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Real-time differentiation of moiré interferometry patterns by incoherent superimposition of lateral shear interferograms

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A practical system for implementing a real-time optical differentiation of in-plane displacement patterns obtained by the moiré interferometry method is described. It performs an in-registry incoherent superimposition of lateral shear interferograms of the specimen grating diffraction orders. Theoretical description and experimental results are presented.

1. Introduction

The moiré interferometry method has already become a well established technique of high sensitivity for the whole field mapping of in-plane displacements of deformed bodies [1]–[3]. A reflective type specimen diffraction grating is fixed to the object under study and illuminated by two mutually coherent collimated beams. First order diffracted beams of opposite sign propagate collinearly along the specimen grating normal and interfere. Since the beams carry mutally conjugate information about the in-plane displacements (corresponding to the departure of specimen grating lines from straightness) the interference pattern provides the map of in-plane displacements with double sensitivity.

From the point of view of strain analysis, not only the displacement fields are desired but their partial derivatives, as well. Several methods proposed for that purpose have been reviewed and grouped in the works by Patorski et al. [4], and Dai et al. [5]. Among non-real time approaches the mechanical shearing [6] and optical shearing [7] techniques are used. In the first case two identical (or positive and negative) copies sufficiently dense displacement fringe patterns are overlapped and laterally shifted. The obtained moiré fringes map the derivative of displacement along the shift direction averaged over the shift distance. In the second case the lateral shear interferograms, with dense tilt fringes, of each diffraction order from the specimen grating are recorded separately and overlapped. Again moiré fringes display the derivative information.

Various optical methods that provide simple real-time differentiation of moiré interferometry fields can be categorized into two main groups called:

i) optical shearing of displacement interferograms [4],

ii) superimposition of lateral shear interferograms [4], [5], [8].
The required incoherence between the overlapped patterns is introduced by polarization effects. The map of displacement derivatives of the object under load is given by additive type moiré fringes, i.e., fringes of the sum of intensity distributions of two quasi-periodic patterns. An experimental demonstration was provided for the case of optical shearing of displacement fringes.

The purpose of this work is to describe a simple implementation of the superimposition of two lateral shear interferograms of wavefronts from the specimen grating. Instead of mutually orthogonal linear polarizations for the illuminating beams [4], [5], [8] the use of two circularly orthogonal states is recommended. In this way there is no more need to introduce high optical quality polarizing elements into illuminating beam. The choice of the shearing unit is discussed and the experimental data are given.

2. Analysis

The method of in-registry superimposition of lateral shear interferograms of specimen grating diffraction orders requires the illumination wavefronts to be mutually incoherent. When the beams are derived from the same source (the use of two independent sources is not practical because of the cost), then the required incoherence may be most simply obtained either by introduction of dissimilar path lengths into the illumination arrangement (exceeding the coherence length of the laser light source) or by orthogonal polarizations. Since the latter approach gives a more compact interferometer system, it will be used in the following analysis.

Figure 1 shows the real time arrangement for overlapping the lateral shear interferograms of the specimen grating diffraction orders, as proposed in the previous paper [4]. Polarizations of illuminating beams A and B are vertical and horizontal, respectively, i.e., parallel and perpendicular to the grating lines. An ordinary nonpolarizing beam splitter is used. The incidence angles of A and B must be

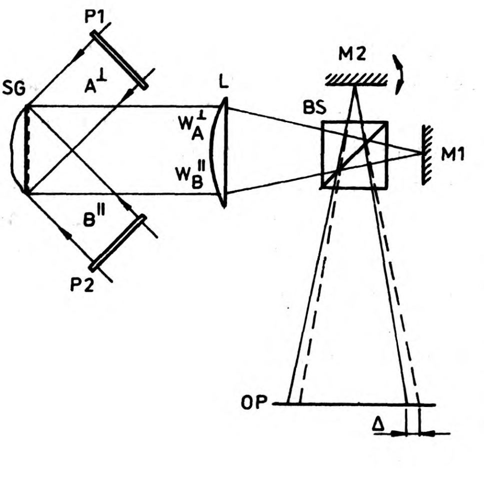

Fig. 1. Real time arrangement for overlapping the lateral shear interferograms of the specimen grating diffraction orders [4]. P1 and P2 – linear polarizers with axes set parallelly and perpendicularly to the lines of specimen grating SG, A and B – illuminating beams, L – imaging lens, BS – beam splitter, M1 and M2 – plane mirrors, OP – observation plane

