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ECONOMIC GROWTH IN IRELAND IN 1980–2014. A THRESHOLD COINTEGRATION APPROACH

The purpose of this paper is to identify the determinants of both the Gross National Product (GNP) growth in Ireland in the long term and the fluctuations around it in the short term using the Threshold Error Correction Model (TECM). The GNP aggregate has been selected as a measure of economic growth because it allows to omit the influence of international companies located in Ireland on the global production level. A TECM approach has been selected for analysis as it is an appropriate tool for the effective modeling of different regimes of growth around the long run which is assumed to be stable. The time series (in levels and in logs) exhibited both non-stationarity and structural breaks. It was possible to find out the determinants of growth in the long as well as short run and the threshold variables that extracted the period of intense economic growth. The best empirical models were obtained with the following thresholds: the GDP deflator and the net exports. The results have been a subject of validation for their robustness.

Keywords: intense economic growth, Ireland, threshold cointegration, TECM, validation
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1. INTRODUCTION

The issue of intense economic growth is one of the most attractive in the macroeconomic perspective. Some authors call it 'an economic miracle' (Selinger, 2010; Best, 2018). So far, economic theory has not worked out the full identification of the causes, conditions, course, and consequences of this phenomenon for socio-economic development, and the functioning of the economies of the countries in which it has occurred. However, a review of the literature in this field indicates considerable interest in this issue by many researchers into the processes of modern economic growth covering the period following the Second World War, who are often very well-known in the area of social sciences and history. Most often, the starting point for research in this area is the historically identified examples of economic miracles in some countries, Ireland being one of them. The focus of the

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research is set on the economic, social and institutional causes and conditions of the economic growth path in Ireland. The case of Ireland has been widely studied in the economic literature (see for example Box, 1998; Barry et al., 2001; Barry, 2002; Kelly and Everett, 2004 and also: Przesławska, 2009; Szczepaniak, 2015). Generally, all the authors agree that institutional development is the main source of the economic and social success of Ireland, although different sources of the success have been stressed. The econometric approach applied in Boehlke et al. (2017) revealed that the nonlinear mechanism of growth of Irish GDP was more likely in the light of empirical tests than the linear one.

The purpose of this paper is to identify the determinants of both: the Gross National Product (GNP) growth in Ireland in the long term and fluctuations around it in the short term using the Threshold Error Correction Model (TECM). The GNP aggregate has been selected as a measure of economic growth because it allows omitting the influence of international companies located in Ireland on the global production level. As the Central Statistical Office in Ireland shows, the GNP gives a more realistic picture of the economy than the GDP because the latter grows higher than the GNP but a significant share of profits does not stay in the domestic economy since they are transferred to the countries of the multinational companies' origin. This problem was widely discussed by FitzGerald (2016) and by Honohan (2016). On the other hand, all international statistics are expressed in terms of GDP, which is a source of some inconvenience. The data coming from the years 1980-2014 have been used for analysis. The starting date has been enforced by the availability of the widest possible set of time series. The final date is related to the comparison of the results obtained for the GDP in Ireland published in Boehlke et al. (2017). The question is whether the dynamics of both aggregates, i.e. GDP and GNP in Ireland, resulted similarly from the same incentives of growth. We refer to this issue in the last section of the paper.

It has been assumed that intense economic growth appears in short or medium periods while the long-term path of economic growth remains stable. Thus, a Threshold Error Correction Model (TECM) has been selected for analysis as it is an appropriate tool for the effective modeling of different regimes of growth around the long run. This fits the situation when the model parameters are stable, but they differ across the regimes that are specified. Furthermore, the set of variables that appear in one regime may be different from another regime taking both the variables and their lags. Following Balcerowicz and Rzońca (Balcerowicz and Rzońca eds., 2014), it

is presumed that when the intense growth is a result of reforms which eliminate at least one barrier of growth, a higher productivity is expected in comparison to the periods of average growth rates. Another characteristic is linked with innovations, which typically re-orient the economy to be an export-driven one. Having the information about the Irish way from an underdeveloped to a developed economy, we check whether the model is able to reveal these processes in the analyzed period of time. Furthermore, the TECM model provides a variety of ways of identification asymmetric adjustment to the long-term equilibrium, if it does exist. The oldest one, introduced by Enders and Siklos (2001), assumes that the long-term equilibrium phenomenon can be measured in two versions, i.e. SETAR and *Momentum* TAR. Other ways of formulating threshold are also possible (see Kapetanios et al., 2006; Bruzda, 2007). In this paper we extended the procedure of the identification of the thresholds by using individual economic variables as a threshold variable, and we used a model with statistically significant parameters as a basis of further testing. The robustness of our results is checked by using the Tsay test for nonlinearity (Tsay, 1998).

The paper is organized as follows. In the second part a brief characteristic of the economy of Ireland has been made within the context of the deployed economic reforms and their consequences. The econometric models used for the empirical analysis of economic growth in Ireland were specified in section three, while in section four the empirical results are presented and discussed. In section five a validation procedure of the results has been performed. The conclusions and discussion are presented in the last part.

2. THE ECONOMY OF IRELAND AS AN EXAMPLE OF AN ECONOMIC MIRACLE

In the case of interpretive epistemologies, the concept of an "economic miracle" used in the literature on the theory of economics, politics, and economic history is usually subject to different interpretations made on the basis of research carried out on the grounds of history and semantics, and formulated within the social sciences and the historical concepts of the course of the process of social change and socio-economic development. In the literature of the subject, this concept is sometimes expressed in quotation marks (cf. e.g. Castels, 2009, p. 204). Under this approach, the concept of the economic miracle, as well as its designatum in empirically knowable reality, are analysed on the ground of hermeneutics, philosophy, the theory

of language, cognitive and developmental psychology, and the science of artificial intelligence.

On the other hand, the definition of the notion of an "economic miracle", as well as an analysis of the causes, conditions of emergence, course, and consequences of this phenomenon on the ground of theory, meaning empirical epistemologies, typically involves the theory of economic growth, contemporary institutionalism, and political theory, including economic policy and a new economic geography. According to one definition of an "economic miracle" proposed on the basis of the theory of economic growth, this is a period of faster-than-expected economic growth (Sharma, 2012). At the same time, it is commonly acknowledged that these expectations are a consequence of the achieved level of GDP per capita, and the standing of the studied country in the world economic system. It follows from the above that in order to define the concept of an "economic miracle" the following factors are essential: expectations regarding the growth of the economy, the actual growth rate, the level of economic development measured by GDP per capita, and time. Taking into account the historical experience of the period after World War II, it should also be added that miracles occur in market economies, and with different levels of economic development. As part of the attempts at a theoretical analysis of economic miracles undertaken by economists, it is worth recalling the works analysing this phenomenon in the context of the processes of economic convergence and divergence (Barro, Sala-i-Martin, 1995), an endogenous increase associated with investment in human capital and learning processes (Lucas, 1993), and industrial policy (Lall, 1996).

Interesting deliberations on understanding the notion of an economic miracle and the characteristics, conditions, and consequences of this phenomenon are included in the work edited by Balcerowicz and Rzońca (Balcerowicz, Rzońca eds., 2014). Economic miracles are here a consequence of internal economic shocks caused by the national economic policy, at the root of which lie institutional determinants of changes in the economic system that could be barriers to, or drivers of, development. The authors of the aforementioned work distinguish between the two types of growth mechanisms, the first of which based on innovations, is potentially sustainable and universal, whereas the "second type comprises specific growth mechanisms contained only in some situations formed by certain types of institutional systems, and/or deforming the economic policy, and they may be activated by respective reforms and, after some time, sometimes a long time, they die out" (Balcerowicz, Rzońca eds., 2014). It is worth

noting that the reported concept of economic reforms is in line with Acemoglu and Robinson (2013). A detailed discussion on the concept of an economic miracle undertaken in top economic publications is provided in Boehlke et al. (2016).

The economy of Ireland as an example of an economic miracle has been the subject of wide economic and statistical analysis over the last few years. The literature on the Irish economy is quite vast. From the perspective of the aim of the paper it is reasonable to divide it into two sub-periods, the first one corresponds to the dynamic growth of up to 2008 and the second relates to the recession phase and after.

