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Gaussian beam diffraction in inhomogeneous media of cylindrical symmetry

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The method of paraxial complex geometrical optics (PCGO) is presented, which describes Gaussian beam (GB) diffraction in smoothly inhomogeneous media of cylindrical symmetry, including fibers. PCGO reduces the problem of Gaussian beam diffraction in inhomogeneous media to the system of the first order ordinary differential equations for the complex curvature of the wave front and for GB amplitude, which can be readily solved both analytically and numerically. As a result, PCGO radically simplifies the description of Gaussian beam diffraction in inhomogeneous media as compared to the numerical methods of wave optics. For the paraxial on-axis Gaussian beam propagation in inhomogeneous fibers, we compare PCGO solutions with numerical results for finite differences beam propagation method (FD-BPM). The PCGO method is shown to provide over 100-times higher rate of calculation than FD-BPM at comparable accuracy. This paper presents PCGO analytical solutions for width evolution of cylindrically symmetric GB in quadratic graded-index fiber, which is obtained in less complicated way comparing to the methods of wave optics. Besides, the influence of initial curvature of the wave front on GB evolution in graded-index fiber is discussed in this paper.

Keywords: Gaussian beam diffraction, paraxial complex geometrical optics, inhomogeneous media of cylindrical symmetry.

1. Introduction

Paraxial complex geometrical optics (PCGO) has two equivalent forms: the ray-based form, which deals with complex rays [1–6], that is with trajectories in a complex space, and the eikonal-based form, which uses complex eikonal instead of complex rays [6–10]. A surprising feature of PCGO is its ability to describe Gaussian beam (GB) diffraction in both ray-based and eikonal-based approaches.

This paper describes the advantages of the eikonal-based form of PCGO for description of Gaussian beam diffraction in inhomogeneous fibers. Section 2 presents the basic equations of PCGO for inhomogeneous medium of cylindrical symmetry. Analytical solution for beam width evolution in quadratic graded-index fiber is presented in Section 3. High efficiency of PCGO algorithms for numerical solutions of diffraction problems as compared with finite differences beam propagation method (FD-BPM) approach is demonstrated in Section 4 for multimode inhomogeneous fibers. Finally Section 5 outlines the ability of PCGO to describe the influence of initial curvature of the wave front on the beam width evolution in graded-index fiber.

2. Basic equations of paraxial complex geometrical optics for inhomogeneous medium of cylindrical symmetry

2.1. Riccati equation for complex parameter *B*

For an axially symmetric wave beam propagating along z direction in axially symmetric medium, PCGO suggests the following form of the solution

$$u(\eta, z) = A \exp(ik_0 \psi) = A(z) \exp\left[ik_0\left(n_0 z + \frac{B(z)\eta^2}{2}\right)\right] \tag{1}$$

where $k_0 = 2\pi/\lambda_0$, where λ_0 is the wavelength of the beam in vacuum and ψ is complex-valued eikonal, which in accordance with (1) has the form

$$\psi = n_0 z + \frac{B(z)\eta^2}{2} \tag{2}$$

where $\eta = \sqrt{x^2 + y^2}$ is the distance from the axis z. In above equation B is the complex curvature of the wave front (see [11]) and n_0 is the refractive index of the medium measured along z axis. The real and imaginary parts of the parameter $B = B_R + iB_I$ determine the real curvature κ of the wave front and the beam width w ($1/e^2$ point of the wave field) correspondingly:

$$B_R = n_0 \kappa, \qquad B_I = \frac{4}{k_0 w^2} \tag{3}$$

The eikonal equation:

$$(\nabla\psi)^2 = n^2 \tag{4}$$

in (η, z) coordinates takes the form:

$$\left(\frac{\partial\psi}{\partial\eta}\right)^2 + \left(\frac{\partial\psi}{\partial z}\right)^2 = n^2(z, \eta) \tag{5}$$