adjusted to have W_A and W_B propagating collinearly. When tilting, for example, M2 the lateral shear interferograms of W_A and W_B appear simultaneously. No carrier fringes are encountered when M1 and M2 are at the back focal plane of the lens L. Carrier fringes should be introduced to density the interferograms, thereby to facilitate further formation of moiré fringes. In the arrangement shown in Fig. 1, this can be done by displacing axially the beam splitter and mirrors toward the lens L. Then the beams in both arms of the interferometer are no longer focused at M1 and M2. If one of them rotates, the carrier fringes running perpendicularly to the shear direction are encountered. The shear and tilt values are not independent in this configuration.

A simultaneous displacement of BS, M1 and M2 made in order to introduce quasi-linear carrier fringes (the interference of two spherical wavefronts results in fringes of hyperbolic shape) requires a special compact design of the shearing unit. Such a design is also required to minimize the influence of external vibrations, a factor of basic importance in experimental mechanics studies. Therefore, the use of a common path lateral shear interferometer is recommended.

Figure 2 shows real-time arrangement for differentiating moiré interferometry in-plane displacement patterns used in our recent experiments. Besides the different lateral shearing interferometry unit (double grating shearing interferometer that is less sensitive to vibrations than the Twyman–Green configuration of Fig. 1), note the difference in the illumination system. It is the most frequently used configuration requiring a single illuminating beam and a mirror perpendicular to the specimen surface [1]. One part of this beam illuminates the specimen grating directly, while the other one reaches the grating via the mirror M. In this way a symmetrical illumination is provided. Although a wide-aperture illuminating beam is required, the interferometer itself becomes very compact and can be fixed to the specimen under study. This results in further substantial reduction of the influence of external vibrations on the interferogram.

The proposed illumination system calls for a more effective method for providing mutual incoherence between the diffraction orders from the specimen grating.

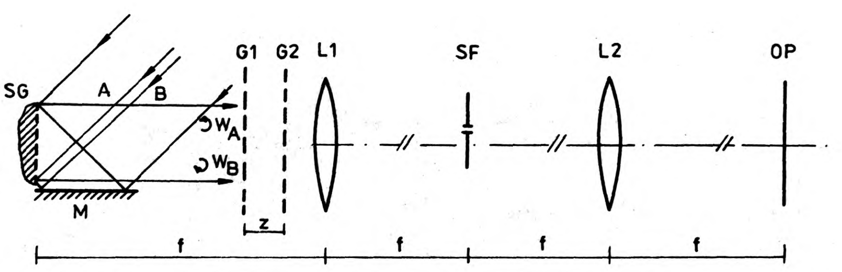

Fig. 2. Modified real time arrangement for overlapping the lateral shear interferograms of the specimen grating diffraction orders. Parts A and B of the illuminating beam are identically circularly polarized. Mirror M provides the symmetrical illumination beam. Diffraction gratings G1 and G2 together with a spatial filtering system L1–SF–L2 provide a double grating shearing interferometer. Specimen grating SG is imaged onto the observation plane OP

Instead of introducing linear polarizers with mutually perpendicular axes into the two parts of the illuminating wavefront, or a half-wave plate in one beam (in both cases the polarizing elements must be of interferometric quality, otherwise extraneous and unequal distortions are introduced into the diffraction orders), the use of circularly polarized illuminating beam is recommended. Because of the presence of mirror M the diffraction order wavefronts W_A and W_B have opposite handness of polarization. In this approach the circular polarizer, common to both illuminating beams, can be of small diameter, since it can be positioned upstream of the beam expanding optics.