The factors of the great economic success of Ireland which started in the 1980s are manifold. Bradley and Hannan (2001) analyzed, among others, the role of structural funds in Ireland's recent economic growth and concluded that neither the single market nor the structural funds are likely to account fully for this increase in Ireland's share. It has been emphasized that it is fundamentally related with the social partnership agreements and the timing of the structural funds' use. On the other hand, Bradley and Birnie (2001) try to answer the question of whether a common united economy on the island of Ireland is possible using synergy coming from the great reforms that were introduced in both Northern Ireland and the Republic of Ireland. As the source of the economic success of the Irish economy some economists emphasized the role of the Irish economic policy, especially the stabilization policy, and since the middle of the 1990s, the institutional reforms – changes in economic law, tax system, education (Honohan and Walsh, 2002). Others focused on the influence of the FDI and financial support from the EU funds (Barry, 2002) or explaining the Irish case as the effective industrialization of the 1990s (Piński, 2013) and the combination of economic policy, institutional reforms (especially regulatory reforms) and membership of the EU (Szczepaniak, 2015). According to Cassidy (2004), there are five important considerations in the discussion about the Irish economy: solid macroeconomic fundamentals, general regulatory environment supporting and encouraging business and entrepreneurship development, good access to risk capital, educational attainment of the workforce and conditions to R&D activity. Many factors of economic success in Ireland are the subject of a broad and detailed discussion undertaken by Box (1998). She indicates that the intensity of growth in Ireland results from labor, capital and total factor productivity. The decomposition suggests that Ireland's TFP contribution to growth equaled 59% over the period 1970-1996, which is above average compared with studies of other countries. Ireland experienced an average

annual TFP growth of 2.4% over the period, with a labor growth of 0.55% and capital growth of 1.16%. Among the main factors of growth, Box named being an English-speaking nation which allows easy opening of the economy, and the historical connections arising from past migration to the USA as an attractive basis for foreign capital inflow. Such incentives as special low tax rates and high-profile country marketing, Ireland's EU membership and geographic position, managed to locate the investments in high-tech, export-oriented sectors which created job opportunities for highly skilled workers in Ireland and pushed Ireland's current account into a surplus. But foreign direct investment was only one of the key drivers of growth and increases in productivity. The others were: macroeconomic stability achieved partly due to the internal reforms and partly due to the ERM membership, high labor force growth and positive demographic processes such as a young population with increasing female participation, as well as the rising level of immigrants and finally, structural grants and agricultural support from the EU. Most of the literature is devoted to the pre-crisis period of economic development in Ireland. In more recent literature the causes of the crisis are broadly analyzed. Conefrey and FitzGerald (2010) analyzed the causes of the bubble in the housing market in Ireland that caused a huge deterioration in the entire economy. The most recent book by O'Leary (2015) developed a long and multi-aspect discussion on such factors of growth as industrial policy, innovation policy and business competitiveness in Ireland from1970 up to now. The author gives a precise insight into the sources of growth when sectoral productivity is analyzed. He uses the proportionality rule when the sectors dominated by multinational companies are considered showing that talking about the productivity of the Irish economy when multinationals are omitted may be strongly misleading. Armingeon and Baccaro (2012) analyzed the economic policy in the period 2007-2010 in the so-called GIIPS countries, including Ireland, compared with Germany. They emphasized the orthodox liberal policy line that the Irish government chose when they decided to guarantee the debt held by foreign lenders, slash public expenditures, increase taxes and cut public sector wages, the minimum wage and unemployment benefits. This caused the deterioration of public finances and consequently imposed severe conditions under which financial support from the troika composed of the ECB, EU and IMF was transferred to Ireland.

Let us have a look at some statistical information. The facts about intense economic growth period until 2008 are convincing. Treated as a peripheral economy, Ireland managed to change its image into a developed one in little

more than three decades. The average GDP annual growth rate in 1980-2008 was at 6.6%, while the GNP growth rate was on average one percent lower which still gives a very good result. The highest GDP level in Ireland, i.e. USD274.71 billion, was achieved in 2008. In the same period GDP per capita in terms of PPP in 1980 started from the level of USD13,434 and reached USD37,276 in 2008. Thus, Ireland could be characterized as a rapidly growing economy before the last financial crisis and the incredible GDP decrease after 2008. In the years 2008-2010 the average GDP growth rate was at minus 3.5%. The situation started to improve after 2013. When comparing with the other developed economies in Europe it is necessary to treat Ireland as quite typical after the period of crisis. Not typical is that, firstly, Ireland – together with Greece and Portugal – was forced to ask for financial support from international institutions and, secondly, unlikely the above-mentioned countries, it managed to start a new line of its development converging with the core countries. After negotiations with the European Union, IMF and the World Bank, Ireland has implemented an economic reforms program. Since 2010 the Irish economy has again started to grow. In 2014 the GDP per capita amounted to USD46,633. According to the World Bank Group, the GDP value of Ireland in 2014 represented 0.40 percent of the world economy, amounting to USD250.81 billion. The dynamics of GDP between 1980-2014 is presented in Figure 1 and its comparison to GNP is shown in Figure 2.

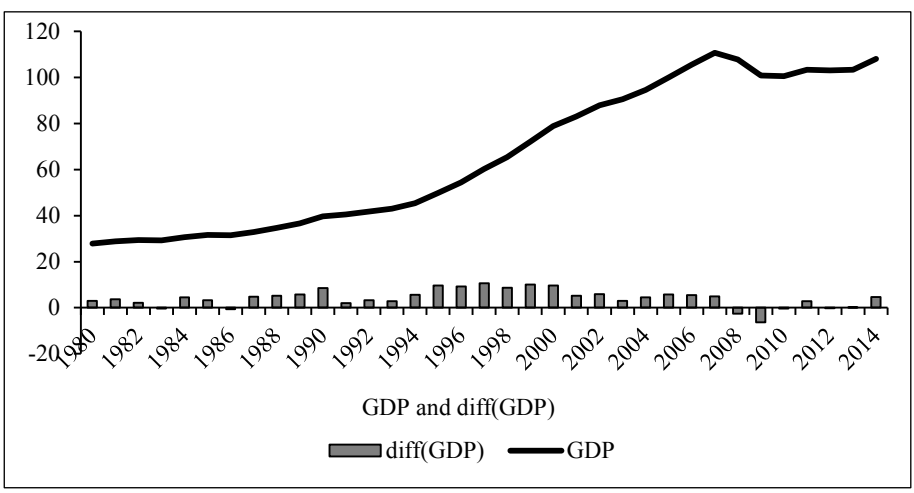

Figure 1. GDP level and its dynamics in 1980-2014

Source: based on the data from Eurostat, http://ec.europa.eu/eurostat.

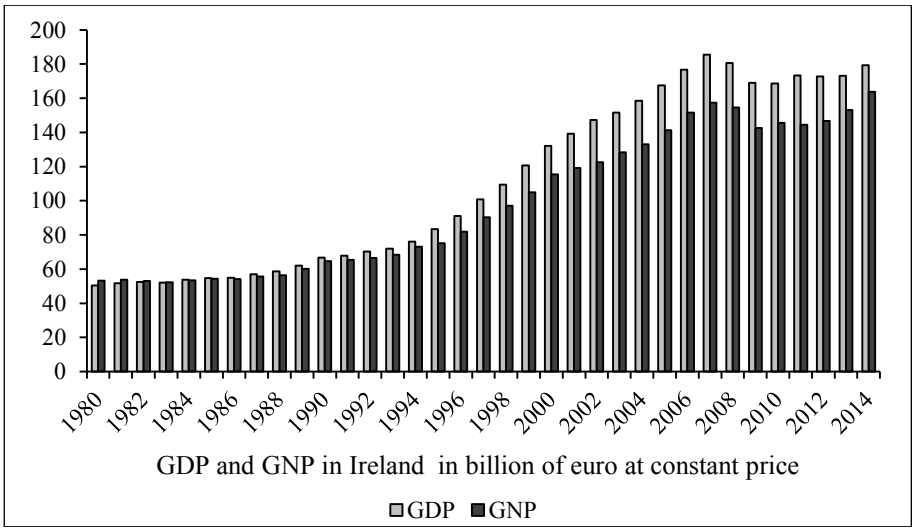

Figure 2. GDP and GNP level in 1998-2012

Source: based on the data from Eurostat, http://ec.europa.eu/eurostat.

