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Solutions of problems of optical diffraction in anisotropic media by use of distributions

This article presents the solutions of the optical diffraction and of the diffraction of the mutual coherence in an anisotropic medium from the optical information processing viewpoint. The resulting relations and practical example are given.

1. Introduction

Optical diffraction phenomena represent a basis to modern optical imaging methods. The holographical methods are especially perspective, since they allow high capacity and density of optical information storage, and also high reliability of the record [1–6].

So far, optical diffraction has not been broadened to anisotropic media, in a manner as it is required in the field of the optical information processing. A study of optical phenomena in these media was restricted mainly to methods of geometrical optics which cannot give satisfactory answers to many problems of practical importance.

In this article some problems of the optical diffraction in anisotropic media will be delt with on the base of mathematical theory of distributions. The tempered distributions from the space $S'(R_n)$, used in this paper, are linear continuous functionals defined on the space $S(R_n)$ of basic functions [7]. The space $S(R_n)$ is produced by functions having all partial derivatives. The basic function $\varphi \in S(R_n)$ and their all partial derivatives decrease to zero as $[\boldsymbol{x}] \to \infty$ ($[\boldsymbol{x}]$ is a norm of a vector $\boldsymbol{x}$) more quickly than an arbitrary power of $[\boldsymbol{x}]^{-1}$ does.

In the theory of distributions the Fourier transform is defined as an operator which transforms the distribution $f \in S'$ to the distribution $g = F[f] \in S'$, according to the relation $(F[f], \varphi) = (f, F[\varphi])$, where

$$F[\varphi](\xi) = \int_{R_n} \varphi(\boldsymbol{x}) \exp(i\boldsymbol{x}\boldsymbol{\xi}) dx_1 dx_2 \ldots dx_n.$$

It is known that the Fourier transform of any distribution from S' as well as its inversion belong to S'. Both transformations map the space S' on itself in a mutually unique way, so that no other mathematical concepts e.g. ultradistributions etc., need not be introduced.

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2. Formulation of diffraction problem and diffraction equations

We shall consider a nonconductive but optically anisotropic medium ($\sigma = 0$). Let it be expressed by a dielectric tensor

$$[\varepsilon] = \begin{bmatrix} \varepsilon_1 & 0 & 0 \\ 0 & \varepsilon_2 & 0 \\ 0 & 0 & \varepsilon_3 \end{bmatrix} = \begin{bmatrix} n_1^2 & 0 & 0 \\ 0 & n_2^2 & 0 \\ 0 & 0 & n_3^2 \end{bmatrix}, \tag{1}$$

where

ε_i — are the principal permittivities (which are assumed to be constants),

n_i — are principal indices of refraction.

We shall not consider a magnetic anisotropy or another else.

As it is well known the Maxwell equations for electromagnetic field may be put in the following form

$$\Delta H_j + \varkappa_j H_j = 0, \tag{2}$$

$$\Delta E_j - \nabla_j \sum_{i=1}^{3} \nabla_i E_i + \varkappa_j E_j = 0, \quad j = 1, 2, 3, \tag{3}$$

where

$E_i = E_i(x_1, x_2, x_3, \tau)$ (for the both E_i and H_i, a sinusoidal time dependence is assumed),

$\varkappa_i = \mu \varepsilon_j \dfrac{\omega^2}{c^2} = \mu n_j^2 \dfrac{\omega^2}{c^2}$,

$\tau = ct$.

The two systems of equation (2) and (3), as a whole, form a set of linear differential equations representing an electromagnetic field in a nonconductive anisotropic medium. Equations (2) and (3) are Helmholtz equations.

The formulation of the diffraction problem consists in expressing the electric and magnetic field intensities satisfying equations (2) and (3) by their boundary values. Mathematical model of the optical diffraction assumes the electric and magnetic field intensity vectors to be zero outside the boundary of some region. Now, we shall derive the diffraction equations which govern this kind of optical diffraction in an anisotropic medium.

