Letters to the Editor

Experiments with partially coherent reconstruction of Fourier holograms

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1. Introduction

For reconstruction of holograms the laser light is usually used. Therefore the process of holographic image formation is usually described as a completely coherent diffraction.

It may be interesting to investigate the effects of partially coherent light used for reconstruction of holograms.

As a simplest example let us consider a Fourier-type hologram. Reconstruction of such hologram is essentially a far field diffraction. This type of diffraction in partially coherent light has been investigated by several authors e.g. FUJI, ASAKURA [1], [2], FUJIWARA [3], SIROHI and RAM MOHAN [4] and others. The correlation between partially coherent and completely coherent diffractions as a function of the degree of partial coherence has been discussed by the author in [5]. It can be applied directly in studying the partially coherent reconstruction of Fourier holograms.

The generalized Schell's theorem, as formulated in [5], expresses the intensity distribution in a partially coherent diffraction pattern as a convolution of the intensity distribution in a respective diffraction pattern, produced by a point source and the Fourier transform of the mutual coherence function of illuminating light.

$$I_{\mathrm{p.coh}}(x,y) = \mathcal{F}\left\{\Gamma\left(\frac{x}{\lambda z},\frac{y}{\lambda z}\right)\right\}\otimes I_0(x,y) \tag{1}$$

z — is a diffraction distance, F — means Fourier transform, — means convolution.

Here, the paraxial approximation and quasimonochromacy are assumed.

2. Theoretical remarks

Let us consider a Fourier-type hologram of a transilluminated object of amplitude transmittance $t(x_0,y_0)$, taken in a typical setup (fig. 1) [6].

The intensity distribution in (x_2,y_2) plane (on a photoplate) is then:

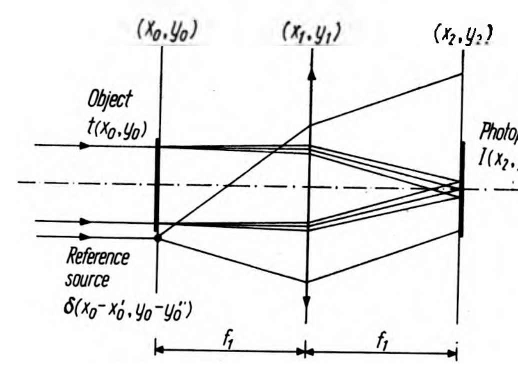

Fig. 1. Recording of Fourier-type hologram

$$I_{\mathrm{hol}}(x_2,y_2) = 1+|\mathcal{F}\{t(x_2,y_2)\}|^2+\mathcal{F}\{t(x_2,y_2)\}\exp[2\pi i(x_2\cdot x_0'+y_2\cdot y_0')]+\mathcal{F}\{t^*(x_2,y_2)\}\exp[-2\pi i(x_2\cdot x_0'+y_2\cdot y_0')]. \tag{2}$$

In the reconstruction step the far field diffraction of completely coherent light on such a hologram, of amplitude transmittance:

$$t_{\mathrm{hol}}(x_2,y_2) \cong I_{\mathrm{hol}}(x_2,y_2) \tag{3}$$

gives a „coherent" image, the intensity distribution in this image being:

$$I_0(x_4,y_4) = \delta(x_4,y_4)+\mathcal{F}\{|\mathcal{F}\{t(x_4,y_4)\}|^2\}+ +t[-(x_4+x_0'),-(y_4+y_0')]+t^*[-(x_4-x_0'),-(y_4-y_0')]. \tag{4}$$

If the reference point source $\delta(x_0-x_0',y_0-y_0')$ is far enough from the optical axis, then both the conjugate images $t[-(x_4+x_0'),\ -(y_4+y_0')]$ and $t^*[-(x_4-x_0'),\ -(y_4-y_0')]$ are spatially separated and can be treated as independent diffraction patterns.

Application of the cited generalized Schell's theorem enables to find the intensity distribution in the image reconstructed from the hologram with partially coherent light.

Let the hologram be illuminated by a light beam originating from an extended source, as it is shown in fig. 2. If such flat, quasimonochromatic and incoherent source is placed in a back focal plane of a collimating lens of focal length f_1, then — according to the VAN CITTERT-ZERNICKE theorem [7] — the mutual coherence function in a front focal plane of this lens is given in the form:

$$\Gamma(x_2'-x_2'',y_2'-y_2'') \cong \mathcal{F}\left\{I_s\left(\frac{x_2'-x_2''}{\lambda f_1}-\frac{y_2'-y_2''}{\lambda f_1}\right)\right\} \tag{5}$$

I_s denotes the intensity distribution on the source.