As far as GNP level is concerned, it is deliberately lower and its growth rate is also smaller due to transfers from foreign companies located in Ireland to abroad. The scale of differences can be compared in Figure 3, where official reserves are presented together with the gap between GDP and GNP growing over time. It should be noted that between 1973 and 1988 the GNP level exceeded the GDP, but starting from 1989 the situation has changed. Furthermore, official reserves reached the level comparable to the magnitude of the difference only in 1995-1998. Since 1999 the gap increased up to 150%, while the magnitude of the reserves remained the same until 2003 and then decreased, reaching the lowest values during the period of crisis. It seems clear that the level of reserves does not compensate for the increasing gap between both aggregates, and that the gap results from the money transfers abroad. One can ask a question about the transfer prices. It should be noted that in 2010 Ireland introduced the Finance Act, which states, among others, that after 1 July 2010 the supply or acquisition of goods, services, money or intangible assets and relating to trading activities is within the charge to Irish corporate tax at the trading rate of 12.5% (see: www.pwc.com/gx/en/international-transfer-pricing/assets/ireland.pdf).
However, this fact is not expected to improve the situation for two reasons. Firstly, small and medium enterprises are the subject of exemption from this

rule, and secondly, it applies only for the arrangements entered into between related parties after 1 July 2010.

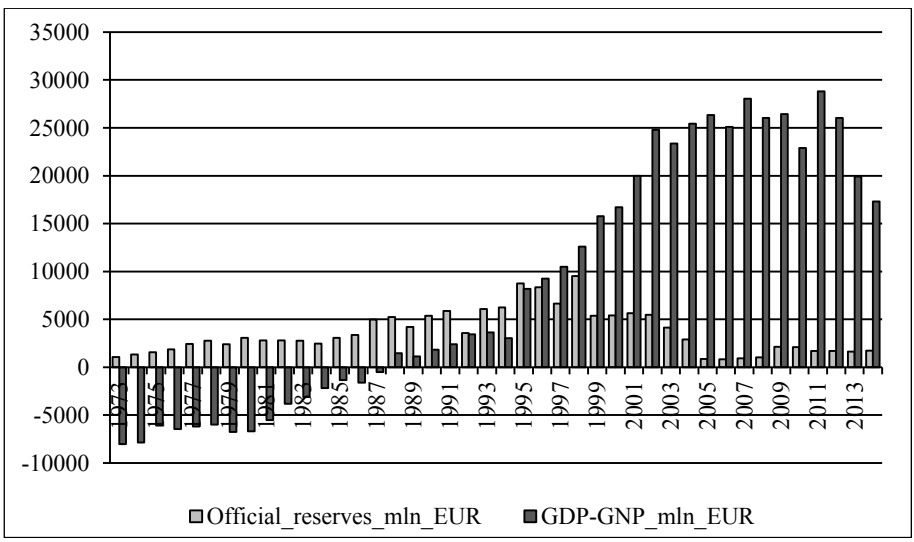

Figure 3. Official reserves and the difference between GDP and GNP in EUR millions
http://www.centralbank.ie/polstats/stats/Pages/default.aspx

Source: based on the data from Central Bank of Ireland.

Investigating the process of Irish economy development of 1980-2014 it could be observed that traditional factors behind the relatively strong growth remained important. In particular, these include the economically efficient utilization of labor and capital in order to keep the economy operating at close to its economic potential. These were determined by changes in both production factor resources and in their productivity. The unemployment rate in Ireland averaged 10.92 percent from 1983 until 2016, reaching an all-time high of 17.30 percent in December of 1985 and a record low of 3.70 percent in December of 2000. After the crisis it considerably increased but in 2016 it stabilized at the level of 7.8 percent. The economic growth of the Irish economy of 1980-2014 was mainly influenced by changes in multi-factor productivity (see Figure 4 for illustration).

It is obvious that aggregate productivity growth depends on the productivity of firms operating in Irish economy, but from the economic policy perspective the most important issue is to ensure institutional structures and policy settings supporting investment processes, innovations,

good managerial practices, efficient working incentives, entrepreneurship and risk taking.

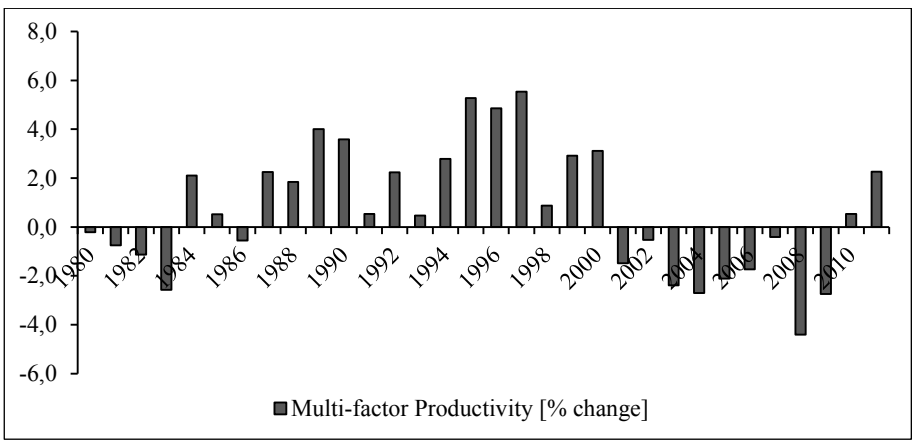

Figure 4. Multifactor productivity in Ireland 1985-2011

Source: based on the data from OECD, http://www.oecd.org.

Over the last 30 years the strength of the Irish economy could be compared with the most developed countries. In the beginning of its road to the high level of development and welfare Ireland was a peripheral country but having two important advantages which may not be available for all developing economies and regions – the English language and the existence of a sophisticated service sector demanding a skilled and active workforce. The relatively high level of human capital value as a result of the centrality of investment in the education sector had important implications for macroeconomic and regional strategy and policy. In the case of an English-speaking society it is no accident that almost 50% of foreign direct investments from the United States and Great Britain had been allocated in that very internationally oriented country, especially in the information technology sector and financial and legal services. After 1993 Ireland became one of the most industrialized economies in the world. The most spectacular achievements were noted in the period 1980–2007. In 1992, the output of the Irish industry per capita was only of 60% of German industry output per capita but in 1997 it was more than 100%. As a result of the FDI allocation, total employment in Ireland increased by 72% in the period 1990–2005. Information technology sector development, the high value of foreign direct investments, international experience in legal and financial

services, and consistency in industrial policy in the long run as an important part of national socio-economic strategy and culture have all played an important role in Irish economic success (O'Hearn, 1998, p. 38). Concerning the industrial policy, it could be possible to note failures as well as some successes. For example, the concentration on tax policy and tax incentives favored foreign capital over the domestic one. Also, the preferences for particular sectors were probably too risky. The Irish industrial policy in the period of 1980-2014 focused on sectors which were in the ascendant so the scope of regulations to attract investments in them was relatively limited. It is necessary to emphasize the strategic position of the Irish Industry Development Agency and a composite country marketing, which resulted in the FDI inflow. The Human Development Index (HDI) of Ireland in 1980 was 89% HDI of the USA. According to the Human Development Report (HDR, 2007, p. 229) in 2007 the HDI of Ireland was 104% of the average HDI of the EU15 and 100% of the HDI of the USA. As a result, after 1980 Irish economic development was accelerated because of the strong impact of foreign direct investment and changes of the internal economic policy stimulating rapid growth of exports, especially to the USA and the European Union, and thus building social confidence.

Presenting the basic macroeconomic data of the Ireland economy in the period of 1980-2014 it is necessary to emphasize the significant changes in the magnitude of the Government Debt and its ratio to the GDP of Ireland, recording a Government Debt to GDP of 107.50 percent of the country's GDP in 2014. In 2015 this was reduced to 93.80 percent. From 1980 until 2015 Government Debt to GDP in Ireland averaged 73.56 percent, reaching an all-the-time high of 120.10 percent in 2012 and a record low of 23.60 percent in 2006. Government debt to GDP and its structure in Ireland is reported in Table 1.

A basic element of the Irish socio-political landscape and a very important institution in the Irish contemporary economy is the voluntary "pay pact" signed in 1987 by the government, trade unions and employers to reduce public debt and wages. It is still working despite the fact that in 2009 the government temporarily ignored it, decreasing the wages. But in 2010 a new social partnership was signed, which influenced the shape and the implementation mode of the economic policy. Strategic issues for long-term economic growth exposed changes in the structure of the Irish economy by investment in infrastructure, especially in public transport, new housing, IT sector and protection of the natural environment (O'Hagan and Newman, 2005).