Let us consider a closed region G in an Euclidian three-dimensional space R_3, bounded by a surface P that is continuous in parts. If $G_1 = R_3 - G$, and the function f has all partial derivatives continuous on $\bar{G}$ and $f = 0$ for $\boldsymbol{x} \in G_1$, then [8]

$$(\nabla_i f, \varphi) = -(f, \nabla_i \varphi) = -\int_{R_3} f \nabla_i \varphi \, dx_1 dx_2 dx_3 = \int_G \{\nabla_i f\} \varphi \, dx_1 dx_2 dx_3 - \int_P [f]_P \cos(\boldsymbol{n} x_i) \varphi \, dP$$
$$= \left(\{\nabla_i f\} - [f]_P \cos(\boldsymbol{n} x_i) \delta_P \varphi\right), \quad i = 1, 2, 3. \tag{4}$$

where

$\{\nabla_i f\}$ — function in $\bar{G}$, continuous in parts,
$[f]_P$ — function f defined on P.
$(\boldsymbol{n} x_i)$ — angle between the axis x_i and a normal $\boldsymbol{n}$ taken from the outside to the surface P.

It follows from (4)

$$\nabla_i f = \{\nabla_i f\} - [f]_P \cos(\boldsymbol{n} x_i) \delta_P. \tag{5}$$

This expression defines a derivative of the distribution f and is convenient for the representation of our diffraction model. Using this expression once more, we get

$$\nabla_k \nabla_i f = \{\nabla_k \nabla_i f\} - [\{\nabla_i f\}]_P \cos(\boldsymbol{n} x_k) \delta_P - \nabla_k \left([f]_P \cos(\boldsymbol{n} x_i) \delta_P\right). \tag{6}$$

The relation (6) will be used for derivation of the differential expressions involved in equations (2) and (3).

If we put $U_i = f$, then by summation we get from (6)

$$\nabla_k \sum_{i=3}^{3} \nabla_i U_i = \left\{\nabla_k \sum_{i=1}^{3} \nabla_i U_i\right\} - \sum_{i=1}^{3} \left|[\{\nabla_i U_i\}]_P \times \cos(\boldsymbol{n} x_k) \delta_P + \nabla_k \left([U_i]_P \cos(\boldsymbol{n} x_i) \delta_P\right)\right|. \tag{7}$$

Similarly, we get

$$\Delta U_k = \left\{\sum_{i=1}^{3} \nabla_i \nabla_i U_k\right\} - [\{\nabla_n U_k\}]_P \delta_P \nabla_n([U_k]_P \delta_P), \tag{8}$$

where

$$\nabla_n = \nabla_1 \cos(\boldsymbol{n} x_1) + \nabla_2 \cos(\boldsymbol{n} x_2) + \nabla_3 \cos(\boldsymbol{n} x_3).$$

Inserting (7) and (8) into (2) and (3) we obtain the resulting diffraction equations

$$\Delta H_j + \varkappa_j H_j = \bar{H}_j, \tag{9}$$
$$\bar{H}_j = [\{\nabla_n H_j\}]_P \delta_P + \nabla_n([H_j]_P \delta_P),$$
$$\Delta E_j - \nabla_j \sum_{i=1}^{3} \nabla_i E_i + \varkappa_j E_j = \bar{E}_j, \tag{10}$$
$$\bar{E}_j = [\{\nabla_n E_j\}]_P \delta_P + \nabla_n([E_j]_P \delta_P) - \left(\sum_{i=1}^{3} [\{\nabla_i E_i\}]_P \times \cos(\boldsymbol{n} x_j) \delta_P - \nabla_j\left(([\boldsymbol{E}]_P \boldsymbol{n}) \delta_P\right)\right), \quad j = 1, 2, 3.$$

3. Solution of diffraction equations

Diffraction equations (9) and (10) will be solved by the method of elementary solutions. We use the Fourier transform and find a matrix of elementary solutions belonging to these systems. The general solution of these equations will be given by convolution of elementary solutions and right-hand sides of the equations. Let us remark that the solution of diffraction equations obeys the following relations

$$(\Delta H_j + \varkappa_j H_j, \varphi) = (H_j, \Delta\varphi - \varkappa_j \varphi) = (\bar{H}_j, \varphi),$$
$$\left(\Delta E_j - \nabla_j \sum_{i=1}^{3} \nabla_i E_i + \varkappa_j E_j, \varphi\right)$$
$$= \left(E_j, \Delta\varphi - \nabla_j \sum_{i=1}^{3} \nabla_i \varphi - \varkappa_j \varphi\right) = (\bar{E}_j, \varphi).$$

