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The transmission characteristics under the influence of the fifth-order nonlinearity management

XIUJUN HE[1, 2], KANG XIE[1]

[1] School of Optoelectronic Information, University of Electronics Science and Technology of China, Chengdu, 610054, China

[2] Chengdu University of Information Technology of China, Chengdu 610225, China

Starting with the nonlinear Schrödinger (NLS) equation, we have derived the evolution equations for the parameters of soliton pulse with propagation distance in optical fibers, taking into consideration the combined effect of second-order dispersion and the fifth-order nonlinearity by means of variation method. According to nonlinear evolution equations, the evolution of the pulse width with propagation distance is obtained under the influence of the different fifth-order nonlinearity. The results show that the pulse width fluctuates periodically under the influence of the different fifth-order nonlinearity. In the cycle, the negative fifth-order nonlinearity makes the pulse-width greater than the initial value while the positive fifth-order nonlinearity makes the pulse width less than the initial value. However, under the positive and negative fifth-order nonlinearity management, compared to the impact of positive or negative fifth-order nonlinearity only, the fluctuations of the solitons width are greatly reduced, even disappear. In other words, the width maintains almost steady. Therefore, it is possible that the pulse width is to be transmitted without any deformation.

Keywords: fifth-order nonlinearity management, soliton, nonlinear Schrödinger equation, variation method.

1. Introduction

The optical pulse compression and soliton transmission mode have been widely cosidered by researchers [1–3]. At present, under a third-order nonlinearity, the transmission of optical pulse has been intensively investigated, but with the development of high-nonlinear fiber (such as semiconductor doped fiber, organic polymer fiber, *etc.*), fifth-order or higher order nonlinearity can occur in the medium power of light [4]. Therefore, researches began to study the self-phase-modulation, the modulation instability [5], the solitary wave transmission, the bistable behavior [6], ground-state soliton, the propagation properties of the Gaussian pulse in the case of the fifth-order nonlinearity [7]. However, it is still a new field for research on transmission of higher order soliton. With the use of variation method and from the extended nonlinear Schrödinger (NLS) equation of the fifth-order nonlinearity, this paper introduces evolution equations of soliton parameters depending on the propagation distance under second-order dispersions and cubic-quintic nonlinearity, simulates the evolution of soliton width on the propagation distance. Nowadays, researchers make greater efforts to apply nonlinearity management to control the transmission of soliton, as with the dispersion-managed soliton propagation [8, 9]. The authors of the present paper focus on effects of the fifth-order nonlinearity management on the width of soliton and their achievements so far appear to be very helpful when investigating the transmission of soliton.

2. Theoretical analysis

Let us start with the nonlinear Schrödinger (NLS) equation with the fifth-order nonlinearity [5]:

$$i\frac{\partial A(z,t)}{\partial z} - \frac{\beta_2}{2}\frac{\partial^2 A(z,t)}{\partial T^2} + \gamma|A(z,t)|^2 A(z,t) + \gamma'|A(z,t)|^4 A(z,t) = 0 \tag{1}$$

where $i = (-1)^{1/2}$ and $A(z, T)$ is the slowly varying envelope of the optical pulse and T the temporal coordinate frame that moves at the group velocity v_g of the pulse, and z the spatial coordinate representing the transmission distance, β_2 represents the second-order dispersion parameter, γ and γ' are respectively the nonlinear Kerr effect coefficient and the fifth-order nonlinearity coefficient.

We proceed to the normalization of Eq. (1) in the following way:

$$i\frac{\partial u}{\partial \xi} + \frac{s}{2}\frac{\partial^2 u}{\partial \tau^2} + g_1|u|^2 u + g_2|u|^4 u = 0 \tag{2}$$

Here, $u = A/P_0^{1/2}$, $\xi = z/L_D$, $s = -\mathrm{sgn}(\beta_2)$, $\tau = T/T_0$, $g_1 = L_D/L_{\mathrm{NL1}}$, and $g_2 = L_D/L_{\mathrm{NL2}}$, while $L_D = T_0^2/|\beta_2|$, $L_{\mathrm{NL1}} = 1/(\gamma P_0)$ and $L_{\mathrm{NL2}} = 1/(\gamma' P_0)$.