In accordance with the paraxial approximation, the radius η should be small enough. Therefore the parameter $n^2(z, \eta)$ in Eq. (5) can be expanded in Taylor series in η in the vicinity of symmetry axis z:

$$n^2(z, \eta) = n^2(\eta = 0) + \left(\frac{\partial n^2}{\partial \eta} \bigg|_{\eta = 0} \right) \eta + \left(\frac{\partial^2 n^2}{\partial \eta^2} \bigg|_{\eta = 0} \right) \frac{\eta^2}{2} \tag{6}$$

Substituting (2) and (6) into eikonal equation (5) and comparing coefficients of η^0, η and η^2, we obtain the following relations:

$$n^2(\eta = 0) = n_0^2, \qquad \frac{\partial n^2}{\partial \eta} \bigg|_{\eta = 0} = 0 \tag{7}$$

and the Riccati equation for complex curvature B:

$$n_0 \frac{\mathrm{d}B}{\mathrm{d}z} + B^2 = \beta \tag{8}$$

The parameter β for axially symmetric medium equals:

$$\beta = \frac{1}{2} \frac{\partial^2 n^2}{\partial \eta^2} \bigg|_{\eta = 0} \tag{9}$$

Substituting (3) into Eq. (1), we obtain the Gaussian beam of the form

$$u(\eta, z) = A(z) \exp\left(-\frac{2\eta^2}{w^2} \right) \exp\left[ik\left(z + \kappa \frac{\eta^2}{2} \right) \right] \tag{10}$$

where $k = 2\pi n_0/\lambda_0 = 2\pi/\lambda$ is the wave number for the beam propagating in the z direction. Solution (10) reflects the general feature of PCGO, which in fact deals with the Gaussian beams. The general form of the Gaussian beams in the 3D inhomogeneous media, as well as Riccati equation for the complex curvature parameter B, can be found in [10] and in the review paper [6].

2.2. The equation for GB complex amplitude

In the frame of paraxial approximation, the amplitude $A = A(z)$ is complex-valued and satisfies the transport equation

$$\mathrm{div}(A^2 \nabla \psi) = 0 \tag{11}$$

which for the axially symmetric beam in (η, z) coordinates takes the following form:

$$\frac{\mathrm{d}A^2}{\mathrm{d}z} \frac{\partial \psi}{\partial z} + \left[\frac{1}{\eta} \frac{\partial}{\partial \eta} \left(\eta \frac{\partial \psi}{\partial \eta} \right) + \frac{\partial^2 \psi}{\partial z^2} \right] A^2 = 0 \tag{12}$$

In accordance with Eq. (2), assuming that η is a small parameter, we obtain

$$\frac{\partial \psi}{\partial z} = n_0, \qquad \frac{1}{\eta}\frac{\partial}{\partial \eta}\left(\eta \frac{\partial \psi}{\partial \eta}\right) = 2B \tag{13}$$

As a result, Eq. (12) reduces to the ordinary differential equation in the form:

$$n_0 \frac{\mathrm{d}A}{\mathrm{d}z} + BA = 0 \tag{14}$$

The above equation for GB complex amplitude, as well as the Riccati equation for complex curvature B, are the basic PCGO equations. PCGO reduces the problem of GB diffraction to the domain of an ordinary differential equation. Having calculated the complex parameter B from Riccati equation (8), one can readily determine the complex amplitude A by integration of Eq. (14). As a result, the complex amplitude of cylindrically symmetric GB takes the form

$$A(z') = A_0 \exp\left(-\int B(z')\mathrm{d}z'\right) \tag{15}$$

where $A_0 = A(0)$ is the initial amplitude and $z' = z/n_0$.