The lateral shearing unit proposed in Fig. 2 consists of two linear diffraction gratings G1 and G2, and an imaging optical system L1–L2 including a spatial filter SF. The filter passes, for example, the doubly diffracted beams (0, +1) and (+1, 0), where the numbers in parenthesis indicate the diffraction order numbers at the first and second grating, respectively. The shear amount is equal to $z\tan\theta$, where z indicates the axial distance between G1 and G2, and θ is the first order diffraction angle of the gratings. The shear amount can be adjusted continuously by changing the distance z. By in-plane rotation of G1 and G2 in opposite directions (from the position of the grating lines set mutually parallel) the carrier fringes are introduced. They run parallely to the shear direction. The fringe spacing is equal to $d/2\sin(\alpha/2)$, where d denotes the spatial period of G1 and G2, and α is the angle between lines of the two gratings.

As mentioned in the previous paper [4], the carrier fringes are introduced by the shearing device. They are identical in the two lateral shear interferograms. The no-load condition must give a null field of moiré interferometry fringes, no tilt between the wavefronts W_A and W_B being allowed.

Mathematical description of the intensity distribution in the observation plane is given by adding the intensity distributions of two shearing interferograms. They are given by:

$$\begin{aligned} I_1(x, y) = \Bigg|&\exp\left\{i\left[k\beta y+\frac{2\pi}{d_s}u(x, y)+kw(x, y)\right]\right\} \\ &+\exp\left\{i\left[-k\beta y+\frac{2\pi}{d_s}u(x-\Delta, y)+kw(x-\Delta, y)\right]\right\}\Bigg|^2 \\ =\ & 2\left\{1+\cos\left[2k\beta y+\Delta\frac{2\pi}{d_s}\frac{\partial u(x, y)}{\partial x}+k\Delta\frac{\partial w(x, y)}{\partial x}\right]\right\}, \end{aligned} \tag{1}$$

$$\begin{aligned} I_2(x, y) = \Bigg|&\exp\left\{i\left[k\beta y-\frac{2\pi}{d_s}u(x, y)+kw(x, y)\right]\right\} \\ &+\exp\left\{i\left[-k\beta y-\frac{2\pi}{d_s}u(x-\Delta, y)+kw(x-\Delta, y)\right]\right\}\Bigg|^2 \\ =\ & 2\left\{1+\cos\left[2k\beta y-\Delta\frac{2\pi}{d_s}\frac{\partial u(x, y)}{\partial x}+k\Delta\frac{\partial w(x, y)}{\partial x}\right]\right\} \end{aligned} \tag{2}$$

where $u(x, y)$ and $w(x, y)$ denote the in-plane displacement function (corresponding to the departure from straightness of the specimen grating lines) and out-of-plane displacement function (describing the change of the specimen surface due to the load), respectively, d_s is the specimen grating period, $k = 2\pi/\lambda$, Δ is the shear value and β is the angle between the propagation direction of the interfering beams and the optical axis coinciding with the specimen grating normal. In our system we have

$$\beta = (\lambda/d)\sin(\alpha/2) \tag{3}$$

where, as before, d is the spatial period of shearing interferometer gratings G1 and G2, and α denotes the angle between the lines of G1 and G2. In the observation plane I_1 and I_2 are detected simultaneously, i.e.

$$I_1(x, y)+I_2(x, y) = 2\left\{1+\cos\left[\Delta\frac{2\pi}{d_s}\frac{\partial u(x, y)}{\partial x}\right]\cos\left[2k\beta y+k\Delta\frac{\partial w(x, y)}{\partial x}\right]\right\}. \tag{4}$$

High spatial frequency fringes described by a second cosine term are amplitude modulated by a first cosine term carring the information about the first derivative of in-plane displacement function $u(x, y)$. As it is well known, the visibility of additive-type moiré is enhanced by applying a nonlinear detection.

