTF_{DISPLAY}}\mathrm{MTF_{EYE}} \tag{9}$$

where:
$\mathrm{MTF_{OPT}}$ – optics modulation transfer function,
$\mathrm{MTF_{DET}}$ – detector modulation transfer function,
$\mathrm{MTF_{ELECT}}$ – electronics modulation transfer function,
$\mathrm{MTF_{DISPLAY}}$ – display modulation transfer function,
$\mathrm{MTF_{EYE}}$ – observer modulation transfer function.

The device components are: optics, detector, electronics and display. Thus the expression for the $\mathrm{MTF_{TIS}}$ can be written

$$\mathrm{MTF_{TIS}} = \mathrm{MTF_{DEV}}\mathrm{MTF_{EYE}} \tag{10}$$

where $\mathrm{MTF_{DEV}}$ is the device modulation transfer function.

The well known central limit theorem of probability and statistics has an analog in linear filter theory, which is that the product of N bandlimited continuous MTFs tends to a Gaussian form as N becomes large. All the devices have at least four component MTFs, so that the device line spread function can be adequately approximated by

$$\mathrm{LSF_{DEV}} = \exp-(f/\delta)^2 \tag{11}$$

where $\mathrm{LSF_{DEV}}$ is the device line spread function, and δ is standard deviation. The device modulation transfer function is Fourier transform of the device line spread function. The corresponding device modulation transfer function is

$$\mathrm{MTF_{DEV}}(f) = \exp(-\pi^2 \delta^2 f^2). \tag{12}$$

The relationship between the standard deviation δ and the device spatial resolution ω can be written as [9]

$$\omega = 0.96\delta \tag{13}$$

where ω is the device spatial resolution. Therefore, taking Eq. (13) into consideration we obtain

$$\mathrm{MTF_{DEV}}(f) = \exp(-10.71\omega^2 f^2). \tag{14}$$

The observer is the last component of a thermal imaging system. The eye modulation transfer function has a simplified form [10]

$$\mathrm{MTF_{EYE}}(f) = \exp[-\pi^2 \delta^2 (f/M)^2] \tag{15}$$

where M is the device visual magnification. For typical display brightness $\delta = 0.28$–0.43. Taking the average value $\delta = 0.35$ we get

$$\mathrm{MTF_{EYE}}(f) = \exp[-1.21(f/M)^2] \tag{16}$$

Assuming that the display angular subtenses for the observer are equal to typical field of view of the human eye, we obtain

$$M = \frac{\mathrm{VFOV_{EYE}}}{\mathrm{VFOV_{DEV}}} \tag{17}$$

where $\mathrm{VFOV_{EYE}}$ is typical field of view of the human eye in vertical direction 20–30 degrees (the average value VFOV = 25 degrees is employed), $\mathrm{VFOV_{DEV}}$ is device field of view in vertical direction. Finally, after employing Eqs. (10), (14), (16), (17) the $\mathrm{MTF_{TIS}}$ has the form

$$\mathrm{MTF_{TIS}}(f) = \exp[-(10.71\omega^2 + 0.001936\ \mathrm{VFOV_{DEV}^2}) f^2]. \tag{18}$$

Now the $\mathrm{MTF_{TIS}}$ is known. Let us determine the limit between the low- and the middle-frequency ranges the frequency f_1. After employing Eqs. (4) and (18), we get

$$f_1 = \sqrt{\frac{\ln(0.6)}{-(10.71\omega^2 + 1936\,10^{-6}\mathrm{VFOV_{DEV}^2}) f^2}}. \tag{19}$$

Let us determine the normalized TIS noise equivalent bandwidth. A typical plot of K_{SP} versus MTF_N is shown in Fig. 1, for the case when MTF_N is Gaussian and the observer psychophysical response is considered to be $\mathrm{sinc}(\pi k/2\ f)$ [11]. The relationship between K_{SP} and MTF_N can be approximated by polynomial

$$K_{SP} = 0.571 + 0.40182\,\mathrm{MTF}_N \tag{20}$$

where MTF_N is TIS components between detector output and the observer modulation transfer function. Because the components are electronics, display and