Table 1

Government debt to GDP and its structure in Ireland in 1980–2014

Ireland	Last (2014)	Previous (2013)	Highest	Lowest	Unit
Government debt to GDP	93.80	107.50	120.10	23.60	%
Government budget	–2.30	–3.80	4.90	–32.30	% of GDP
Government budget value	–1213.00	1194.00	6733.00	–4927.00	EUR million
Government spending	6604.00	6699.00	7534.00	4298.00	EUR million
Government spending to GDP	35.10	38.60	65.70	30.90	%
Government revenues	12799.00	17602.00	17602.00	315.40	EUR million
Government debt	203187.00	215539.00	215539.00	36004.00	EUR million
Fiscal expenditure	13969.00	17560.00	19435.00	410.00	EUR million
Asylum applications	140.00	190.00	405.00	55.00	Persons
Credit rating	70.38				
Military expenditure	1188.10	1191.30	1454.80	916.90	USD million

Source: www.tradingeconomics.com.

The breakthrough in Irish economic history was the accession to the EEC in 1973. It is obvious that the economic development of this country was systematically supported by financial transfers from the European Union, but the most important role was played by the Irish government's economic policy. In 2008 Ireland had the highest level of household debt relative to disposable income at about190%. Today, after the last financial crisis, Ireland is the only one of the so-called GIIPS countries growing very fast (over 7% in 2015) with a low national debt (about 1.5% in 2015). This has happened because of the swiftly implemented sweeping reforms, mainly in the financial sector. In 2014 the Index of Economic Freedom and Heritage Foundation Index ranked Ireland ninth in the world among the "economically free" economies.

The export-dominated economy has also been lifted by the relatively weak euro and the strong resumption of domestic demand from its once heavily indebted consumers. The case of Ireland shows the importance of opening up to the global economy, but from another perspective the fact remains that the cultural diversity within the country, the scope and quality of economic policy, and political consensus, were also great advantages in its socio-economic success.

3. THE THRESHOLD ERROR-CORRECTION CLASS OF MODELS

Economic growth has rarely been analysed by means of nonlinear equilibrium. After the Washington consensus, the economists have moved away from the issue of real growth factors and replaced it with the institutional framework that is necessary for growth to be operated. However, Durlauf (2000) showed his concern about the possibility of the effective modelling of legal, social or political factors in the context of economic growth because most of them are endogenously related with the level of growth. Durlauf et al. (2005) argued that modelling economic growth based on time series is limited due to the short series of data, sensitivity of growth to business cycles and other short-term instabilities. However, nonlinearity and multi-regimes in the growth patterns seem to be a good alternative. On the other hand, after the economic recession observed in the first decade of 21st century it became clear that creating economic growth is still the economic policy's priority. Having in mind all these concerns, remarks and needs, we tried to model the economic growth in Ireland using a threshold approach (see Tong, 1990) which is in fact a nonlinear one.

We took into account that before the 1980s Ireland was a really poor country and the growth factors must have been introduced exogenously, possibly using a discrete threshold. This assumption determined the set of threshold variables analysed in the research. The modelled variable is GNP in Ireland. We assume that its lagged values can also serve as a threshold. Moreover, the following explanatory variables and possible thresholds are divided into two groups: external factors based on the institutional system in Ireland such as foreign direct investment and net income from the EU, and internal factors resulting from the economy and economic policy i.e. employment, investment, multifactor productivity, GDP deflator, net exports, short and long term interest rates and public debt/GDP ratio. Taking into account the data availability, the above mentioned categories cover mostly the emphasized factors of the Irish economic growth. The set of time series taken into account has been summarized in Table 2.

As was already mentioned, the key assumption is that intense economic growth appears in short or medium periods while the long-term path of economic growth remains stable. Thus a Threshold Error Correction Model (TECM) has been selected for analysis as it is an appropriate tool for the effective modeling of different regimes of growth around the long run. It fits a situation when the model parameters are stable but they differ across the regimes that are specified *via* estimation. Furthermore, it copes with the structural breaks which are typically observed when the policy regimes change.

Table 2

Variables used in the study (constant prices)

Variable	Variable name	Variable unit
FDI_t	Foreign direct investment	millions of euro
NI_EU_t	Net income from EU	millions of euro
EMP_t	Employment	thousands
PD_t	Public debt	% of GDP
$Deflator_t$	GDP deflator	index
I_t	Investments	billions of euro
N_Ex_t	Net exports	billions of euro
SR_t	Short-term interest rate	%
LR_t	Long-term interest rate	%
MFP_t*	Multi-factor productivity	% change
GNP_t	Gross National Product	billions of euro

http://www.cso.ie/en/statistics/, http://stats.oecd.org/

*MFP_t was observed only in 1980-2011, thereafter replaced by Total Factor Productivity (TFP_t).

Source: the data were downloaded from http://www.economywatch.com/economic-statistics/country/Ireland/.

At the very beginning we assumed a long-term dynamics that means a long-term equilibrium path and short-term adjustment. We based our research on the TAR and M-TAR approaches defined by Balke and Fomby (1997) and developed by Enders and Siklos (2001). The starting point of the Enders and Siklos procedure is the following long-term equation:

$$Y_t = \alpha_0 + \sum_{i=1}^{k} \alpha_i X_{it} + u_t, \tag{1}$$

where all variables Y_t, X_{it} for $i = 1,...,k$ are assumed to be I(1). Stationarity of the adjustment process (residual process) satisfies a threshold cointegration (TEC) if:

$$\Delta u_t = I_t \rho_1 u_{t-1} + (1 - I_t) \rho_2 u_{t-1} + \sum_{i=1}^{p} \beta_i \Delta u_{t-i} + \varepsilon_t, \tag{2}$$

where:

$$I_t = \begin{cases} 1 & \text{for} \quad u_{t-1} \geq \gamma \\ 0 & \text{for} \quad u_{t-1} < \gamma \end{cases} \tag{3}$$

in the case of TAR-type adjustment

or
$$I_t = \begin{cases} 1 & \text{for} \quad \Delta u_{t-1} \geq \gamma \\ 0 & \text{for} \quad \Delta u_{t-1} < \gamma \end{cases} \tag{4}$$

in the case of M-TAR -type adjustment, while $\gamma = 0$ in both cases.

The set of two null hypotheses to be tested is as follows:

$$H_0^1 : \rho_1 = \rho_2 = 0$$

$$H_0^2 : \rho_1 - \rho_2 = 0$$

The interpretation of the hypotheses is the same as in Balke, Fomby (1997). H_0^1 is for the case of no threshold cointegration then the Engle-Granger linear cointegration (Engle and Granger, 1987) is confirmed, while H_0^2 assumes a symmetric reaction, so confirming a linear cointegration. If the Enders and Siklos procedure indicates the threshold type of cointegration around the long-term equilibrium, it means that the short-term adjustment is asymmetric respectively for positive and negative changes. When the results of testing seem too general, the reasonable solution is to ask about single variables that diversify the mechanism of a short-term adjustment. Thus, the second approach applied in the research is the model proposed by Kapetanios et al. (2006) and modified by Bruzda (2007). Kapetanios, Shin and Snall (2006) proposed other types of indication function than (3-4) and assumed a maximum one co-integration vector. Bruzda (2007) implemented equation (5) to test the threshold co-integration, considering that in the case of common factors (COMFAC), model (5) can be reduced to the form given by (2). When the COMFAC condition does not hold a test equation in the form (5) improves the power of the procedure. Furthermore, Bruzda showed the way of testing for threshold co-integration based on (5) without the autoregressive component using the bootstrap technique and extended the original procedure by the Enders and Siklos case for three regimes. In the reported research, model (5) has been used as a basis of testing but the threshold variables were taken individually from the following: the long-term regression of the form (1) and their first differences. Having (1) unchanged the testing equation (2) is the subject of re-formulation to the form:

$$\Delta Y_t = I_t \, \rho_1 \, u_{t-1} + \left(1 - I_t\right) \rho_2 \, u_{t-1} + \omega \Delta X_t + \sum_{j=1}^{p} \psi_{yj} \Delta Z_{t-j} + e_t, \tag{5}$$

where: $Z_t = \left(Y_t, X_{1t}, X_{2t}, \ldots, X_{kt}\right)'$, $X_t = \left(X_{1t}, X_{2t}, \ldots, X_{kt}\right)'$, u_t is a residuals process from eq. (1) and I_t is the Heaviside function (3) or (4) and γ is a subject of estimation. The criterion of selection is the minimum value of the Akaike Information Criterion AIC for given γ $\left(-\infty < \hat{\gamma} < \infty ; \hat{\gamma} = arg \min_{\gamma} AIC(\gamma)\right)$. The set of hypotheses to be tested remains the same.