Variational method was used to solve Eq. (2) and the Lagrangian density is obtained from Eq. (2),

$$L = -\frac{i}{2}\left(uu_\xi^* - u^* u_\xi\right) - \frac{s}{2}|u_\tau|^2 + \frac{g_1}{2}|u|^4 + \frac{g_2}{3}|u|^6 \tag{3}$$

The following generic ansatz for the stationary solution of Eq. (2) shall be considered

$$u = a\,\mathrm{sech}\left[r(t-q)\right]\exp\left[i\varphi + ic(t-q)^2 - i\omega(t-q)\right] \tag{4}$$

where a, r, q, φ, c and ω represent the pulse amplitude, width inversion, central position, the phase, chirp and frequency shift, which are all real functions of ξ

$$\delta\int\langle L\rangle \mathrm{d}\xi = 0 \tag{5}$$

$\langle L\rangle$ is the average Lagrange density function, that is,

$$\langle L\rangle = \int_{-\infty}^{+\infty} L\,\mathrm{d}\tau =$$

$$= \frac{a^2}{r^3}\left(-\frac{1}{6}\pi^2\frac{\mathrm{d}c}{\mathrm{d}\xi} - \frac{1}{3}\pi^2 c^2 - r^2\omega^2 - 2r^2\omega\frac{\mathrm{d}q}{\mathrm{d}\xi} - \frac{1}{3}r^4 + \right.$$

$$\left. + \frac{2}{3}g_1 r^2 a^2 + \frac{16}{45}g_2 r^2 a^4 - 2r^2\frac{\mathrm{d}\varphi}{\mathrm{d}\xi}\right) \tag{6}$$

From Equation (5), the parameter variation equation can be obtained

$$\frac{\mathrm{d}\langle L\rangle}{\mathrm{d}y_i} = 0 \tag{7}$$

where y_i represents parameters a, r, q, φ, c and ω. Parameter evolution equations can be derived by applying Eqs. (6) and (7)

$$\frac{\mathrm{d}a}{\mathrm{d}\xi} = -asc \tag{8}$$

$$\frac{\mathrm{d}r}{\mathrm{d}\xi} = -2rsc \tag{9}$$

$$\frac{\mathrm{d}q}{\mathrm{d}\xi} = -s\omega \tag{10}$$

$$\frac{\mathrm{d}\omega}{\mathrm{d}\xi} = 0 \tag{11}$$

$$\frac{\mathrm{d}c}{\mathrm{d}\xi} = -2sc^2 + \frac{1}{\pi^2}\left(2r^4 s - 2g_1 r^2 a^2 - \frac{32}{15}g_2 r^2 a^4\right) \tag{12}$$

$$\frac{\mathrm{d}\varphi}{\mathrm{d}\xi} = \frac{1}{3}r^2 s - \frac{5}{6}g_1 a^2 - \frac{32}{45}g_2 a^4 - \frac{1}{2}\omega^2 s \tag{13}$$

3. Computational results and analyses

3.1. Soliton propagation in different fifth-order nonlinearity situations

Using the above evolution equations of soliton parameters, the change of soliton width is as shown in Fig. 1. So, here $g_1 = 1$, $s = 1$ and initial values of solitons, $a_0 = 1$, $r_0 = 1$, $c_0 = 0$. When $g_2 < 0$, the pulse width changes in a periodic and fluctuating way and it is greater than the fluctuation of initial width, that is to say, when the initial pulse width starts to increase gradually, eventually increasing to the soliton width peak, it begins to gradually diminish, eventually reducing to the initial pulse width and starts to increase again. Then, these changes continue to repeat. The larger the $|g_2|$ is, the longer it lasts, and the more times it is repeated, the larger its fluctuation is. When $g_2 > 0$, the pulse width also changes in a periodic and fluctuating way and it is smaller than the fluctuation of initial width, that is to say, when the initial pulse width starts to decrease gradually, eventually decreasing to the lowest value of the width, it begins to gradually increase, eventually increasing to the initial pulse width and starts to decrease again. Then, these changes continue to repeat. The larger the $|g_2|$ is, the shorter it lasts, and the more times it is repeated, the larger its fluctuation is. As for the equivalent positive and negative fifth-order nonlinearity, the negative fifth--order nonlinearity of influence on the pulse width is much bigger than the positive fifth-order nonlinearity.