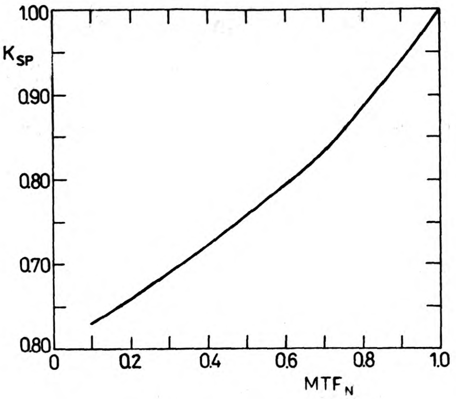

Fig. 1. Typical values of normalized noise equivalent bandwidth

observer, therefore

$$MTF_N = MTF_{ELECT} MTF_{DISPLAY} MTF_{EYE}. \tag{21}$$

All the device components are optics, detector, electronics and display. Thus, MTF_{DEV} has the form

$$MTF_{DEV} = MTF_{OPT} MTF_{DET} MTF_{ELECT} MTF_{DISPLAY}. \tag{22}$$

For the optimally designed TIS quality of the device components should be identical. For example quality of the optics should be approximate to quality of the detector. Thus we can assume

$$MTF_{OPT} = MTF_{DET} = MTF_{ELECT} = MTF_{DISPLAY}. \tag{23}$$

Therefore, we obtain (from Eqs. (21)–(23))

$$MTF_N(f) = \sqrt{MTF_{DEV}(f)}\, MTF_{EYE}(f). \tag{24}$$

Using Equations (14), (16), (17) and (23), MTF_N has the form

$$MTF_N(f) = \sqrt{\exp - 10.71\omega^2 f^2}\, \exp[-0.001936 f^2 VFOV^2_{DEV}]. \tag{24}$$

Finally, after employing Eqs. (20) and (24), the normalized TIS noise equivalent bandwidth K_{SP} has the form

$$K_{SP}(f) = 0.571 + 0.40182\sqrt{\exp - 10.71\omega^2 f^2}\, \exp[-0.001936 f^2 VFOV^2_{DEV}]. \tag{25}$$

Now all the missing data: MTF_{TIS}, f_1 and K_{SP} are known and MRT can be determined using Eqs. (8), (18), (19) and (25). As in has been shown, MRT can be determined from typical catalogue data: spatial resolution ω [mrad], temperature resolution ΔT [deg], field of view [deg].

3. Discussion and conclusions

Let us now examine the MRT model, developed in this work, by comparing the calculated MRTs of a few known systems to the measured values. The systems, of which the measured values of MRT are known (the MRTs have been published), are: AGA–780 (Sweden) [12], Rubin MT (USSR) [13], Raduga MT (USSR) [4]. Their catalogue data taken from [14] are:

$\Delta T = 0.1$ K, $\omega = 3.4$ mrad, VFOV = 20° for AGA–780,
$\Delta T = 0.05$ K, $\omega = 2.04$ mrad, VFOV = 10° for Rubin MT,
$\Delta T = 0.2$ K, $\omega = 2.04$ mrad, VFOV = 17° for Raduga MT.

The MRT curves (measured and calculated) are plotted in Fig. 2**a**, **b**, **c**. For AGA–780 the differences between the measured and the calculated values are quite significant for the spatial frequencies from 0.075 to 0.12 cycle/mrad and over 0.13 cycle/mrad. For example, at the spatial frequency of 0.1 cycle/mrad the measured value is 2/3 of that calculated by the derived model. For Raduga MT the derived model yields the lower MRT values (more optimistic values) than the measured values in the high-frequency range for frequencies over 0.176 cycle/mrad. The most significant differences between the measured and the calculated values are for Rubin MT. For example, at the spatial frequency of 0.2 cycle/mrad the calculated value is 1/2 the measured one. But at the spatial frequency of 0.2 cycle/mrad it is conversely so and the measured value is 6/10 of the calculated one. As it is seen (Fig. 2), the differences between the measured and calculated MRTs are sometimes quite significant. But the so far derived MRT models such as the R–L model or the adaptive matched filter (AMF) model accuracy for many TIS can be worse than the derived model accuracy. For example, in the low-frequency range the MRT values calculated by R–L model are 1/10 the measured values and the MRTs calculated by the AMF model are 10 times or more higher than the measured values in the middle- and the high-frequency range [1]. Thus, for a typical TIS the accuracy of the derived model is really good, compared to that of other models such as R–L and AMF ones.