3.1. Elementary solution

Elementary solutions belonging to (9) form the diagonal matrix

$$h = \|h_{ij}\|, \tag{11}$$

where nonzero elements are solutions of the following equations

$$\Delta h_{ij} + \varkappa_i h_{jj} = \delta. \tag{12}$$

It is known [7] that these equations have the solution

$$h_{jj} = \frac{i \sqrt[4]{\varkappa_j}}{4\sqrt{2\pi r}} H_{1/2}^{(2)}(\sqrt{\varkappa_j} r), \tag{13}$$

where $H_{1/2}^{(2)}$ is the Hankel function,

$$r^2 = x_1^2 + x_2^2 + x_3^2.$$

Elementary solutions of (10) form the matrix

$$e = \|e_{ij}\|, \tag{14}$$

the elements of which are solutions of the equations

$$\sum_{k=1}^{3} [-\nabla_i \nabla_k + \delta_{ik}(\Delta + \varkappa_i)] e_{kj} = \delta_{ij}\delta, \quad (15)$$

where δ_{ij} is the Kronecker symbol.

By applying the Fourier transform to the preceding system of equations, we get the following system of algebraical relations

$$\sum_{k=1}^{3} [k_i k_k + \delta_{ik}(-k^2 + \varkappa_i)] \tilde{e}_{kj} = \delta_{ij}, \quad (16)$$

where

$$\tilde{e}_{ij} = F\{e_{ij}(x_1, x_2, x_3)\}.$$

The solution of the system (16) is given by the following relation

$$e_{ij}(x_1, x_2, x_3) = F^{-1}\left\{\frac{d_{ij}}{d_0}\right\}, \quad (17)$$

where

$$d_0 = (-k^2+k_1^2+\varkappa_1)(-k^2+k_2^2+\varkappa_2)(-k^2+k_3^2+\varkappa_3)+2k_1^2k_2^2k_3^2-(-k^2+k_1^2+\varkappa_1)k_2^2k_3^2-(-k^2+k_2^2+\varkappa_2)k_1^2k_3^2-$$
$$-(-k^2+k_3^2+\varkappa_3)k_1^2k_2^2,$$

$$d_{11} = (-k^2+k_2^2+\varkappa_2)(-k^2+k_3^2+\varkappa_3)-k_2^2k_3^2,$$
$$d_{12} = k_1k_2k_3^2-k_1k_2(-k^2+k_3^2+\varkappa_3),$$
$$d_{13} = k_1k_2^2k_3-k_1k_3(-k^2+k_2^2+\varkappa_2),$$
$$d_{22} = (-k^2+k_1^2+\varkappa_1)(-k^2+k_3^2+\varkappa_3)-k_1^2k_3^2,$$
$$d_{23} = k_1^2k_2k_3-k_2k_3(-k^2+k_1^2+\varkappa_1),$$
$$d_{33} = (-k^2+k_1^2+\varkappa_1)(-k^2+k_2^2+\varkappa_2)-k_1^2k_2^2,$$
$$d_{21} = d_{12},\ d_{31} = d_{13},\ d_{32} = d_{23},$$
$$k^2 = k_1^2+k_2^2+k_3^2.$$

3.2. Resulting solution of diffraction equations

The resulting solution may be found by convolving the elementary solutions and right-hand sides of diffraction equations. We get

$$H_k(x_1, x_2, x_3) = (h_{kk} * \overline{H}_k)(x_1, x_2, x_3). \quad (18)$$

Substituting (13) and right-hand sides of (9) into (18), we get

$$H_k(x_1, x_2, x_3)$$
$$= \frac{i\sqrt[4]{\varkappa_k}}{4\sqrt{2\pi}} \int_P \Bigg(\frac{[\{\nabla_n^{(\xi)} H_k(\xi_1, \xi_2, \xi_3)\}]_P}{[(x_1-\xi_1)^2+(x_2-\xi_2)^2+(x_3-\xi_3)^2]^{1/2}} H_{1/2}^{(2)}\left(\sqrt{\varkappa_k}\left((x_1-\xi_1)^2+(x_2-\xi_2)^2+(x_3-\xi_3)^2\right)^{1/2}\right)+$$
$$+[H_k(\xi_1, \xi_2, \xi_3)]_P \nabla_n^{(x)} \cdot \frac{H_{1/2}^{(2)}\left(\sqrt{\varkappa_k}\left((x_1-\xi_1)^2+(x_2-\xi_2)^2+(x_3-\xi_3)^2\right)^{1/2}\right)}{[(x_1-\xi_1)^2+(x_2-\xi_2)^2+(x_3-\xi_3)^2]^{1/2}} \Bigg) dP_{\xi_1,\xi_2,\xi_3}. \quad (19)$$