2.3. The equation for GB width evolution

Riccati equation (8) is equivalent to the set of two equations for the real and imaginary parts of the complex curvature B:

$$\begin{cases} n_0 \dfrac{\mathrm{d}B_R}{\mathrm{d}z} + B_R^2 - B_I^2 = \beta \\ n_0 \dfrac{\mathrm{d}B_I}{\mathrm{d}z} + 2 B_R B_I = 0 \end{cases} \tag{16}$$

Substituting (3) into (16), one obtains

$$\frac{\mathrm{d}}{\mathrm{d}z}\left(\frac{1}{w^2}\right) = -\frac{2\kappa}{w^2} \tag{17}$$

This leads to the known relation between the beam width w and the wave front curvature κ, derived in [12]:

$$\kappa = \frac{1}{w}\frac{\mathrm{d}w}{\mathrm{d}z} \tag{18}$$

Substituting now relation (18) into the first equation of the system (16), we obtain the ordinary differential equation of the second order for GB width

$$\frac{d^2 w}{dz^2} - \gamma w = \frac{16}{k^2 w^3} \tag{19}$$

where

$$\gamma = \frac{1}{2n_0^2} \frac{\partial^2 n^2}{\partial \eta^2}\bigg|_{\eta = 0}$$

and

$$k^2 = k_0^2 n_0^2$$

The identical equation was obtained in the frame of diffraction approach, dealing with the truncated parabolic wave equation [13, 14].

3. Solution for GB diffraction in quadratic graded-index fibers

In this section, the PCGO method is applied for beam propagation in inhomogeneous medium of cylindrical symmetry with refractive index of the form

$$n = n_0 - \frac{\eta^2}{L^2} \tag{20}$$

where η is a distance from the fiber axis and L is the characteristic inhomogeneity scale of the medium. For the refractive index (20), the Riccati equation (8) takes the form

$$\frac{dB}{dz} + B^2 = -\frac{2n_0}{L^2} \tag{21}$$

The corresponding equation (19) for the beam width evolution in such a medium becomes

$$\frac{d^2 w}{dz^2} + \frac{w}{L_0^2} = \frac{16}{k^2 w^3} \tag{22}$$

where

$$L_0 = \sqrt{\frac{n_0}{2}}\, L$$

Introducing new variable $F = w/w_0$, Eq. (22) can be rewritten as

$$\frac{d^2F}{dz^2} + \frac{F}{L_0^2} - \frac{1}{L_D^2}\frac{1}{F^3} = 0 \tag{23}$$

where $L_D = k w_0^2/4$ is the diffraction (Rayleigh) distance and $w_0 = w(0)$ is the initial width. The integration of Eq. (23) with initial conditions: $F(0) = 1$ and $dF(0)/dz = 0$, what corresponds to the GB with a plane initial wave front, yields

$$\left(\frac{dF}{dz}\right)^2 + \frac{F^2}{L_0^2} + \frac{1}{L_D^2}\frac{1}{F^2} = \frac{1}{L_0^2} + \frac{1}{L_D^2} \tag{24}$$

Taking advantage of differential relation $\left[(F^2)'\right]^2 = 4F^2F'^2$ and differentiating once the Eq. (24), we obtain:

$$\frac{d^2F^2}{dz^2} + \frac{4F^2}{L_0^2} = 2\left(\frac{1}{L_0^2} + \frac{1}{L_D^2}\right) \tag{25}$$

which solution is

$$w = w_0\sqrt{1 + \left(\frac{L_0^2}{L_D^2} - 1\right)\sin^2\left(\frac{z}{L_0}\right)} \tag{26}$$

4. Numerical solutions for multimode inhomogeneous fibers: A comparison of PCGO and FD-BPM results

PCGO method deals with ordinary differential equations, which can be easily solved numerically. To show efficiency and accuracy of PCGO algorithm, we solve numerically Eq. (19) by the Runge–Kutta method for chosen inhomogeneous fibers.