In the next step, a new testing equation of the form (6) is proposed when regimes are split not only by lagged ECMs but also by the lagged values of exogenous and endogenous variables. The intuition for composing equation 6 lies in the fact that asymmetry in the adjustment process can occur in the short run, and can result from the changes of variables other than the ECM. A test analogical to the procedure defined by Enders and Siklos is conducted using the statistically significant equation (6). Then the proposed TECM model takes the following form:

$$\begin{aligned}\Delta Y_t = {} & I_t \rho_1 u_{t-1} + (1 - I_t)\rho_2 u_{t-1} + I_t \omega_1 \Delta X_t + (1 - I_t)\omega_2 \Delta X_t + \ldots \\ & + \sum_{j=1}^{\max\{p;q\}} I_t \psi_{1yj} \Delta Z_{t-j} + \sum_{j=1}^{\max\{p;q\}} (1 - I_t)\psi_{2yj} \Delta Z_{t-j} + e_t,\end{aligned} \tag{6}$$

where: Z_t, X_t, u_t and γ are defined as in (5) and I_t is a function defined by one of the following (3), (4), (7) or (8)
where:

$$I_t = \begin{cases} 1 & \text{for} \quad z_{t-i} \geq \hat{\gamma} \\ 0 & \text{for} \quad z_{t-i} < \hat{\gamma} \end{cases} \tag{7}$$

or

$$I_t = \begin{cases} 1 & \text{for} \quad \Delta z_{t-i} \geq \hat{\gamma} \\ 0 & \text{for} \quad \Delta z_{t-i} < \hat{\gamma} \end{cases} \tag{8}$$

where $z_t \in Z_t$ and $-\infty < \hat{\gamma} < \infty ; \hat{\gamma} = arg \min_{\gamma} AIC(\gamma)$.

In the proposed model the short-term equations differ between the regimes taking both a vector of explanatory variables and parameters estimate. The advantage of such an approach is that in the final TECM a different set of variables can act in different regimes leaving the long-term

relationship unchanged. Equation (6) shows the possibility of a discrete threshold coming from different sources of the long and short term regime changes and a short-term asymmetry. The empirical indication of the threshold variable allows identifying the most likely model (or a set of models) of economic growth that dominated over the analyzed period.

4. EMPIRICAL STUDY

Considering the remarks on the GDP and GNP processes in Ireland the empirical research was focused on modelling the GNP observed in 1980-2014 as yearly observations. The analysis prepared for the GDP has been discussed in Boehlke et al. (2017). Here our aim is to identify the possible structural breaks and to explain the causes for structural breaks with threshold models. As discussed in Durlauf et al. (2005), we met the problem of short time series as the total number of observations is $n=35$. This fact limited the estimated TECMs of type (6). The threshold values of the variables were then estimated separately using the Chan approach (Chan and Tong, 1996).

The data for analysis were taken from the Eurostat, OECD and the Central Bank of Ireland databases. All the calculations were made for both: levels and logs. The first operation conducted for the GNP series was filtering out a long-term component from the original series which was the subject of the long-term regression estimation, which was then denoted by HP_GNP_t.

The data exhibited non-stationarity, thus at the first step we tested the time series for unit roots using the Philips and Perron (1988) test and for stationarity using the KPSS (Kwiatkowski et al. 1992) test. The first one is a modification of the traditional ADF test (Dickey and Fuller, 1974) when the series is correlated at higher order lags. Phillips and Perron (1988) proposed a method of dealing with correlation when testing for a unit root. In the PP method the DF test equation is estimated, and the test statistic is modified such that serial correlation does not affect the asymptotic distribution of the test. The second one is a stationarity test. The results of testing confirmed the unit roots hypothesis in all the analysed cases (see Table 3). The next step of analysis was aimed at finding structural breaks in the time series in interest. When a structural break takes place, it is another argument to consider a threshold-type model. Perron (1989) argued that in the presence of a structural break, the standard unit root tests (i.e. the ADF test) are biased

Table 3

Results of PP test H_0: Z_t~I(1) and KPSS test H_0: Z_t~I(0)

Test	**PP**				**KPSS**			
Time series	**Original data**		**Logs**		**Original data**		**Logs**	
Variable	**Levels**	**Differences**	**Levels**	**Differences**	**Levels**	**Differences**	**Levels**	**Differences**
FDI_t	I(1)	I(0)	I(1)	I(0)	I(1)	I(0)	I(0)	I(0)
$Deflator_t$	I(2)	I(1)	I(1)	I(0)	I(1)	I(0)	I(1)	I(0)
PD_t	I(2)	I(1)	I(1)	I(0)	I(1)	I(0)	I(1)	I(0)
NI_EU_t	I(1)	I(0)	I(1)	I(0)	I(0)	I(0)	I(0)	I(0)
N_Ex_t	I(1)	I(0)	I(1)	I(0)	I(1)	I(0)	I(1)	I(0)
EMP_t	I(2)	I(1)	I(2)	I(1)	I(1)	I(0)	I(1)	I(0)
Lr_t	I(1)	I(0)	I(1)	I(0)	I(1)	I(0)	I(1)	I(0)
Sr_t	I(1)	I(0)	I(1)	I(0)	I(1)	I(0)	I(1)	I(0)
EMP_t	I(1)	I(0)	I(1)	I(0)	I(1)	I(0)	I(1)	I(0)
MFP_t	-	I(0)	-	-	I(0)	I(0)	-	-
GNP_t	I(1)	I(0)	I(1)	I(0)	I(1)	I(0)	I(1)	I(0)
HP_GNP_t	I(2)	I(1)	I(2)	I(1)	I(1)	I(0)	I(1)	I(0)

GNP_t and HP_GNP_t denote GNP process with and without cyclical component – after the Hodrick-Prescott type of filtration, respectively.

Source: own calculations.

towards the non-rejection of the null hypothesis. Perron showed that most macroeconomic series were not characterized by a unit root but rather that persistence arises only from large and infrequent shocks, and that the economy returns to a deterministic trend after small and frequent shocks. In the research, the Andrews and Zivot (1992) test (AZ) for an endogenous structural break has been applied. It is a sequential test which utilizes the full sample and uses different dummy variables for each possible break date. The break date is selected where the t-statistic from the augmented DF test of unit root is at a minimum value. The results of the AZ test are presented in Table 4.

At the next stage the testing procedure described in section 4 (equations 1-8) has been applied. The results are presented in Tables 5, 6, 7 and 8. First, the long-term equations have been estimated for HP_GNP in levels and in logs. The long-term equations included all the variables used in the analysis (see Table 2) which were systematically eliminated from general to specific. The estimated long-term equations are presented in Table 5.

Table 4

Andrews and Zivot (AZ) test results

Variable	Levels	Date of change	Differences	Date of change	Levels	Date of change	Differences	Date of change
	Original data				Logs			
FDI_t	-3.365	2004	-8.213***	2007	-5.468***	2004	-10.248***	2007
$Deflator_t$	-3.769	2000	-3.453	2008	-4.318	2008	-4.1837	1997
PD_t	-9.625***	2008	-6.981***	2008	-7.815***	2008	-4.990**	2008
N_Ex_t	-3.730	2001	-6.171***	2005	-2.775	1987	-13.444***	1995
Lr_t	-3.294	2007	-6.279***	1990	-5.276**	2009	-3.010	2006
Sr_t	-4.993**	1994	-5.899***	1993	-0.942	2009	-7.123***	2008
NI_EU_t	-2.939	1990	-5.353***	1994	-3.246	2005	-5.374***	1998
EMP_t	-3.721	2009	-4.968***	2008	-3.913	2009	-4.561	2008
I_t	-3.742	2002	-6.358***	2006	-3.408	2008	-3.836	2006
MFP_t	-	-	-4.991**	2001	-	-	-	-
GNP_t	-3.305	1999	-4.426	2008	-4.142	2008	-4.212	2007
HP_GNP_t	-4.557	2008	-3.235	1991	-3.872	2000	-4.027	1991

H_0: Z_t ~ a non-stationary variable with drift and a structural change in intercept

H_1: Z_t ~ a stationary variable and a structural change in intercept

Rejecting H_0 at the level 1%. 5% and 10% respectively ***. ** and *.

Source: own calculations.