The resulting solution of (10) is

$$E_k(x_1, x_2, x_3) = \Big(\sum_{j=1}^{3} e_{kj} * \bar{E}_j\Big)(x_1, x_2, x_3). \quad (20)$$

According to the same procedure, we get

$$E_k(x_1, x_2, x_3) = \sum_{j=1}^{3} \int_P \Big(\Big([\{\nabla_n^{(\xi)} E_j(\xi_1, \xi_2, \xi_3)\}]_P - \sum_{i=1}^{3} [\{\nabla_i^{(\xi)} E_i(\xi_1, \xi_2, \xi_3)\}]_P \cos(\boldsymbol{n}\xi_j) \Big) \times$$
$$\times e_{kj}(x_1-\xi_1, x_2-\xi_2, x_3-\xi_3) + [E_j(\xi_1, \xi_2, \xi_3)]_P \nabla_n^{(x)} e_{kj}(x_1-\xi_1, x_2-\xi_2, x_3-\xi_3) - \big([\boldsymbol{E}(\xi_1, \xi_2, \xi_3)]_P \boldsymbol{n}\big) \times$$
$$\times \nabla_j^{(x)} e_{kj}(x_1-\xi_1, x_2-\xi_2, x_3-\xi_3)\Big) dP_{\xi_1,\xi_2,\xi_3}. \quad (21)$$

The magnetic (W_m) and electric (W_e) energies of the electromagnetic field involved in a region V of an anisotropic medium may be determined by integrating the corresponding energy densities, expressed by the components H_i and E_i:

$$W_m = \int_V w_m dV,$$

$$W_e = \int_V w_e dV,$$

where

$$w_m = 1/2\mu\mu_0(H_1^2+H_2^2+H_3^2),$$
$$w_e = 1/2\varepsilon_0(n_1^2E_1^2+n_2^2E_2^2+n_3^2E_3^2).$$

4. Diffraction of mutually coherent light signals

Mutual coherence forms a basic quantity at an optical imaging with a partially coherent radiation. In case of the anisotropic medium, we shall use the general definition of the mutual coherence Γ in a form of the matrix, the elements of which obey the specified system of differential equations. Thus

$$\Gamma = \|\Gamma_{ij}\|, \tag{22}$$

where

$$\Gamma_{ij} = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} {}^{(1)}E_1(\tau_x+t, x_1, x_2, x_3)\times$$
$$\times {}^{(2)}E_j^*(\tau_y+t, y_1, y_2, y_3)dt,$$

${}^{(2)}E_j^*$ is complex conjugate of ${}^{(2)}E_j$,

$$\Gamma_{ij} = \Gamma_{ij}(\tau_x, \tau_y, x_1, x_2, x_3, y_1, y_2, y_3).$$

4.1. Formulation of diffraction of mutually coherent signals and derivation of diffraction equations

The elements of mutual coherence matrix obey the following two systems of differential equations

$$\begin{aligned} &\Delta\Gamma_{ij} - \nabla_i \sum_{k=1}^{3} \nabla_k \Gamma_{kj} - \mu\varepsilon_i \frac{\partial^2}{\partial\tau_x^2}\Gamma_{ij} = 0, \\ &\Delta^{(y)}\Gamma_{ij} - \nabla_i^{(y)} \sum_{k=1}^{3} \nabla_k^{(y)} \Gamma_{kj} - \mu\varepsilon_i \frac{\partial^2}{\partial\tau_y^2}\Gamma_{ij} = 0, \end{aligned} \tag{23}$$

where

$$\Delta^{(y)} = \frac{\partial^2}{\partial y_1^2} + \frac{\partial^2}{\partial y_2^2} + \frac{\partial^2}{\partial y_3^2},$$

$$\nabla_k^{(y)} = \frac{\partial}{\partial y_k}.$$

To derive the diffraction equations for mutual coherence Γ, we proceed in the same manner as in 2., assuming $\Gamma_{ij} = 0$ outside closed surfaces R_x and R_y in an Euclidian space R_3. The resulting diffraction equations will have the form