3.2. Soliton propagation under the fifth-order nonlinearity management

We know that the dispersion management lowers mean dispersion of the whole transmission line through configuring properly the fibers with opposite dispersion

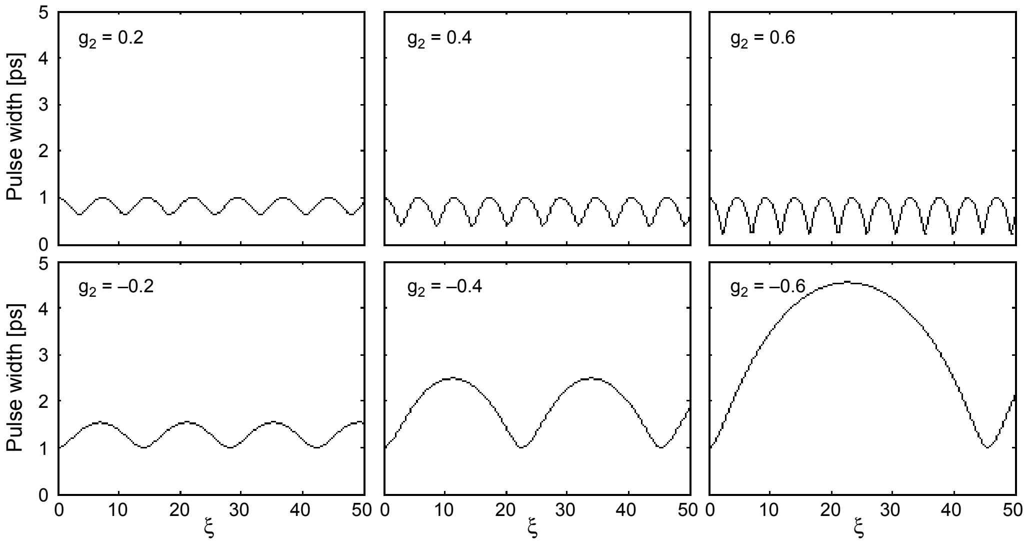

Fig. 1. The evolution of the pulse width with propagation distance under the influence of the different fifth-order nonlinearity.

properties in order to improve transmission performance of solitons. As to the dispersion management principles, nonlinear coefficient is also a function of ξ, which shows periodic change along the fiber length. In a cycle, one situation of nonlinear coefficient is that absolute values are identical but opposite in sign, the other situation is that absolute values ranges and the opposite is the sign, which is called nonlinearity management. As for the fifth-order nonlinearity management $g_2(\xi)$ should satisfy the following equation, that is,

$$g_2(\xi) = \begin{cases} g_+ & nL \le \xi < l_1 + nL \\ g_- & l_1 + nL \le \xi \le (n+1)L \end{cases}, \qquad n = 1, 2, ... \tag{14}$$

Here, $g_+ > 0$, $g_- > 0$, and $L = l_1 + l_2$ are the cycle lengths of the nonlinear management; l_1 and l_2 are the interaction lengths of the positive and negative nonlinearity. Obviously, the positive nonlinearity makes the pulse width decrease and the negative nonlinearity makes it increase. Therefore, using the positive and negative nonlinearity management, their functions can cancel out, which improves drastically the fluctuation of width.