In accordance with [15], the standard multimode (MMF) inhomogeneous fiber can be described by refractive index:

$$n(r) = \frac{n_1}{1-\Delta}\left[1 - \Delta\left(\frac{r}{a}\right)^g\right], \quad r \le a \tag{27a}$$

$$n = n_1, \quad r > a \tag{27b}$$

where a is the radius of the core. For numerical simulations we use the following parameters:

$$\Delta = 0.008, \quad n_1 = 1.6, \quad a = 31\ \mu\text{m}, \quad \lambda = 1\ \mu\text{m}, \quad w_0 = w(0) = 15\ \mu\text{m}, \quad g = 2 \tag{28}$$

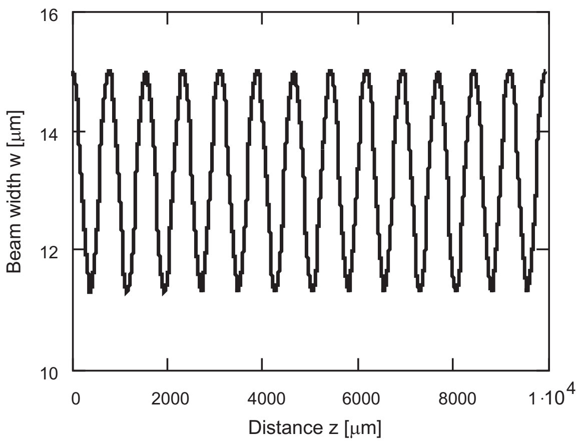

Fig. 1. Numerical solution of Eq. (19) for GB width in standard MMF.

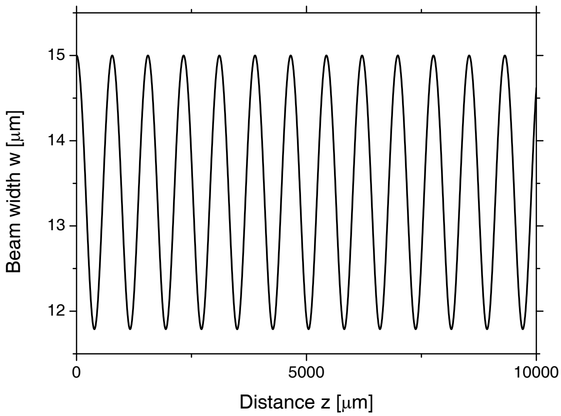

Fig. 2. FD-BPM numerical solution for GB width in standard MMF.

The solution for the GB width, obtained by PCGO algorithm for a standard MMF, is shown in Fig. 1.

Let us compare the PCGO results with the finite differences beam propagation method, based on a parabolic wave equation in the frame of the Crank–Nicolson scheme [16–19]. The FD-BPM numerical solution for GB with parameters (28) is shown in Fig. 2, where the discrepancy between PCGO and FD-BPM results is estimated according to the formula:

$$\delta = \left| \frac{w_{\mathrm{PCGO}} - w_{\mathrm{BPM}}}{w_{\mathrm{PCGO}}} \right| \tag{29}$$

It follows from Tab. 1 that the relative difference δ between PCGO and FD-BPM results never exceeds 4%. Besides, we have compared time of calculations, provided

Table 1. The results of calculations of the beam width by FD-BPM and PCGO methods for standard MMF.

Propagation distance z [μm]	Beam width w_{PCGO} [μm]	Beam width $w_{\mathrm{FD\text{-}BPM}}$ [μm]	$\delta\times100\%$
0	15	15	0
100	14.49	14.55	0.4
400	11.32	11.79	4
800	14.97	14.97	0
1000	12.84	13.11	2
2000	11.56	11.99	3.7
3000	14.44	14.5	0.4
4000	14.28	14.37	0.6
5000	11.46	11.91	3.9
6000	13.06	13.27	1.58
7000	14.98	14.99	0.009
8000	12.62	12.93	2.4
9000	11.68	12.08	3.34
10000	14.59	14.61	0.16

by the PCGO with that of FD-BPM method for a standard MMF. The lengths of steps under numerical calculations for the axes x, y, z were as follows:

$$\Delta x = 100 \text{ nm}, \quad \Delta y = 100 \text{ nm}, \quad \Delta z = 1 \text{ μm} \tag{30}$$

For the propagation distance of 1 cm, the time of numerical calculations for FD-BPM happened to be over 100 times higher as compared with PCGO method. Another cycle of calculations was performed for multimode gradient index polymer optical fiber (GI-POF).