$$\Delta\Gamma_{ij} - \nabla_i \sum_{k=1}^{3} \nabla_k \Gamma_{kj} + \varkappa_i^{(x)}\Gamma_{ij} = {}^{(x)}\Gamma_{ij}, \tag{24}$$

$$\Delta^{(y)}\Gamma_{ij} - \nabla_i^{(y)} \sum_{k=1}^{3} \nabla_k^{(y)} \Gamma_{kj} + \varkappa_i^{(y)}\Gamma_{ij} = {}^{(y)}\Gamma_{ij}, \tag{25}$$

where

$$\begin{aligned} {}^{(u)}\Gamma_{ij} = {} & [\{\nabla_n^{(u)}\Gamma_{ij}\}]_{R_u}\delta_{R_u} + \nabla_n^{(u)}([\Gamma_{ij}]_{R_u}\delta_{R_u}) - \\ & - \sum_{k=1}^{3}\Big([\{\nabla_k^{(u)}\Gamma_{kj}\}]R_u\cos(nu_i)\delta_{R_u} + \\ & + \nabla_i^{(u)}\big([\Gamma_{kj}]_{R_u}\cos(nu_k)\delta_{R_u}\big)\Big). \end{aligned}$$

4.2. Solution of diffraction equations for mutually coherent signals

We shall use again the method of elementary solutions.

4.2.1. Elementary solutions

The elements $\gamma_{ij}^{(x)}$ of the matrix $\gamma^{(x)}$ of elementary solutions

$$\gamma^{(x)} = \|\gamma_{ij}^{(x)}\| \quad i, j = 1, \dots, 9 \tag{26}$$

belonging to the system (24) are given by the following equations written in the matrix form as follows

$$\left\|\begin{matrix} \Delta_1^{(1)} & 0 & 0 & -\nabla_1\nabla_2 & 0 & 0 & -\nabla_1\nabla_3 & 0 & 0 \\ 0 & \Delta_1^{(1)} & 0 & 0 & -\nabla_1\nabla_2 & 0 & 0 & -\nabla_1\nabla_3 & 0 \\ 0 & 0 & \Delta_1^{(1)} & 0 & 0 & -\nabla_1\nabla_2 & 0 & 0 & -\nabla_1\nabla_3 \\ -\nabla_1\nabla_2 & 0 & 0 & \Delta_2^{(1)} & 0 & 0 & -\nabla_2\nabla_3 & 0 & 0 \\ 0 & -\nabla_1\nabla_2 & 0 & 0 & \Delta_2^{(1)} & 0 & 0 & -\nabla_2\nabla_3 & 0 \\ 0 & 0 & -\nabla_1\nabla_2 & 0 & 0 & \Delta_2^{(1)} & 0 & 0 & -\nabla_2\nabla_3 \\ -\nabla_1\nabla_3 & 0 & 0 & -\nabla_2\nabla_3 & 0 & 0 & \Delta_3^{(1)} & 0 & 0 \\ 0 & -\nabla_1\nabla_3 & 0 & 0 & -\nabla_2\nabla_3 & 0 & 0 & \Delta_3^{(1)} & 0 \\ 0 & 0 & -\nabla_1\nabla_3 & 0 & 0 & -\nabla_2\nabla_3 & 0 & 0 & \Delta_3^{(1)} \end{matrix}\right\| \cdot \left\|\begin{matrix} \gamma_{11}\ \gamma_{12} \cdots \gamma_{19} \\ \gamma_{21}\ \gamma_{22} \cdots \gamma_{29} \\ \cdot \\ \cdot \\ \cdot \\ \\ \\ \\ \gamma_{91}\ \gamma_{92} \cdots \gamma_{99} \end{matrix}\right\| = \left\|\begin{matrix} \delta 0 \dots 0 \\ 0 \delta \dots 0 \\ \\ \\ \\ 00 \dots \delta \end{matrix}\right\|, \tag{27}$$

where

$$\Delta_k^{(1)} = \Delta - \nabla_k^2 + \varkappa_k^{(x)}.$$

The solution of equations (27) is not of an easy procedure. Here we give only the result