At first, when $g_+ = g_-$, $l_1 = l_2$, the soliton transmission in the fifth-order nonlinearity management is shown in Figs. 2 and 3. Comparing to Fig. 1, the fluctuations of the soliton width reduce greatly after applying the fifth-order nonlinearity management. The negative fifth-order nonlinearity develops the soliton width above and

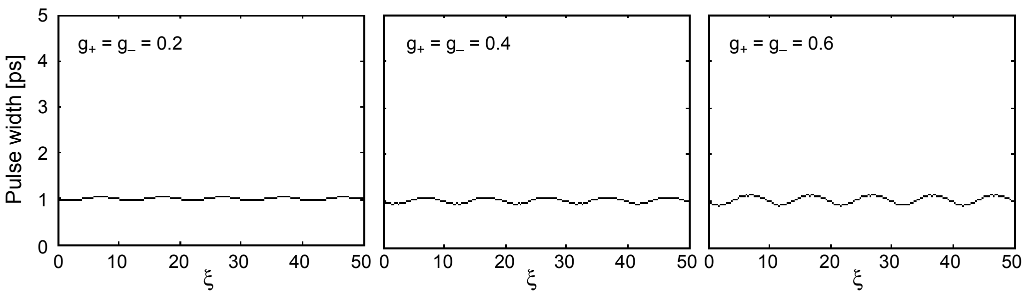

Fig. 2. The evolution of the pulse width with propagation distance under the influence of the fifth-order nonlinearity management ($L = l_1 + l_2 = L_D$).

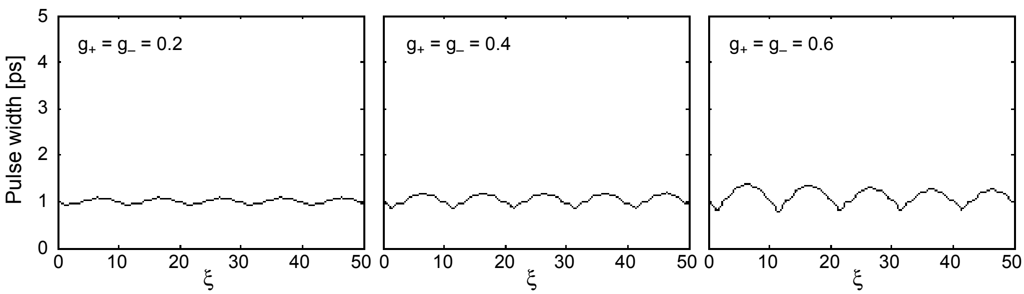

Fig. 3. The evolution of the pulse width with propagation distance under the influence of the fifth-order nonlinearity management.

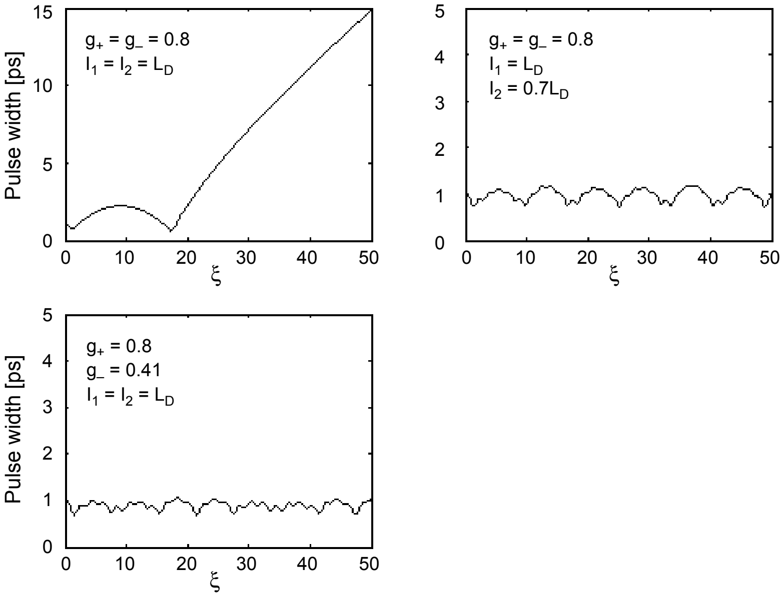

Fig. 4. The evolution of the pulse width with propagation distance under the influence of the fifth-order nonlinearity management.

beyond its initial value fluctuation, while the positive fifth-order nonlinearity develops the soliton width to be less than its initial value fluctuation and they can be cancelled, which decreases drastically the fluctuation. Better results are achieved by using a smaller cycle.