In accordance with [20], the refractive index of multimode GI-POF can be modeled as:

$$n(r) = n_1\left[1 - 2\Delta\left(\frac{r}{a}\right)^g\right], \quad r \le a \tag{31a}$$

$$n = n_1, \quad r > a \tag{31b}$$

For numerical simulations we use the following parameters:

$$n_1 = 1.6, \quad a = 400 \text{ μm}, \quad \lambda = 0.8 \text{ μm}, \quad w_0 = w(0) = 30 \text{ μm}, \quad g = 2 \tag{32}$$

The parameter Δ is the same as in Eq. (28). The numerical solution, obtained by PCGO algorithm for the GB width in GI-POF for parameters (32) is shown in Fig. 3.

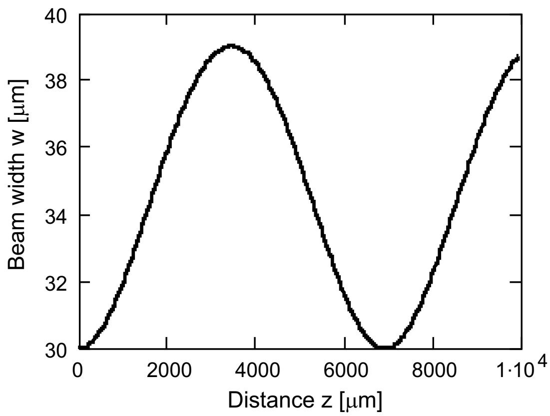

Fig. 3. Numerical solution of Eq. (19) for GB width in GI-POF.

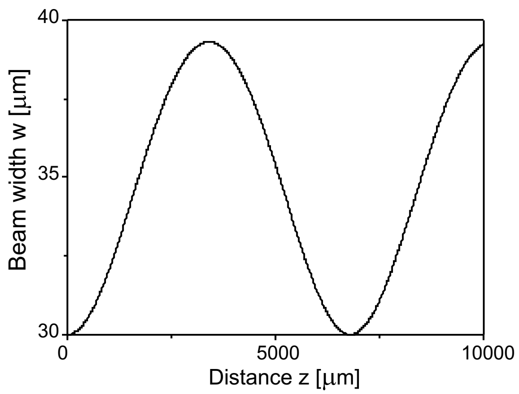

Fig. 4. FD-BPM numerical solution for GB width in GI-POF.

It follows from Tab. 2 that the relative difference δ between PCGO and FD-BPM results is also less than 4%. Additionally, we have compared time of calculations, provided by the PCGO, with that of FD-BPM method for GI-POF (see Fig. 4). For both methods, we have carried out calculations with numerical steps for the axes x, y, z equal to:

$$\Delta x = 250 \text{ nm}, \quad \Delta y = 250 \text{ nm}, \quad \Delta z = 1 \text{ μm} \tag{33}$$

For these calculations, the time of numerical calculations for FD-BPM also happened to be over 100 times higher, comparing to PCGO method for the propagation distance which equals only 1 cm.

Thus, PCGO method for both standard MMF and GI-POF has demonstrated 100-times higher rate of calculations, as compared with FD-BPM method at the accuracy over 96%. The same result as presented in Fig. 1 for MMF can be

T a b l e 2. The results of calculations by FD-BPM and PCGO methods for GI-POF.

Propagation distance z [μm]	Beam width w_{PCGO} [μm]	Beam width $w_{\mathrm{FD\text{-}BPM}}$ [μm]	$\delta\times 100\%$
0	30	30	0
100	30.02	30.03	0.03
400	30.32	30.36	1.11
800	31.25	31.38	0.43
1000	31.88	32.07	0.6
2000	35.75	36.21	1.28
3000	38.6	39.02	1.08
4000	38.64	38.66	0.05
5000	35.85	35.35	1.39
6000	31.97	31.35	1.92
7000	30	30.10	0.33
8000	31.8	32.90	3.47
9000	35.66	37	3.76
10000	38.56	39.23	1.73

obtained from the analytical solution (26), where $L_D = kw_0^2/4 = 327$ μm and $L_0 = a/2\sqrt{\Delta} = 247$ μm. Also, the result for GI-POF presented in Fig. 3 can be obtained from (26), where this time $L_D = 1719$ μm and $L_0 = 2236$ μm.