Thus:

$$\mathcal{F}\left\{\Gamma_{\mathrm{hol}}\left(\frac{x_4}{\lambda f_2},\frac{y_4}{\lambda f_2}\right)\right\} = I_s\left(-\frac{f_2}{f_1}x_4,-\frac{f_2}{f_1}y_4\right). \tag{6}$$

Insertion of this expression into (1) gives:

$$I_{\mathrm{p.coh}}(x_4,y_4) = I_s\left(-\frac{f_2}{f_1}x_4,-\frac{f_2}{f_1}y_4\right)\otimes I_0(x_4,y_4). \tag{7}$$

The last formula may be interpreted as follows: The intensity distribution in an image, obtain from a Fourier hologram reconstructed with the light from an extended source, is equal to the convolution of the intensity distribution in an image, obtained from the same hologram reconstructed with a point source („coherent reconstruction") and the intensity distribution on the light source used.

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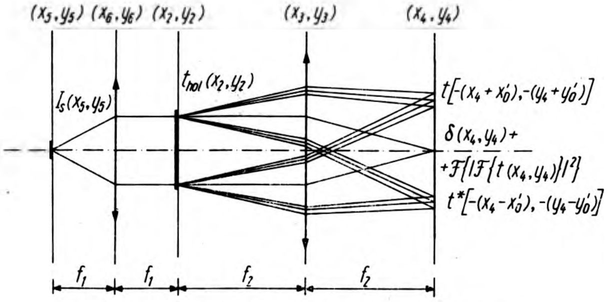

Fig. 2. Reconstruction of Fourier-type hologram

This means that each point of the incoherent source generates an image shifted with respect to those generated by the neighbouring points of the source. All these images superpose incoherently. This causes „blurring" of the reconstructed image.

3. Experimantal results

To illustrate experimantally the described process a setup shown in fig. 3 has been used for reconstruction of Fourier holograms.

A small pinhole P, illuminated by a XBO-101 high pressure mercury lamp S through a system of lenses, represents an incoherent extended source of light. The optical system of illuminator, composed of two lenses (one of them being a 20^X microobjective) and an interference filter (for $\lambda = 546$ nm), images the arc in the mercury lamp onto the pinhole. In this way the pinhole can be treated as a completely incoherent secondary light source. The diameter of this pinhole can be changed to have 5 different values: $d_1 = 125 \approx 1$ μm, $d_2 = 211 \pm$ μm, $d_3 = 314 \pm 1$ μm, $d_4 = 416 \pm 2$ μm, $d_5 = 589 \pm \pm 2$ μm. Collimating lens have focal length $f_1 = 185$ mm, and transforming lens $f_2 = 500$ mm. The reconstructed image was photographed. For comparison a photograph of an image reconstructed from the same hologram in laser light (He–Ne laser, $\lambda = 628$ nm) in the same configuration was also taken. Figures 4 and 5 show one of the conjugate images reconstructed from two exemplary holograms. Photograph a) is an image reconstructed with laser light. Photographs b), c), d), e) and f) show images reconstructed incoherently with increasing diameter of the pinhole.

Effect of „blurring" is easily seen. Images of radial test (fig. 5) show also a contrast inversion in several places. General shape of incoherently reconstructed image of radial test suggests that the effect of „blurring" depends on spatial frequency contained in the image. The same suggestion arises from the equation (7).

This problem in now being investigated.