Table 5

The estimated long-term models

Variable	HP_GNP		HP_log_GNP	
	parameter estimate	p-value	parameter estimate	p-value
const	-12.588	0.049	-4.047	0.0000
NI_EU	0.003	0.000		
EMP	0.052	0.000	1.043	0.0000
N_Ex	0.601	0.000	0.045	0.0005
Sr	-0.358	0.042	-0.006	0.0633
I	0.341	0.006		
PD	0.143	0.000	0.095	0.0000
Deflator			0.119	0.0184
DW test		1.2772	DW test	1.0276
R^2		0.9987	R^2	0.9959
QLR test		0.0008	QLR test	0.0000

QLR test represents the Quandt test for structural breaks (Quandt 1960)

Source: own calculations.

Table 6

The Enders and Siklos test results for HP_GNP and HP_log_GNP based on eq. 2

Threshold variable	Threshold value	H^1: ($\rho_1 = \rho_2 = 0$) p-value	H^2: ($\rho_1 - \rho_2 = 0$) p-value	Remark
ECM(t-1) (HP_GNP)	0	0.0021	0.823	Linear cointegration
d_ECM(t-1) (HP_GNP)	**0**	**0.0004**	**0.102**	**Threshold cointegration**
ECM(t-1) (HP_log_GNP)	**0**	**0.00005**	**0.137**	**Threshold cointegration**
d_ECM(t-1) (HP_log_GNP)	0	0.00009	0.321	Linear cointegration

Source: own calculations.

The specification of the long-term equations differs whether the variables are analyzed in levels and in logs. The vector of variables that entered the long-term equation for levels includes: net income from the EU, employment, net exports, short interest rate, investment and public debt. In the case of logs the corresponding set includes: employment, net exports, short interest rate, public debt and the GDP deflator. What is interesting is that all the parameters, apart from the short interest rate took positive values. This concerns for example the public debt/GDP ratio which positively influenced the growth in the entire sample. The residuals from both regressions show the structural breaks tested using the classic Quandt test (Quandt, 1960).

In Table 6 the results of the original Enders and Siklos test are shown. It can be seen that in two cases out of four, the Enders and Siklos test supports the hypothesis of threshold cointegration. The p-values for asymmetry hypothesis are slightly above 0.1 but in the case of the short time series we decided to take this result as a preliminary indicator. This means that around the long-term path asymmetry of short-term adjustment can be observed. This finding gives the basis for further investigation in order to reveal the possible significant threshold variables that influence the economic growth pattern in Ireland.

The results of testing for threshold cointegration and asymmetry using the approach proposed in the paper are given in Tables 7 and 8. The phrase "long-term equation" in Tables 7-8 denotes the fact of the presence of a threshold variable in the long-term equation.

Table 7

The Enders and Siklos test results based on eq. 6 – threshold variable: levels

Threshold variable	Threshold value	Long-term equation	Hypothesis	p-value	Remarks
ECM(t-1)	0.000	Yes	H^1: ($\rho_1 = \rho_2 = 0$)	0.134	Partial cointegration (one side only)
d_ECM(t-1)	0.000	Yes	H^1: ($\rho_1 = \rho_2 = 0$)	0.056	Partial cointegration (one side only)
NI_EU (t-2)	1627.900	Yes	H^2: ($\rho_1 - \rho_2 = 0$)	0.000	**Threshold cointegration**
defl_GDP(t-3)	64.900	Yes	H^2: ($\rho_1 - \rho_2 = 0$)	0.000	**Threshold cointegration**
hpt_GNP(t-5)	141.581	Yes	H^1: ($\rho_1 = \rho_2 = 0$)		Too few observations in the regime
Emp(t-4)	1901.600	Yes	H^1: ($\rho_1 = \rho_2 = 0$)		Too few observations in the regime
Sr(t-5)	6.250	Yes	H^1: ($\rho_1 = \rho_2 = 0$)	0.096	Linear cointegration
FDI(t-5)	18210.640	Yes	H^1: ($\rho_1 = \rho_2 = 0$)		Too few observations in the regime
MFP(t-1)	2.500	Yes	H^1: ($\rho_1 = \rho_2 = 0$)		Too few observations in the regime
d_defl_GDP(t-1)	1.700	Yes	H^1: ($\rho_1 = \rho_2 = 0$)	0.002	Partial cointegration (one side only)
d_Sr(t-4)	0.580	Yes	H^1 ($\rho_1 = \rho_2 = 0$)	0.020	Partial cointegration (one side only)
d_ NI_EU (t-5)	-11.500	Yes	H^1: ($\rho_1 = \rho_2 = 0$)	0.009	Partial cointegration (one side only)
d_hpt_GNP(t-5)	1.965	Yes	H^1: ($\rho_1 = \rho_2 = 0$)	0.080	Partial cointegration (one side only)
*TFP(t-1)**	1.6	Yes	H^1: ($\rho_1 = \rho_2 = 0$)	0.216	No cointegration

(*) denotes total factor productivity

Source: own calculations.

The results presented in Tables 6-7 show the severe problems that were indicated at the very beginning of the paper, i.e. the low number of observations applicable for the research. The severe limitations resulting from that fact are indicated in the tables by the comment "too few observations in the regime". Although we found out that in cases of net income from the EU lagged by two years, the GDP deflator lagged by three years (for the original data) and d_ECM(-1) and the first difference of the net exports lagged by two (for the logs), a significant threshold cointegration took place. These variables were present in the long-term relationships.

Table 8

The Enders and Siklos test results based on eq. 6 – threshold variable: logarithmic data

Threshold variable	Threshold value	Long-term equation	Hypothesis	p-value	Remarks
ECM(t-1)	0.000	Yes	H^1: ($\rho_1 = \rho_2 = 0$)	0.087	Partial cointegration (one side only)
d_ECM(t-1)	0.000	Yes	H^2: ($\rho_1 - \rho_2 = 0$)	0.083	**Threshold cointegration**
hpt_lnGNP(t-5)	4.692	Yes	H^1: ($\rho_1 = \rho_2 = 0$)	0.569	No cointegration
MFP(t-1)	2.500	Yes	H^1: ($\rho_1 = \rho_2 = 0$)	0.480	No cointegration
d_lnN_Ex$_t$ (t-2)	0.054	Yes	H^2: ($\rho_1 - \rho_2 = 0$)	0.009	**Threshold cointegration**
d_lnI(t-4)	-0.017	Yes	H^1: ($\rho_1 = \rho_2 = 0$)	0.074	Partial cointegration (one side only)
d_hpt_lnGNP(t-4)	0.014	Yes	H^1: ($\rho_1 = \rho_2 = 0$)		Too few observations in the regime
*TFP(t-1)**	1.6	Yes	H^1: ($\rho_1 = \rho_2 = 0$)	0.579	No cointegration

(*) denotes total factor productivity

Source: own calculations.

Table 9

Best TECM models for levels

Dependent variable		Threshold variable		Value of threshold		Threshold variable		Value of threshold	
		NI_EU(t-2)=		1.6		**deflator(t-3)=**		64.9	
Δ*HP_GNP*		**N1=**	10	**N2=**	22	**N1=**	16	**N2=**	14
		BIC=		-31.71		**BIC=**		-117.02	
variable		**I_ regime**	**I_ p-value**	**II_ regime**	**II_ p-value**	**I_ regime**	**I_ p-value**	**II_ regime**	**II_ p-value**
const		3.771	<0.001	-0.660	0.048	0.124	0.509	0.127	0.012
Δ*Sr*				-0.183	0.035	-0.128	0.003	-0.031	<0.001
Δ*Lr*								0.051	0.003
Δ*I*		0.512	<0.001	0.110	0.033			-0.019	0.056
Δ*PD*				0.059	0.001	0.010	0.067	-0.012	0.012
Δ*FDI*				<0.001	0.003			0.000	0.005
Δ*NI_EU*				0.002	0.001			0.000	0.009
Δ*EMP*				0.016	0.001	0.004	0.018	0.002	0.038
Δ*Deflator*				0.209	0.006	0.173	<0.001	0.053	0.010
Δ*N_Ex*		0.167	0.032	0.195	<0.001	0.040	0.026		
Δ*ECM(t-1)*		-0.220	0.001	-0.312	0.005	-0.119	0.003	-0.038	0.001
Δ*HP_GNP(t-1)*				0.723	<0.001	0.742	<0.001	1.068	<0.001
ARCH LM(4)		5.014	(0.286)	2.359	(0.67)	4.058	(0.398)	4.606	(0.33)
Ljung-	*Q(2)*	0.7382	(0.691)					1.946	(0.378)
Box	*Q(3)*					1.0898	(0.78)		
max	*Q(4)*			1.7183	(0.787)				

For ARCH LM and the Ljung-Box tests, p-values are given in brackets.