3. Experimental work

For the experimental investigations we have used the moiré interferometry system shown schematically in Fig. 2. The cross-type grating SG applied to the specimen had a spatial frequency of 1200 lines/mm resulting in the basic sensitivity of in-plane

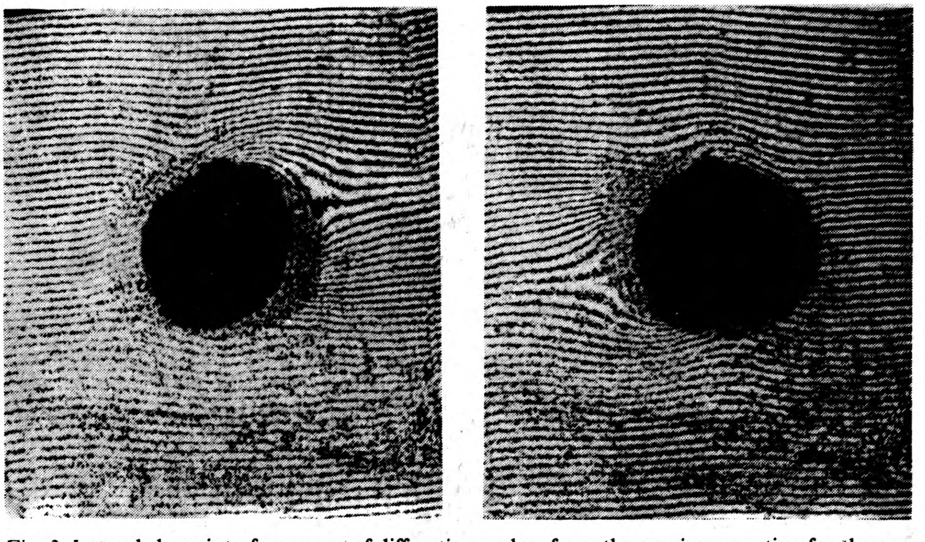

Fig. 3. Lateral shear interferograms of diffraction orders from the specimen grating for the case of a tensile test of an alluminium specimen with a central hole. The intensity distributions are described by Eqs. (1) and (2)

displacement detection equal to 0.417 μm per fringe order. For the He–Ne laser as a light source the illumination angle is equal to 49.4 deg. Binary amplitude diffraction gratings G1 and G2 used for the shearing interferometer unit were of spatial frequency of 25 lines/mm.

An alluminium specimen with a central hole was subject to tensile test. The specimen was 14 mm wide, 3 mm thick and the hole diameter was 4 mm. Figure 3 shows lateral shear interferograms of the two diffraction orders with wavefronts W_A and W_B, Fig. 2, from the specimen grating. Illuminating beams A and B were impinging in a horizontal plane. Horizontal carrier fringes were added, the shear being along the x direction. The interferograms in Fig. 3 correspond to Eqs. (1) and

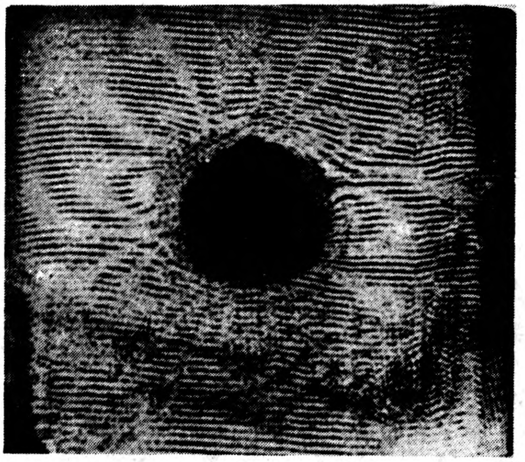

Fig. 4. Real time superimposition of interferograms shown in Fig. 3 realized in a modified moiré interferometry configuration, Fig. 2. Moiré fringes map the derivative $\varepsilon_x = \partial u(x, y)/\partial x$