The differences between the measured and the calculated MRT curves for the reference systems can be due to the following reasons:

– the assumptions concerning R–L model and its simplifications,

– for the low-frequency range the MRT values are not exactly equal to the temperature resolution ΔT, being dependent on the frame rate f_R, and overscan ratio c, angular scan velocity v, electronic noise band-pass Δf, which have not been taken into consideration,

– $\mathrm{MTF}_{\mathrm{DEV}}$ of a real device is not exactly approximated by Gaussian function,

– in the presented model a constant M is assumed, but an observer can optimize his distance to screen; he can change the visual magnification M,

– in Figure 1 only typical values of normalized noise equivalent bandwidth are presented, moreover the relationship between K_{SP} and MTF_N is approximated,

– device parameters can be different for different copies and are time dependent.

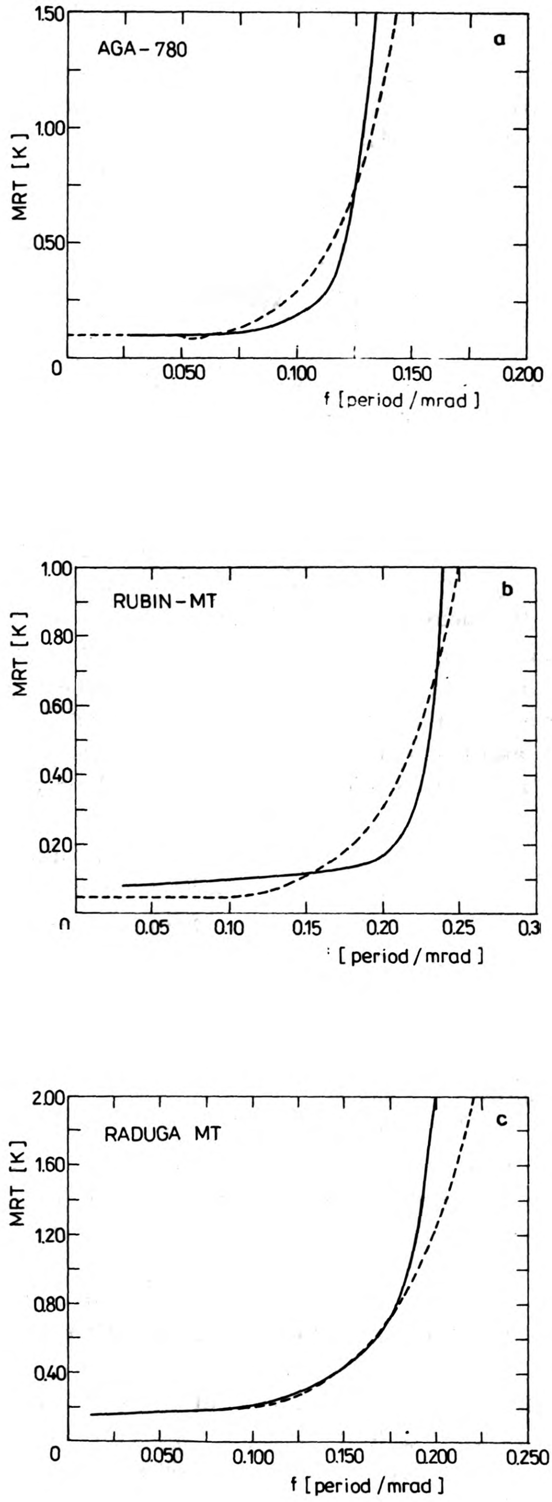

Fig. 2. Calculated and measured MRT for the reference systems: **a** – for AGA 780, **b** – for Rubin MT, **c** – for Raduga MT (——— measured values, — — — — calculated ones)

Relationship (11) is satisfied only when the output signal is the convolution of the spread function of the device with the input signal. Therefore, using the derived model we can determine MRT in the scanning direction (when the bars are oriented perpendicularly to the scan direction) for line scanned TIS. At present the line scanned TIS are the most important group of thermal equipment.

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Простая модель минимального разрешаемого температурного напора тепловизионных систем

В статье представлена новая, простая модель минимального разрешаемого температурного напора (MRT) основной характеристики тепловизионных систем. Разработанные до сих пор модели MRT предназначены для конструкторов тепловизионных систем. Для определения MRT необходимо знание многих конструкционных параметров, которые неизвестны пользователям этих устройств. Представленная модель позволяет определить MRT на основе типичных каталоговых данных тепловизионных систем (угловое разрешение, порог температурной чувствительности, поле обзора). Модель проверена посредством сравнения рассчитанных и измеренных характеристик MRT для нескольких известных тепловизионных систем.