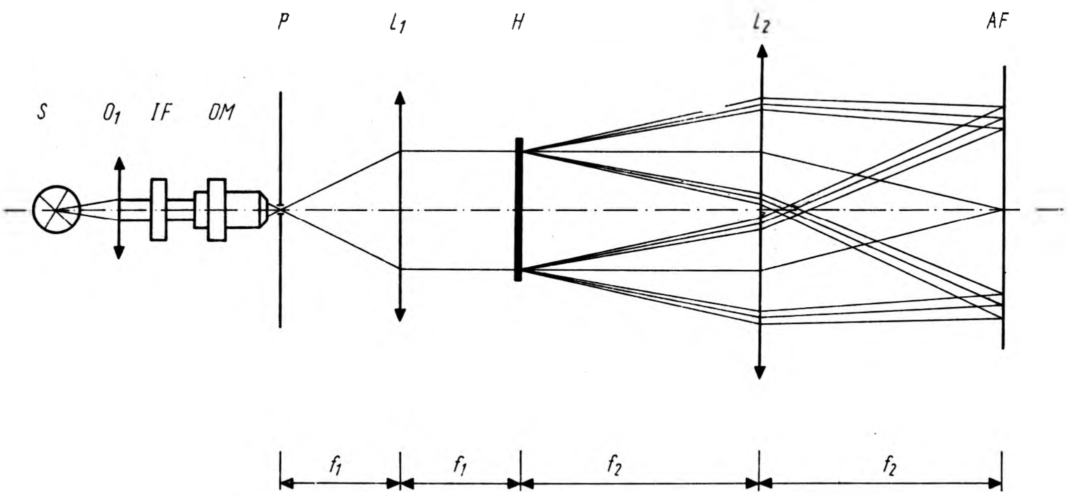

Fig. 3. Diagram of an experimental setup for partially coherent reconstruction of Fourier holograns

S – XBO-101 mercury lamp, O_1 – lens, IF – interference filter for $\lambda = 546$ nm, OM – 20X microobjective, P – exchangeable pinhole, L_1 – collimating lens, $f_1 = 185$ mm, H – hologram, L_2 – transforming lens, f_2 – 500 mm, AF – observing screen or photographic camera

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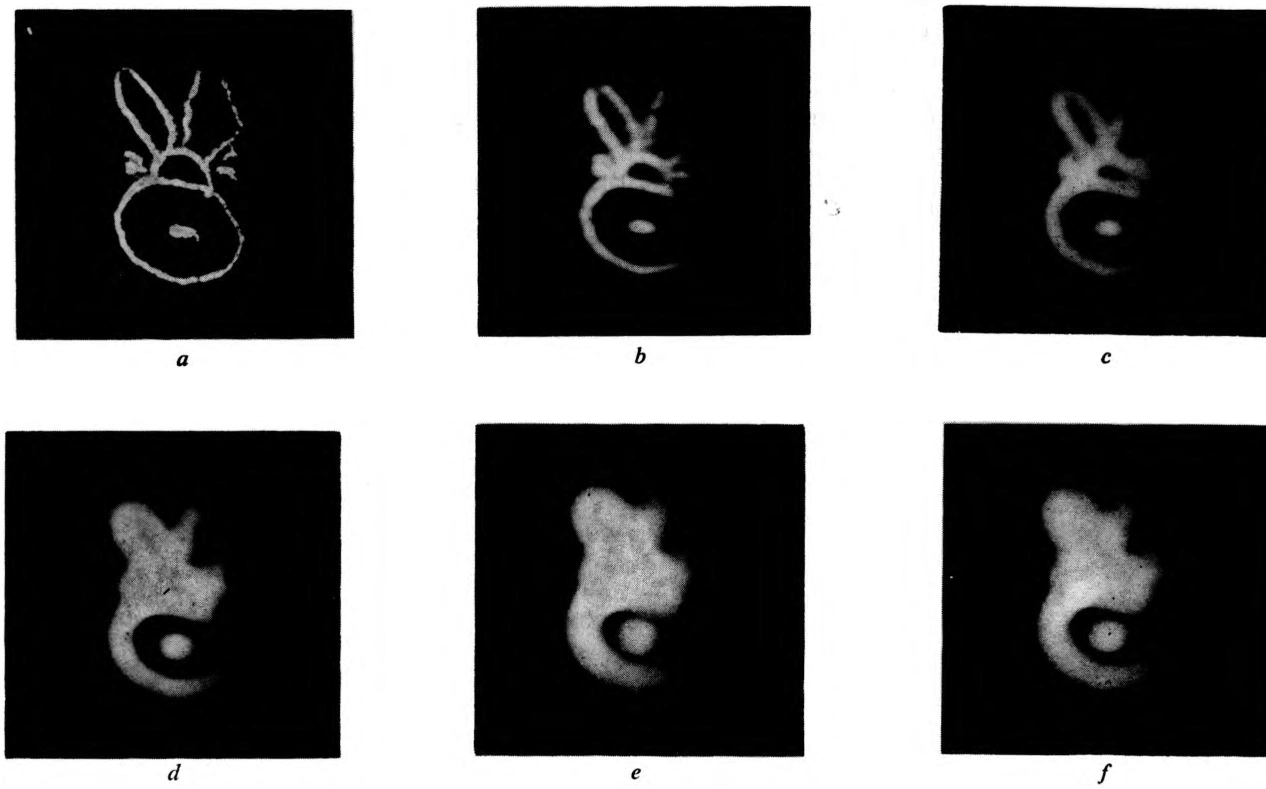