Source: own calculations.

The estimation of the TECM parameters and testing for threshold cointegration and asymmetry of adjustment was carried out as the last step of the analysis. The aim was to establish the most likely thresholds that influence the fluctuations around the growth in the short term and then to estimate the (symmetric or asymmetric) reaction of the first differences of GNP on the growth factors within the regimes. The long-term models (ECM-terms) are shown in Table 5, while the best TECM models indicated by minimum values of BIC are presented in Tables 9 and 10. The nominal values of threshold variables are also shown in the tables.

Table 10

Best TECM models for logarithmic data

Dependent variable		**Threshold variable**		**Value of threshold**		**Threshold variable**		**Value of threshold**	
		Δ*ECM (t-1)*		0		**Δ*N_Ex(t-2)***		0.054	
Δ*HP_ln_GNP*		**N1=**	18	**N2=**	14	**N1=**	21	**N2=**	10
		BIC =		-284.47		**BIC =**		-313.43	
variable		**I_ regime**	**I_ p-value**	**II_ regime**	**II_ p-value**	**I_ regime**	**I_ p-value**	**II_ regime**	**II_ p-value**
const		0.041	<0.001	0.009	0.017	0.004	0.063	0.031	<0.001
Δ*Sr*		-0.003	0.066	-0.006	<0.001	-0.002	<0.001		
Δ*Lr*				0.008	0.002				
Δ*I*				0.065	0.001	0.028	0.019		
Δ*PD*		-0.148	<0.001					-0.047	0.012
Δ*FDI*									
Δ*NI_EU*		-0.015	0.052	0.022	0.030	0.005	0.066		
Δ*EMP*						0.109	0.025		
Δ*Deflator*								-0.168	0.050
Δ*N_Ex*		0.018	0.024			0.019	0.020	0.011	0.047
Δ*ECM(t-1)*		-0.359	0.004	-0.088	0.156	-0.052	0.155	-0.235	0.103
Δ*HP_GNP(t-1)*				0.639	<0.001	0.713	<0.001		
ARCH LM(4)		1.065	(0.9)	1.145	(0.887)	5.000	(0.287)	5.805	(0.214)
Ljung-Box max	*Q(2)*			2.3519	(0.309)			0.2049	(0.903)
	Q(3)	4.3113	(0.23)						
	Q(4)					2.6142	(0.624)		

For ARCH LM and the Ljung-Box tests, p-values are given in brackets.

Source: own calculations.

The results of both the long-term equations and TECM estimation show that the reasonable parameters estimates were obtained. In general, parameter estimates in the long-term equations were greater in magnitude than the corresponding short-term adjustment coefficients for both logarithmic and non-logarithmic data. The parameter estimates standing for the adjustment to the long-term path are different for the negative and

positive side of the long-term equilibrium. They are presented in the tables as *d_ECM(t-1)* in the I regime and in the II regime. The division into two regimes according to the threshold variable (net income from the EU) is illustrated in Figure 5.

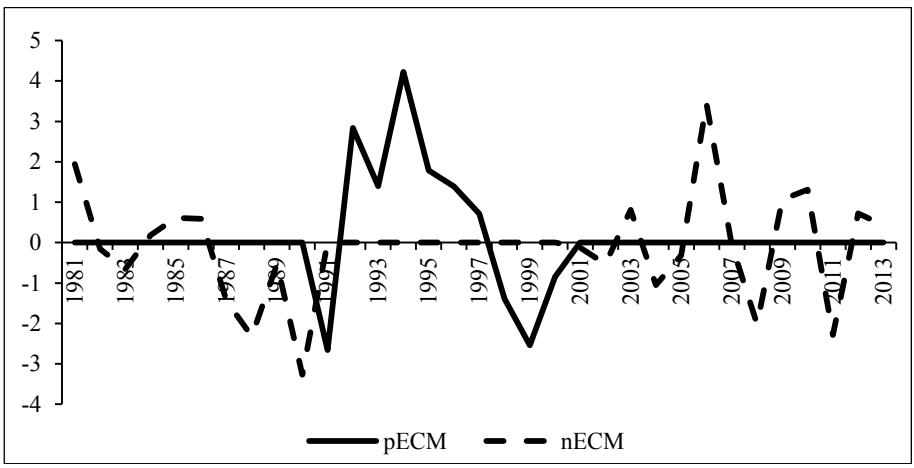

Figure 5. ECM for threshold variable: NI_EU(*t*–2)

Source: own calculations.

It is worth noting that the net income from the EU playing a role of the threshold variable is present only in the second regime covering the years 1991-2000 This period has been identified as the years when adjustment to the equilibrium level was made from the positive side over the level of the EU grants lagged by two years, amounted to 1.6 billion Euro. In the short run only, the economy was driven by the first differences of investments and net exports. In the years 1981-1990 and 2001-2013 the short run of GNP has been adjusted from the negative side. In the second regime a wide set of factors was moved into operation than in the first one. The corresponding *d_ECM(t-1)* coefficients are -0.220 and -0.312, which means that the adjustment from the negative side was faster than from the positive one.

Another threshold variable is the GDP deflator. In this case the magnitude of the short-term adjustment was as follows: -0.119 from the positive side and -0.038 from the negative one. For logarithmic data in the case of *d_ECM(t-1)* periods, the magnitude was equal to: -0.359 and -0.088, respectively in the first (positive) and in the second (negative) regimes; and in the case of d_net export the values are equal: -0.052 and -0.235. These

values of short-term adjustment show that the regimes are constructed in different ways based upon the threshold. The careful analysis of the results shows that the factors of growth had a positive impact in short run. Only the short interest rate, which was partly responsible for domestic investment and inflation limitation, has a negative sign in all the estimated TECM models which is in line with the expectations. It is worth noting that since 2008 its role was very much limited due to the liquidity trap (Eggertsson and Krugman, 2012). The role of changes in public debt/GDP ratio is relatively diversified because positive and negative signs appear in the estimated TECM models. The lowest values of BIC indicate that the model with GDP deflator lagged by three years is preferred by the data when levels are analysed, while in the case of logs – the model with net exports lagged by two years seems to be the best one.

5. ROBUSTNESS OF THE ESTIMATED TECM MODELS

In this part of the paper we described briefly the procedure of checking the robustness of the results obtained above. To do so, we employed the Tsay test (Tsay, 1998). This test indicates whether the variable is generated by a linear or non-linear process. In the null it is assumed that Y_t is linear versus the alternative, i.e. that it follows the multivariate threshold model (9).

$$Y_t = c_j + \sum_{i=1}^{p} \varphi_i^{(j)} Y_{t-i} + \sum_{i=1}^{q} \beta_i^{(j)} X_{t-i} + \varepsilon_t^{(j)} \text{ if } r_{j-1} < z_{t-d} \le r_j \qquad (9)$$

where: c_j – a vector of constants, Y_t . – k-dimensional time series, $Y_t = (Y_{1t},\ldots,Y_{kt})'$, X_t – v-dimensional exogenous variables, $X_t = (X_{1t},\ldots,X_{vt})'$, z_{t-d} a threshold variable with delay d, $-\infty = r_0 < r_1 < \ldots < r_{s-1} < r_s = \infty$, p and q are non-negative integers and $j = 1, \ldots, s$, $t = 1, \ldots, n$.

The innovations satisfy $\varepsilon_t^{(j)} = \Sigma_j^{1/2} a_t$, where $\Sigma_j^{1/2}$ are symmetric positive definite matrices and $\{a_t\}$ is a sequence of serially uncorrelated random vectors with mean 0 and covariance matrix I, which is the identity matrix. The threshold variable z_t have a continuous distribution and it is assumed to be stationary. It is assumed that the threshold viable z_t is known, but the delay d, the number of regimes s, and the thresholds r_i remains unk.