$$\gamma^{(x)} = \|\gamma_{ij}^{(x)}\|$$

$$= \left\| \begin{matrix} e_{11} & 0 & 0 & e_{12} & 0 & 0 & e_{13} & 0 & 0 \\ 0 & e_{11} & 0 & 0 & e_{12} & 0 & 0 & e_{13} & 0 \\ 0 & 0 & e_{11} & 0 & 0 & e_{12} & 0 & 0 & e_{13} \\ e_{21} & 0 & 0 & e_{22} & 0 & 0 & e_{23} & 0 & 0 \\ 0 & e_{21} & 0 & 0 & e_{22} & 0 & 0 & e_{23} & 0 \\ 0 & 0 & e_{21} & 0 & 0 & e_{22} & 0 & 0 & e_{23} \\ e_{31} & 0 & 0 & e_{32} & 0 & 0 & e_{33} & 0 & 0 \\ 0 & e_{31} & 0 & 0 & e_{32} & 0 & 0 & e_{33} & 0 \\ 0 & 0 & e_{31} & 0 & 0 & e_{32} & 0 & 0 & e_{33} \end{matrix} \right\| \qquad (28)$$

The matrix of the elementary solution $\gamma^{(y)}$ differs from $\gamma^{(x)}$ only by the constant. The elements $\gamma_{ij}^{(y)}$ can be obtained from $\gamma_{ij}^{(x)}$ by substitution of $\varkappa_i^{(y)}$ for $\varkappa_i^{(x)}$. The resulting solution will be derived by using the matrix (28).

4.2.2. Resulting solution of diffraction equations for mutually coherent light

After some rearrangements we get the following result

$$\Gamma_{ij}(x_1, x_2, x_3, y_1, y_2, y_3) = \left(\sum_{k=1}^{3} e_{ik}^{(x)} * \left(\sum_{l=1}^{3} e_{kl}^{(y)} * \overline{\Gamma}_{lj}\right)\right) \times$$

$$\times (x_1, x_2, x_3, y_1, y_2, y_3). \qquad (29)$$

The expression (29) holds for sinusoidal time signal and the integration is performed on surfaces in a tree-dimensional space R_3. At special tasks, these surfaces can be simplified so that the computation be performed on computer. If this expression is interpreted as an analytic signal, then the mean value of the electric energy density in anisotropic medium is proportional to the sum of the diagonal elements of the matrix (22) multiplied by the squares of principal indices of refraction [9]. In case of optical information processing the relative values of the optical energy should be known, the absolute value of it being usually not required.

5. Point imaging in free space

The preceding theoretical results will now be made clearer by introducing the imaging of a point in a free space as an example. The point imaging is given by diffraction equations (9) and (10); the right-hand sides are the δ-distributions. Hence it follows that the corresponding elementary solution represents mathematically a point imaging, and is the solution of the given task.

The elements of the matrix (16) can be found by employing the inverse Fourier transform. The following holds

$$e_{11} = \left[\frac{\partial^4}{\partial x_1^4} + \frac{\partial^2}{\partial x_1^2}\left(\frac{\partial^2}{\partial x_2^2} + \frac{\partial^2}{\partial x_3^2} + \varkappa_2 + \varkappa_3\right) + \right.$$
$$\left. + \varkappa_2 \frac{\partial^2}{\partial x_2^2} + \varkappa_3 \frac{\partial^2}{\partial x_3^2} + \varkappa_2 \varkappa_3\right] J,$$

$$e_{12} = \frac{\partial^2}{\partial x_1 \partial x_2} (\Delta + \varkappa_3) J,$$

$$e_{13} = \frac{\partial^2}{\partial x_1 \partial x_3} (\Delta + \varkappa_2) J,$$

$$e_{23} = \left[\frac{\partial^4}{\partial x_2^4} + \frac{\partial^2}{\partial x_2^2}\left(\frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_3^2} + \varkappa_1 + \varkappa_3\right) + \right.$$
$$\left. + \varkappa_1 \frac{\partial^2}{\partial x_1^2} + \varkappa_3 \frac{\partial^2}{\partial x_3^2} + \varkappa_1 \varkappa_3\right] J,$$

$$e_{23} = \frac{\partial^2}{\partial x_2 \partial x_3} (\Delta + \varkappa_1) J,$$