5. Influence of initial curvature of the wave front on beam width evolution in graded-index fiber

Integrating Eq. (23), we obtain the relation:

$$\frac{1}{2}\left(\frac{\mathrm{d}F}{\mathrm{d}z}\right)^2 + \frac{F^2}{2L_0^2} + \frac{1}{2F^2L_D^2} = C \tag{34}$$

We notice that the initial condition for the first derivative in Eq. (34), in accordance with (18), denotes the initial curvature of the wave front:

$$\left(\frac{\mathrm{d}F(0)}{\mathrm{d}z}\right)^2 = \frac{1}{w_0^2}\left(\frac{\mathrm{d}w(0)}{\mathrm{d}z}\right)^2 = \kappa_0^2 \tag{35}$$

In accordance with the initial condition $F(0) = 1$, the integration constant in Eq. (34) equals

$$C = \frac{1}{2}\kappa_0^2 + \frac{1}{2L_0^2} + \frac{1}{2L_D^2} \tag{36}$$

Substituting (36) into (34) and multiplying both sides by F^2, we obtain:

$$F^2\left(\frac{\mathrm{d}F}{\mathrm{d}z}\right)^2 + \frac{F^4}{L_0^2} + \frac{1}{L_D^2} = \left(\kappa_0^2 + \frac{1}{L_0^2} + \frac{1}{L_D^2}\right)F^2 \tag{37}$$

Taking advantage of the differential relation $\left[(F^2)'\right]^2 = 4F^2F'^2$ and differentiating once Eq. (37), we obtain:

$$\frac{\mathrm{d}^2F^2}{\mathrm{d}z^2} + \frac{4F^2}{L_0^2} = 2\left(\kappa_0^2 + \frac{1}{L_0^2} + \frac{1}{L_D^2}\right) \tag{38}$$

The solution of this equation has a form

$$w = w_0\sqrt{1 + \left(\kappa_0^2 L_0^2 + \frac{L_0^2}{L_D^2} - 1\right)\sin^2\left(\frac{z}{L_0}\right) + \kappa_0 L_0 \sin\left(\frac{2z}{L_0}\right)} \tag{39}$$

It depends on the parameters $\kappa_0 = \kappa(0)$, L_D and L_0. This solution can be also obtained from the truncated parabolic equation [14], but in more complicated way. Figure 5 shows evolution of the beam width in the case $L_0 = 1.5L_D$, when diffraction widening of GB dominates over the fiber focusing process. Figure 6 depicts the inverse situation $L_0 = 0.5L_D$, when focusing dominates, as compared with diffraction effects. For simplicity, the above solution is illustrated for the case when $n_0 = 1$.

According to Fig. 5, the width of the initially converging beam ($\kappa_0 < 0$) first decreases, approaching the minimum value $w_{\min} = 0.7w_0$. Then diffraction widening

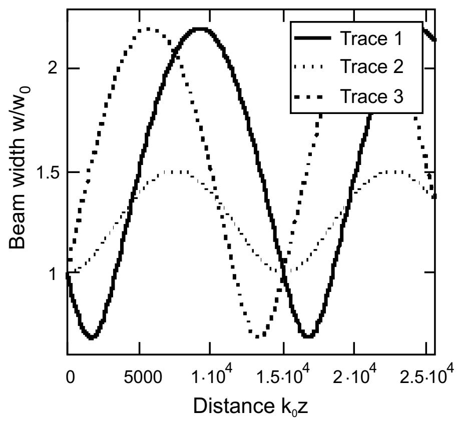

Fig. 5. GB width evolution for different values of initial wave front curvature in the case when $L_0 = 1.5L_D$ ($k_0L_D = 3198$, $w(0) = 9\lambda$). Trace 1 corresponds to the GB with initial curvature of the wave front equal to: $\kappa_{10}/k_0 = -3\times10^{-4}$, trace 2: $\kappa_{20}/k_0 = 0$, trace 3: $\kappa_{30}/k_0 = 3\times10^{-4}$.