Fig. 3. Photographs of the reconstructed images

a) reconstruction with laser light, b) reconstruction with incoherent, circular source of diameter $d_1 = 125$ μm, c) as b), $d_2 = 211$ μm, incoherent, circular source of diameter $d_2 = 125$ μm, d) as b), $d_3 = 314$ μm, incoherent, circular source of diameter $d_3 = 125$ μm e) as b), $d_4 = 416$ μm, incoherent, circular source of diameter $d_4 = 125$ μm f) as b), $d_5 = 589$ μm, incoherent, circular source of diameter $d_5 = 125$ μm

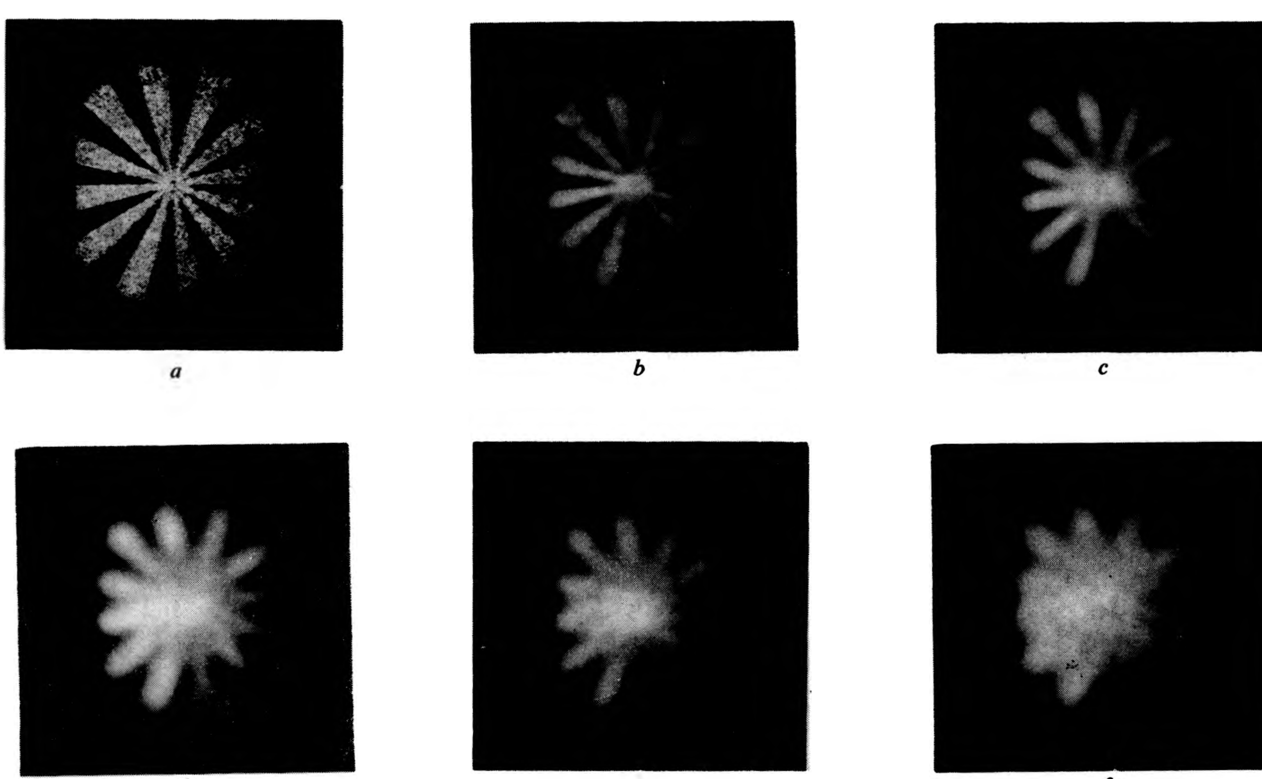

Fig. 5. Photographs of the reconstructed images

a) reconstruction with laser light, b) reconstruction with incoherent, circle source of diameter $d_1 = 125$ μm, c) as b), but $d_2 = 211$ μm, d) as b), but $d_3 = 314$ μm, e) as b), but $d_4 = 416$ μm, f) as b), but $d_5 = 589$ μm.

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