$$e_{33} = \left[\frac{\partial^4}{\partial x_3^4} + \frac{\partial^2}{\partial x_3^2}\left(\frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} + \varkappa_1 + \varkappa_2\right) + \right.$$
$$\left. + \varkappa_1 \frac{\partial^2}{\partial x_1^2} + \varkappa_2 \frac{\partial^2}{\partial x_2^2} + \varkappa_1 \varkappa_2\right] J, \qquad (30)$$

where

$$J = \frac{1}{8\pi^3} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{[\cos(k_1x_1 + k_2x_2 + k_3x_3) - i\sin(k_1x_1 + k_2x_2 + k_3x_3)]\, dk_1\, dk_2\, dk_3}{\varkappa_1 k_1^4 + \varkappa_2 k_2^4 + \varkappa_3 k_3^4 + (\varkappa_1 + \varkappa_2) k_1^2 k_2^2 + (\varkappa_1 + \varkappa_3) k_1^2 k_3^2 + (\varkappa_2 + \varkappa_3) k_2^2 k_3^2 - \varkappa_1(\varkappa_2 + \varkappa_3) k_1^2 - \varkappa_2(\varkappa_1 + \varkappa_3) k_2^2 - \varkappa_3(\varkappa_1 + \varkappa_2) k_3^2 + \varkappa_1 \varkappa_2 \varkappa_3}.$$

The projection of the solution, e.g. the $e_{ij} - r$ dependence is shown in figs. 1 and 2. The computation was performed for an uniaxial crystal (ADP) with $n_1 = n_2 = 1.5254$, $n_3 = 1.4798$, $n = 1$, and $\omega = c$, using relations (30). The integral $J(x_1, x_2, x_3)$ was estimated on computer, while the derivatives were calculated numerically.

6. Conclusions

The optical diffraction in anisotropic media has not been solved yet in the manner, shown in this article. The task requires a special mathematical procedure

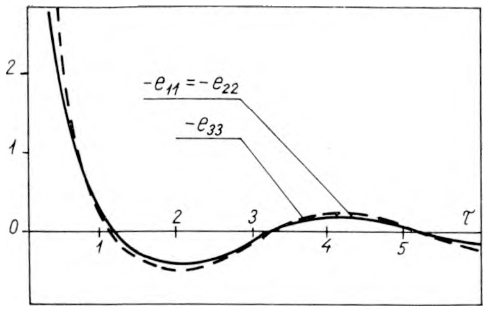

Fig. 1. The dependences $e_{11}-\tau$, $e_{22}-\tau$, and $e_{33}-\tau$

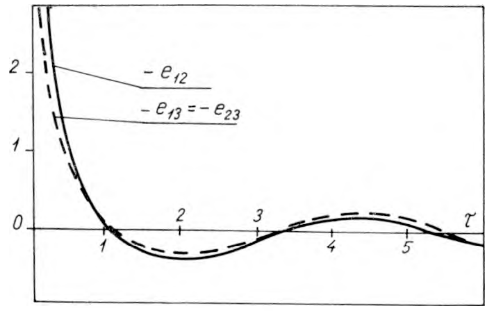

Fig. 2. The dependences $e_{12}-\tau$, $e_{13}-\tau$, and $e_{23}-\tau$

and cannot be solved by the methods of classical mathematical analysis. This is due to the fact that the solution of diffraction equations does not exist as a function but as a distribution. In the field of the distributions the diffraction equations have a clear meaning, their solutions exists and — as it is shown — can be found by the ordinary methods of derivation and integration.

The solution of the propagation of the mutual coherence in an anisotropic medium is much more complex. The elements of the mutual coherence matrix obey two partial differential equations, and the elements of the matrix of the elementary solutions satisfy the matrix equation (27). However, then solutions can be easily found. The solutions of diffractions equations for mutual coherence are given by the elements of elementary solution of diffraction equation for electric intensity vector. As an example the optical imaging of the radiation point in an anisotropic medium is shown. It can be seen in figs. 1 and 2 that it has the same character as that in the isotropic medium, where it is expressed by the Green function.

Решение проблемы оптической дифракции в анизотропных средах при использовании дистрибуции

В работе описывается решение проблемы оптической дифракции для случая когерентного и частично когерентного света в анизотропных средах с точки зрения обработки оптической информации. Выведены зависимости и приведены иллюстрации.

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