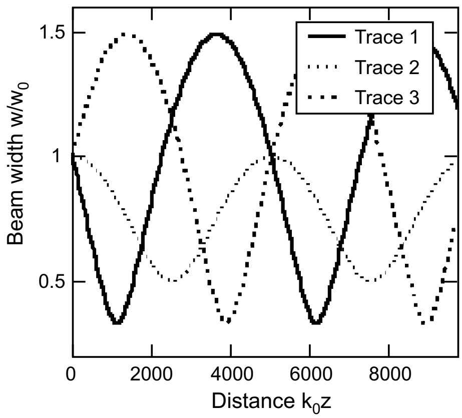

Fig. 6. GB width evolution for different values of initial phase front curvature in the case when $L_0 = 0.5 L_D$ ($k_0 L_D = 3198$, $w(0) = 9\lambda$). Trace 1 corresponds to the GB with initial curvature of the wave front equal to: $\kappa_{10}/k_0 = -6.5\times10^{-4}$, trace 2: $\kappa_{20}/k_0 = 0$, trace 3: $\kappa_{30}/k_0 = 6.5\times10^{-4}$.

dominates and the beam width increases to the maximum value $w_{max} = 2.25\, w_0$. For the initially diverging beam ($\kappa_0 > 0$), the GB width first increases to the maximum value $w_{max} = 2.25\, w_0$. Then focusing in the fiber dominates and, as a result, the beam width decreases to the minimum value, comparable with the case when $\kappa_0 < 0$.

Figure 6, related to the case $\kappa_0 < 0$, shows that the GB width first decreases, approaching the minimum value $w_{min} = 0.3\, w_0$. Then diffraction widening dominates and beam width increases to the maximum value $w_{max} = 1.5\, w_0$. For the GB with positive value of initial curvature of the wave front ($\kappa_0 > 0$), the beam width first increases to the maximum value $w_{max} = 1.5\, w_0$, next focusing in the fiber dominates. As a result, the beam width decreases to the minimum value $w_{min} = 0.3\, w_0$.

For both cases illustrated above, the maximum value of GB width for both initially diverging and converging beams happens to be 1.5 times higher, as compared with maximum values, answering to plane wave front $\kappa_0 = 0$.

6. Conclusions

The simple and effective method to calculate Gaussian beam wave field, diffracted in arbitrary smoothly inhomogeneous media, is presented. The method, based on paraxial complex geometrical optics, reduces the diffraction problem to the ordinary differential equations for complex curvature of the wave front and for GB amplitude. These equations can be readily solved both analytically and numerically by the Runge–Kutta method. In this paper we present PCGO analytical solution for width evolution of cylindrically symmetric GB in quadratic graded-index fiber. This solution is obtained in less complicated way, as compared with the methods of wave optics and quasi-optics. Besides, the influence of initial curvature of the wave front on GB evolution in graded-index fiber is discussed. For the paraxial on-axis Gaussian beam, propagating in standard MMF and GI-POF, PCGO solutions are compared in accuracy with FD-BPM approach. At the same time, the PCGO method is shown to provide 100-times higher rate of calculation than FD-BPM at comparable accuracy. Thereby, the paraxial complex geometrical optics greatly simplifies the description of Gaussian beam diffraction, as compared with the traditional numerical methods of the wave theory, reducing radically the time of numerical calculations.

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References

[1] KRAVTSOV YU.A., *Complex rays and complex caustics*, Radiophysics and Quantum Electronics **10**(9–10), 1967, pp. 719–730.