The starting point of the test is the auxiliary regression (11). Assuming that

$$Y_t^{'} = x_t^{'}\Phi + \varepsilon_t^{'}, \qquad t = h+1,\ldots,n, \tag{10}$$

where $h = max(p,\ q,\ d)$, $x_t^{'} = \left(1, Y_{t-1}^{'},\ldots,Y_{t-p}^{'}, X_{t-1}^{'},\ldots,X_{t-q}^{'}\right)$ is a ($pk + qv + 1$)- dimensional regressor, Φ is the matrix of parameters, the least squares estimates of (10) are useful. The auxiliary regression (11) based on the increasing order of the threshold variable z_{t-d} is as follows

$$Y_{t(i)+d}^{'} = x_{t(i)+d}^{'}\Phi + \varepsilon_{t(i)+d}^{'}, \quad i = 1,\ldots,n-h, \tag{11}$$

where $t(i)$ is the time index of $z(i)$, $z(i)$ is the i^{th} smallest element of S, S is the threshold variables with regression (10), $S = \left\{ z_{h+1-d},\ldots,z_{n-d} \right\}$.

If Y_t follows a linear process, then the recursive least squares estimator of the regression (11) is consistent, so that the predictive residuals approach the white noise. In effect, the predictive residuals are uncorrelated with the regressors $x_{t(i)+d}^{'}$. On thether hand, if Y_t follows a threshold model, then the predictive residuals are no longer the white noise, because the least squares estimator is biased. In this case, the predictive residuals are correlated with the regressor $x_{t(i)+d}^{'}$. This constitutes the philosophy of the test. Let $\hat{\Phi}_m$ be the least squares estimate of Φ in the equation (11) with $i = 1, ..., m$. This signifies the estimate of the regression (11) using data points associated with the m smallest values of z_{t-d}.

Let us denote

$$\hat{e}_{t(m+1)+d} = Y_{t(m+1)+d} - \hat{\Phi}_m^{'} x_{t(m+1)+d} \tag{12}$$

is the predictive residuals and

$$\hat{\eta}_{t(m+1)+d} = \frac{\hat{e}_{t(m+1)+d}}{\left[1 + x_{t(m+1)+d}^{'} V_m x_{t(m+1)+d}\right]^{1/2}}, \tag{13}$$

where

$$V_m = \left[\sum_{i=1}^{m} x_{t(i)+d} x_{t(i)+d}^{'} \right]^{-1}$$

is the standardized predictive residual of regression (11). These quantities can be efficiently obtained by the recursive least squares algorithm. Next, consider the regression

$$\hat{\eta}'_{t(l)+d} = x'_{t(l)+d}\Psi + \mathrm{w}'_{\mathrm{t(l)+d}},\ \mathrm{l} = \mathrm{m}_0 + 1, \ldots, \mathrm{n} - \mathrm{h}, \tag{14}$$

where m_0 means the starting point of the recursive least squares estimation.

The hypotheses refer to equation (14)

$$H_0 : \Psi = 0$$

$$H_1 : \Psi \neq 0 .$$

Then Tsay proposes the following test statistic:

$$C(d) = \left[n - h - m_0 - (kp + vq + 1)\right]\left\{\ln\left[\det(S_0)\right] - \ln\left[\det(S_1)\right]\right\}, \tag{15}$$

where the delay d signifies that the test depends on the threshold variable z_{t-d}, det(A) indicates the determinant of the matrix A, and

$$S_0 = \left[\frac{1}{n - h - m_0}\right] \sum_{l=m_o+1}^{n-h} \hat{\eta}_{t(l)+d} \hat{\eta}'_{t(l)+d},$$

and

$$S_1 = \left[\frac{1}{n - h - m_0}\right] \sum_{l=m_o+1}^{n-h} \hat{w}_{t(l)+d} \hat{w}'_{t(l)+d},$$

where $\hat{w}_t$ is the least squares residual of regression (14). Under the null hypothesis that Y_t is linear and some regularity conditions, $C(d)$ is asymptotically a chi-squared random variable with $k(pk + qv + 1)$ degrees of freedom.

In the case of the GNP in Ireland, the results of the Tsay test are presented in Table 11.

Table 11

The results of the Tsay test for selected thresholds

Dependent variable	Threshold variable	C(d)	p-value
hpt_GNP$_t$	*NI_EU(t-2)*	13.6825	0.00107
hpt_GNP$_t$	*deflator(t-3)*	61.7186	< 0.00001
hpt_ln_GNP$_t$	*d_ECM(t-1)*	38.1546	< 0.00001
hpt_ln_GNP$_t$	*d_ln_N_Ex(t-2)*	19.8055	0.00001

Source: own calculations.

The results refer to the cases of TECM models which were previously confirmed by the modified version of the Enders and Siklos procedure described in section III.

It can be concluded that in all cases of the identified thresholds the dependent variable GNP_t follows a threshold model. Thus, all the identified threshold variables, such as *NI_EU(t-2)*, *Deflator(t-3)* in the case of levels and *d_ECM(t-1)* and *d_ln_N_EX(t-2)* in the case of logs were confirmed by the Tsay test that validates the results. This means that funds incoming from the European Union and GDP *Deflator* (in levels) as well as *d_ECM(t-1)* and *d_ln_N_EX(t-2)* delimit the regimes of the fluctuations around the long-term growth path in Ireland.

CONCLUSIONS

The aim of the paper was to identify the determinants of both: the Gross National Product (GNP) growth in Ireland in the long term and fluctuations around it in the short term using the Threshold Error Correction Model (TECM). This related to the empirical identification of the accelerated growth period in Ireland which pushed the country's economy from the periphery towards the core. The literature on the Irish economic success analysis is quite broad which allowed indicating its main factors such as: favourable demographic processes, well-educated labour force, English speaking society, historical links with the USA and Great Britain through the Irish diaspora in those countries, good conditions for the foreign investments location, locating new companies in highly technological sectors, accession to the European Union and EMU, and political, social and economic reforms that established a contemporary image of Ireland. Taking these into account we try to identify the periods and factors of economic growth in Ireland using empirical Threshold Error Correction Models (TECM). We used yearly data from the years 1980–2014 in both forms: levels and logs. As we showed, the time series are non-stationary and exhibit structural change in different moments in time. The structural changes were most often identified in the period of recession of 2008–2009 (eight cases), however, they were not observed in the GNP growth series. Further procedure was based on the Enders and Siklos test, but we modified it letting individual series serve as threshold variables. Such a construction allows to identify those variables which determined the short-term regimes along the same stable growth path in the long term. It is important to notice that threshold cointegration was confirmed in two cases using the original Enders-Siklos test. Moreover,

individual threshold variables confirmed the asymmetry of the short-term adjustment and the possibility that the threshold variables are not necessarily present in the long-term equation. The following threshold variables were identified such as: net income from the EU (*NI_EU*), GDP deflator (*deflator*), the first difference of ECM and of net exports (*d_N_Exp*). The best models are obtained with the following thresholds: GDP *deflator* (for levels) and net exports (for logs). These findings were fully validated by the Tsay test results.

The researchers often emphasize the important difference between GDP and GNP in Ireland due to the high ratio of foreign direct investment in GDP and transfers of revenues abroad (Barry et al., 2001). In the paper the period of intense economic growth in Ireland was of particular interest. Although the small number of observations was an important limitation for empirical research, we managed to indicate the most important thresholds and asymmetries in the GNP growth rate in Ireland in 1980–2014. Comparing the results to the analysis made for GDP (see Boehlke et al., 2017), it should be emphasized that the identified threshold variables were the same as concerns the variables observed in levels, i.e. net income from the EU and GDP deflator (deflator) and different for logs, i.e. lagged GDP and multifactor productivity. The best models are obtained with the following thresholds: net income from EU and lagged *HP_GDP*, respectively. The results for logarithmic variables give important information since the regimes' change is more evident in comparison to levels which are more vulnerable to the impact of outliers. In the case of the GDP it seems that multinational companies enforced the increase in the factors' productivity (MFP as the threshold) and the process once put into operation was self-exacting (*HP_GDP* as the threshold). In the case of the GNP the role of net exports is emphasized which results from the preceding FDI and economic institutions.

The estimated models allow to identify periods of intense growth, i.e. that period when fluctuations around the long-term path were adjusted from the positive side. These periods indicated the 1990s and the beginning of 2000s which is in line Irish economic policy of the past. Since putting the reforms into operation and getting more money from abroad (net income from the EU, FDI) in the 1980s, it took a decade to build the mechanism of growth. This type of growth was definitely of a mixed nature, exogenous funds from the EU and foreign investment were influenced by institutional changes and social reforms. In the decade of 1991–2002 the most spectacular economic growth took place, ranging at 8% year by year. In this period greater

economic force acted to push up the economy and to enable the catching-up process. In the last twelve years the growth slowed down and then the economic recession occurred. It was a hard time for the economy that suffered greatly from the recession. However, after 2010 Irish economy has started its recovery and managed to become successful.

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