In Figure 4, due to dissymmetry of the function of positive and negative fifth-order nonlinearity, an effect of the fifth-order nonlinearity management is not good enough when the fifth-order nonlinear coefficient is bigger under the condition of $g_+ = g_-$ and $l_1 = l_2$. According to the function of equal negative fifth-order nonlinearity above and

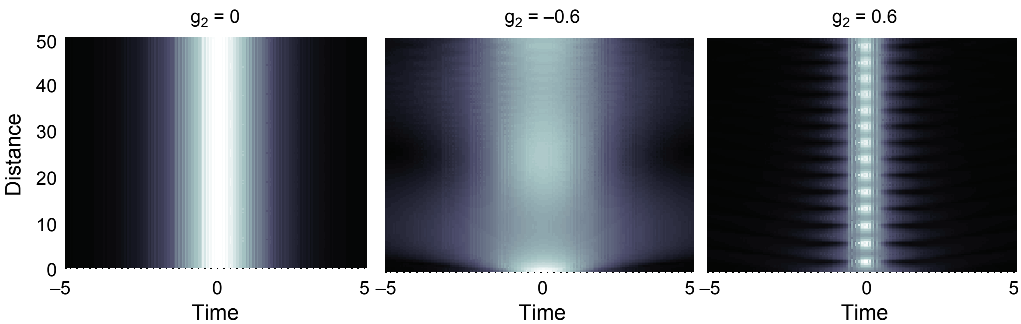

Fig. 5. The evolution of the soliton with propagation distance under the influence of the different fifth-order nonlinearity.

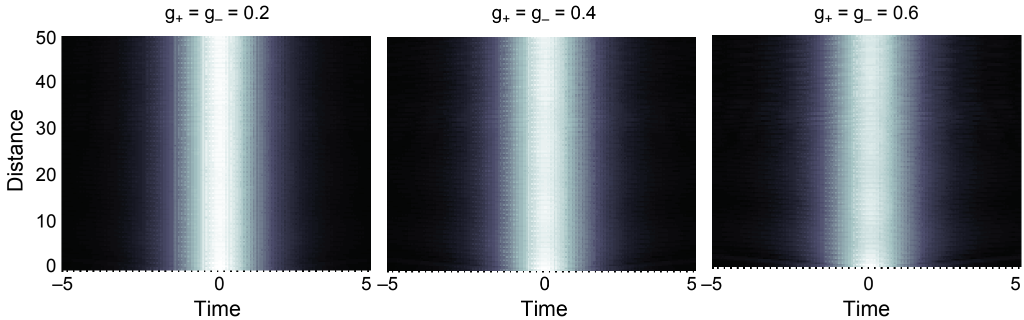

Fig. 6. The evolution of the soliton with propagation distance under the influence of the fifth-order nonlinearity management.

beyond the positive fifth-order nonlinearity and under the condition of $g_+ > g_-$ and $l_1 > l_2$, choosing a suitable ratio we can get a good performance through the fifth-order nonlinearity management.

Finally, the soliton transmission as a function of the fifth-order nonlinearity is calculated by applying numerical methods. The results are consistent with those of theoretical calculations, see in Figs. 5 and 6.

4. Conclusions

This paper applies the variation method to research effect of the fifth-order nonlinearity on the soliton transmission. The result shows that pulse width of soliton has a cyclical fluctuation under the influence of the fifth-order nonlinearity. The negative fifth-order nonlinearity develops the soliton width above and beyond its initial value fluctuation while the positive fifth-order nonlinearity develops the soliton width to be less than its initial value fluctuation in a cycle, and the bigger the absolute value of the nonlinear coefficient, the greater the fluctuation. On this basis, the influence of the fifth-order nonlinearity management is further researched to find out that the fluctuation of soliton width is greatly reduced under such conditions, and the soliton width is almost not changed, which can make pulses realize transmission of original shape under the function of the fifth-order nonlinearity.

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