[2] KELLER J.B., STREIFER W., *Complex rays with application to Gaussian beams*, Journal of the Optical Society of America **61**(1), 1971, pp. 40–43.

[3] DESCHAMPS G.A., *Gaussian beam as a bundle of complex rays*, Electronics Letters **7**(23), 1971, pp. 684–685.

[4] KRAVTSOV YU.A., FORBES G.W., ASATRYAN A.A., *Theory and applications of complex rays*, [In] *Progress in Optics*, [Ed.] E. Wolf, Vol. 39, Elsevier, Amsterdam, 1999, pp. 3–62.

[5] CHAPMAN S.J., LAWRY J.M.H., OCKENDON J.R., TEW R.H., *On the theory of complex rays*, SIAM Review **41**(3), 1999, pp. 417–509.

[6] KRAVTSOV YU.A., BERCZYNSKI P., *Gaussian beams in inhomogeneous media: A review*, Studia Geophysica et Geodaetica **51**(1), 2007, pp. 1–36.

[7] KRAVTSOV YU.A., *Geometrical Optics in Engineering Physics*, Alpha Science International, UK, 2005.

[8] KRAVTSOV YU.A., BERCZYNSKI P., *Description of the 2D Gaussian beam diffraction in a free space in frame of eikonal-based complex geometric optics*, Wave Motion **40**(1), 2004, pp. 23–27.

[9] BERCZYNSKI P., KRAVTSOV YU.A., *Theory for Gaussian beam diffraction in 2D inhomogeneous medium, based on the eikonal form of complex geometrical optics*, Physics Letters A **331**(3–4), 2004, pp. 265–268.

[10] BERCZYNSKI P., BLIOKH K.YU., KRAVTSOV YU.A., STATECZNY A., *Diffraction of Gaussian beam in three-dimensional smoothly inhomogeneous medium: Eikonal-based complex geometrical optics approach*, Journal of the Optical Society of America A **23**(6), 2006, pp. 1442–1451.

[11] ARNAUD J.A., *Beams and Fiber Optics*, Academic Press, New York, 1976.

[12] KOGELNIK H., *On the propagation of Gaussian beams of light through lenslike media including those with a loss or gain variation*, Applied Optics **4**(12), 1965, pp. 1562–1569.

[13] BABICH V.M., BULDYREV V.S., *Asymptotic Methods in Problem of Diffraction of Short Waves*, Nauka, Moscow, 1972; (English translation: BABICH V.M., BULDYREV V.S., *Short-Wavelength Diffraction Theory: Asymptotic Methods*, Springer Verlag, Berlin, Heidelberg, 1991).

[14] PERMITIN G.V., SMIRNOV A.I., *Quasioptics of smoothly inhomogeneous isotropic media*, Journal of Experimental and Theoretical Physics (JETP) **82**(3), 1996, pp. 395–402.

[15] FEIT M.D., FLECK J.A., *Light propagation in graded-index optical fibers*, Applied Optics **17**(24), 1978, pp. 3990–3998.

[16] CHUNG Y., DAGLI N., *An assessment of finite difference beam propagation method*, IEEE Journal of Quantum Electronics **26**(8), 1990, pp. 1335–1339.

[17] HADLEY G.R., *Transparent boundary condition for beam propagation*, Optics Letters **16**(9), 1991, pp. 624–626.

[18] HADLEY G.R., *Transparent boundary condition for the beam propagation method*, IEEE Journal of Quantum Electronics **28**(1), 1992, pp. 363–370.
[19] YEVICK D., HERMANSSON B., *Efficient beam propagation techniques*, IEEE Journal of Quantum Electronics **26**(1), 1990, pp. 109–112.
[20] OHDOKO K., ISHIGURE T., KOIKE Y., *Propagating mode analysis and design of waveguide parameters of GI-POF for very short-reach network use*, IEEE Photonics Technology Letters **17**(1), 2005, pp. 79–81.

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