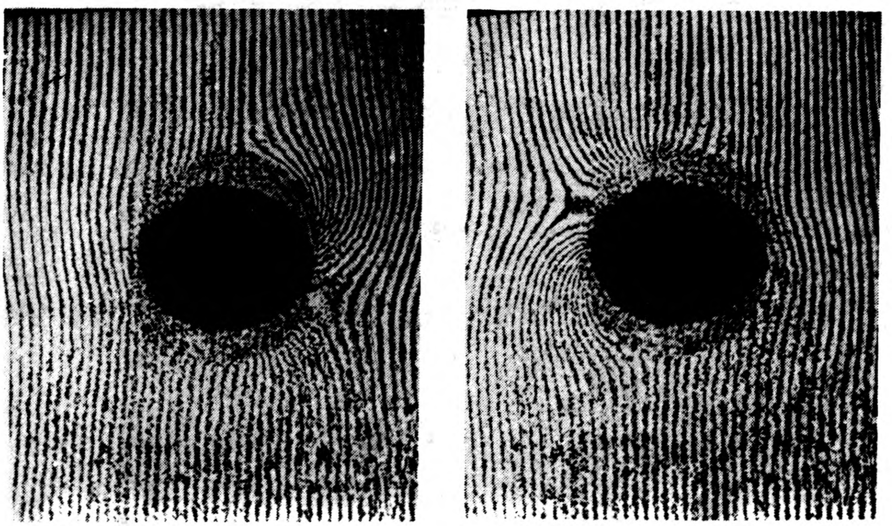

Fig. 5. Lateral shear interferograms of specimen grating diffraction orders carrying the information about in-plane displacement $u(x, y)$. The shear direction coincides with the y axis

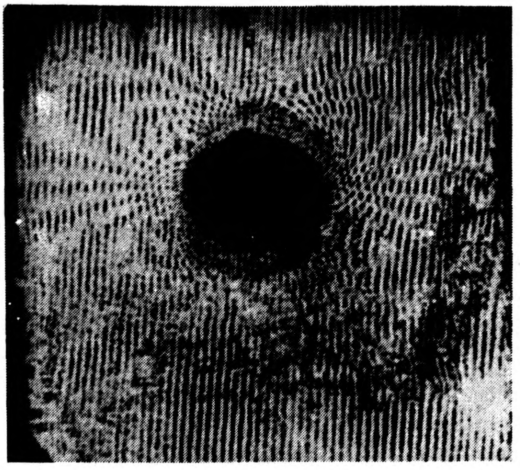

Fig. 6. Moiré fringes formed by a real time superimposition of interferograms shown in Fig. 5. Moiré fringes map the derivative $\partial u(x, y)/\partial y$

(2). The result of their simultaneous in-registry superimposition and observation is shown in Fig. 4. The shape of in-plane displacement derivative fringes $\partial u(x, y)/\partial x$ coincides well with the theoretical and experimental results of KATO [9] and WEISSMAN and POST [6].

Figure 5 shows shear type interferograms of the same diffraction orders with the shear direction along the y axis. The result of their real time overlap corresponding to the map of $\partial u(x, y)/\partial y$ is shown in Fig. 6.

4. Conclusions

A practical system implementing a real time optical differentiation of in-plane displacement patterns obtained by the moiré interferometry method has been described. It utilizes the approach of in-registry noncoherent superimposition of lateral shear interferograms of the specimen grating diffraction orders. The orders have orthogonal circular polarizations. The system proposed is simple, compact and stable against vibrations. Its cost is reduced with comparison to the previously described configurations requiring costly polarizing elements, such as a polarizing beam splitter and large diameter polarizers or half-wave plates of interferometric quality. The cost of rotational stages of double grating shearing interferometer is comparable with that of tilting mechanisms used in other shearing units.

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Дифференцирование в реальном времени интерференционных полос, полученных интерференционным методом Муара (некогерентная суперпозиция интерферограмм с поперечным раздвоением волнового фронта)

В настоящей работе представлена практическая система, реализующая оптическое дифференцирование перемещений в плоскости в реальном времени, полученных в системе интерференционного метода Муара. Система реализует суперпозицию дифференциальных интерферограмм с поперечным раздвоением волновых фронтов для дифракционных рядов предметной решетки. Представили теоретическое описание и результаты экспериментальных исследований.